

MODERN BUSINESS MATHEMATICS

VAN TUYL

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MODERN BUSINESS MATHEMATICS

BY

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VAN TUYL'S BUS. MATH.

W. P. 8



MADE IN U.S.A.

PREFACE

THIS book contains sufficient material for two 30-hour courses in "Business Mathematics" for university classes.

The first half of the text, through foreign exchange, comprises the topics generally known as "Business Arithmetic." Non-essentials have been omitted; but the fundamental topics and operations so important to the successful accountant have been included. The short methods presented are those which have stood the test of experience and are really usable.

The second half of the text embraces the more important fundamental operations of the mathematics of finance. This part of the text will serve as an introduction to the study of actuarial mathematics.

But little real progress in the understanding and solution of problems of finance can be made without a knowledge of progressions and logarithms. These topics, therefore, are presented at the beginning of this part of the text.

In these days of large financial undertakings the successful man of affairs must understand the effect of compound interest as applied to long-time investments. Annuities, sinking funds, bond investments, etc. are all explained and illustrated.

The problems have been carefully selected with a view to presenting practical business propositions and illustrating the importance of the correct application of mathematical principles in their solution.

At the end of the book review problems are provided, covering the entire range of the text. These include representative problems from C. P. A. examinations of various states as well as from many of the American Institute of Accountants examinations.

Grateful acknowledgment to the American Institute of Accountants for permission to use their problems is here recorded. The interest tables (pages 284-303) have been taken from Rietz, Crathorne, and Rietz's "Mathematics of Finance," by courtesy of the publishers, Henry Holt and Company.

GEORGE H. VAN TUYL

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BUSINESS MATHEMATICS

TO THE STUDENT

The suggestions for shortening the work of calculation, found on the following pages, will be of use to the student only as he masters them and makes constant application of them in his everyday work.

The manufacturer who could reduce the expenses of operation 25% to 50% by the installation of new, up-to-date machinery and refused to do so would soon find himself driven from the field by his competitors. So the clerk or accountant who neglects the up-to-date methods of handling figures cannot hope to keep pace with his more enterprising neighbor.

Every short method presented has underlying it some well-established principle of mathematics. Without a knowledge of why a "short cut" works, we cannot, at all times, be sure of its use. The learner should, therefore, first discover the principle on which the operation is based, and then apply that principle, through drill, until it becomes second nature to make use of it.

Business mathematics is a science. Exact results may be obtained only by care and painstaking effort. All written work should be arranged in a systematic manner. In problem work the result arising from each separate step in the solution should be labeled so that its relation to other steps can be easily recalled.

The illustrative solutions and their explanation should not be neglected. They have been prepared with special reference to the needs of the learner. The solution of exercises and problems should not be attempted until the illustrations have been carefully studied. Many valuable and practical suggestions are contained in these illustrations, which will make the student's work easier and more profitable.

I. UNITED STATES MONEY

United States money consists of the coins, notes, and certificates authorized by Congress to be used as money. It is a decimal system.

The gold dollar, containing 25.8 grains of standard gold .9 fine, is the unit of value.

The silver dollar weighs 15.988 times as much as the gold dollar. That is, the ratio of gold to silver is 15.988 to 1 (commonly called 16 to 1).

DENOMINATIONS, WEIGHT, AND FINENESS OF THE COINS OF THE UNITED STATES

GOLD			
DENOMINATION	FINE GOLD CONTAINED (GRAINS)	ALLOY CONTAINED (GRAINS)	WEIGHT (GRAINS)
One dollar (not coined since 1890)	23.22	2.58	25.80
Quarter eagle (\$2.50)	58.05	6.45	64.5
Half eagle (\$5.00)	116.10	12.90	129.
Eagle (\$10.00)	232.20	25.80	258.
Double eagle (\$20.00)	464.40	51.60	516.
SILVER			
	PURE SILVER CONTAINED		
One dollar	371.25	41.25	412.5
Half dollar	173.61	19.29	192.9
Quarter dollar	86.805	9.645	96.45
Dime	34.722	3.858	38.58
MINOR			
	PURE COPPER CONTAINED		
Five cents * (nickel)	57.87	19.29	77.16
One cent † (copper)	45.60	2.40	48.

*75% copper and 25% nickel.

†95% copper and 5% tin and zinc.

II. ALIQUOT PARTS

An **aliquot part** of a number is an exact divisor of the number. 50, $33\frac{1}{3}$, 25, $16\frac{2}{3}$, and $12\frac{1}{2}$ are aliquot parts of 100.

Other divisors of a number are **aliquant parts** of the number. $62\frac{1}{2}$, 40, and $37\frac{1}{2}$ are aliquant parts of 100.

An aliquot part of a number, expressed as a fraction in its lowest terms, always has 1 for a numerator.

An aliquant part of a number, expressed as a fraction in its lowest terms, has some number other than 1 for a numerator.

Calculation by aliquot parts is much easier and more rapid than by aliquant parts.

The following table of halves, quarters, eighths, and sixteenths is designed to aid in transforming aliquant into aliquot parts.

TABLE OF HALVES, QUARTERS, EIGHTHS, AND SIXTEENTHS

	HALVES	QUARTERS	EIGHTHS	SIXTEENTHS
1				.06 $\frac{1}{4}$
2			.12 $\frac{1}{2}$	
3				.18 $\frac{3}{4}$
4		.25		
5				.31 $\frac{1}{4}$
6			.37 $\frac{1}{2}$	
7				.43 $\frac{3}{4}$
8	.50			
9				.56 $\frac{1}{4}$
10			.62 $\frac{1}{2}$	
11				.68 $\frac{3}{4}$
12		.75		
13				.81 $\frac{1}{4}$
14			.87 $\frac{1}{2}$	
15				.93 $\frac{3}{4}$
16	1.00	1.00	1.00	1.00

A careful study of the table on the preceding page will aid in memorizing the fractional parts, and will reveal the relation of any given value in the table to any other value in the table.

The numbers at the left are the numerators of fractions whose denominator is 16. To illustrate: $.37\frac{1}{2}$ is on line 6, hence $.37\frac{1}{2}$ is $\frac{6}{16}$, or $\frac{3}{8}$, of 1.00; $.68\frac{3}{4}$ is on line 11, hence $.68\frac{3}{4}$ is $\frac{11}{16}$ of 1.00; $.93\frac{3}{4}$ is on line 15, hence $.93\frac{3}{4}$ is $\frac{15}{16}$ of 1.00, etc.

The ratio, or relation, of one value to another, in the table, is the ratio of the numerators at the left. To illustrate: The ratio of $.43\frac{3}{4}$ to .50 is the same as the ratio of 7 to 8, or $\frac{7}{8}$; the ratio of $.68\frac{3}{4}$ to .50 is the same as the ratio of 11 to 8, or $\frac{11}{8}$; the ratio of $.18\frac{3}{4}$ to .75 is the ratio of 3 to 12, or $\frac{1}{4}$.

Calculations involving multiplication by any of the sixteenths are rendered easier and quicker by observing and applying the principles illustrated in the following exercises:

ILLUSTRATIONS

1. Find the value of 1734 yd. of cloth at $43\frac{3}{4}\text{¢}$

$$a. \begin{array}{r|l} \$1734 & = \text{value at } \$1. \end{array}$$

$$b. \begin{array}{r|l} 867 & = \text{value at } \$.50 \end{array}$$

$$c. \begin{array}{r|l} 10837 & = \text{value at } .06\frac{1}{4} \left(\frac{1}{8} \text{ of } \$.50 \right) \end{array}$$

$$d. \begin{array}{r|l} \$ 75863 & = \text{value at } \$.43\frac{3}{4} \text{ (Difference between } b \text{ and } c) \end{array}$$

2. Find the value of $2472\frac{1}{2}$ bu. of corn at $81\frac{1}{4}\text{¢}$.

$$a. \begin{array}{r|l} \$247250 & = \text{value at } \$1. \end{array}$$

$$b. \begin{array}{r|l} 123625 & = \text{value at } \$.50 \end{array}$$

$$c. \begin{array}{r|l} 618125 & = \text{value at } .25 \left(\frac{1}{2} \text{ of } \$.50 \right) \end{array}$$

$$d. \begin{array}{r|l} 154531 & = \text{value at } .06\frac{1}{4} \left(\frac{1}{4} \text{ of } \$.25 \right) \end{array}$$

$$e. \begin{array}{r|l} \$200891 & = \text{value at } \$.81\frac{1}{4} \text{ (Sum of } b, c, \text{ and } d) \end{array}$$

3. Find the value of $3476\frac{3}{4}$ bu. of wheat at $\$1.31\frac{1}{4}$.

$$a. \begin{array}{r|l} \$347675 & = \text{value at } \$1. \end{array}$$

$$b. \begin{array}{r|l} 8691875 & = \text{value at } .25 \end{array}$$

$$c. \begin{array}{r|l} 2172968 & = \text{value at } .06\frac{1}{4} \end{array}$$

$$d. \begin{array}{r|l} \$456323 & = \text{value at } \$1.31\frac{1}{4} \text{ (Sum of } a, b, \text{ and } c) \end{array}$$

In the preceding illustrations note that use has been made of *aliquot* parts only. In the second illustration, for instance, the \$.50 is *one* half of \$1; the \$.25 is *one* half of the \$.50; and the \$.06 $\frac{1}{4}$ is *one* fourth of the \$.25. One should analyze the price and separate it into its component parts, each of which is an aliquot part either of unity or of another preceding part of the price.

In the following exercises make extensions mentally, as far as possible, and find the total value of each group:

1.

16 yd. @ 25¢
 24 yd. @ 25¢
 30 yd. @ 50¢
 16 yd. @ 75¢
 40 yd. @ 50¢

2.

32 yd. @ 50¢
 48 yd. @ 25¢
 54 yd. @ 50¢
 60 yd. @ 75¢
 80 yd. @ 75¢

3.

120 yd. @ 25¢
 140 yd. @ 50¢
 150 yd. @ 25¢
 160 yd. @ 75¢
 126 yd. @ 25¢

4.

13 yd. @ 25¢
 21 yd. @ 50¢
 39 yd. @ 25¢
 31 yd. @ 75¢
 48 yd. @ 75¢

5.

3486 lb. @ 25¢
 1984 lb. @ 50¢
 4821 lb. @ 25¢
 8448 lb. @ 75¢
 1331 lb. @ 50¢

6.

128 lb. @ 75¢
 324 lb. @ 50¢
 233 lb. @ 25¢
 199 lb. @ 75¢
 247 lb. @ 25¢

7.

24 lb. @ 37 $\frac{1}{2}$ ¢
 32 lb. @ 12 $\frac{1}{2}$ ¢
 40 lb. @ 62 $\frac{1}{2}$ ¢
 56 lb. @ 37 $\frac{1}{2}$ ¢
 84 lb. @ 37 $\frac{1}{2}$ ¢

8.

48 lb. @ 12 $\frac{1}{2}$ ¢
 64 lb. @ 25 ¢
 72 lb. @ 37 $\frac{1}{2}$ ¢
 88 lb. @ 87 $\frac{1}{2}$ ¢
 96 lb. @ 62 $\frac{1}{2}$ ¢

9.

120 yd. @ 12 $\frac{1}{2}$ ¢
 164 yd. @ 12 $\frac{1}{2}$ ¢
 200 yd. @ 37 $\frac{1}{2}$ ¢
 240 yd. @ 12 $\frac{1}{2}$ ¢
 288 yd. @ 62 $\frac{1}{2}$ ¢

10.

32 yd. @ 6 $\frac{1}{4}$ ¢
 48 yd. @ 12 $\frac{1}{2}$ ¢
 64 yd. @ 18 $\frac{3}{4}$ ¢
 96 yd. @ 6 $\frac{1}{4}$ ¢
 160 yd. @ 43 $\frac{3}{4}$ ¢

11.

80 yd. @ 31 $\frac{1}{4}$ ¢
 128 yd. @ 37 $\frac{1}{2}$ ¢
 144 yd. @ 56 $\frac{1}{4}$ ¢
 96 yd. @ 93 $\frac{3}{4}$ ¢
 72 yd. @ 37 $\frac{1}{2}$ ¢

12.

47 yd. @ 25 ¢
 53 yd. @ 12 $\frac{1}{2}$ ¢
 58 yd. @ 6 $\frac{1}{4}$ ¢
 63 yd. @ 50 ¢
 75 yd @ 37 $\frac{1}{2}$ ¢

Find the total value of each group, as on page 10:

13.

576 yd. @ $37\frac{1}{2}\text{¢}$ 865 yd. @ $18\frac{3}{4}\text{¢}$ 973 yd. @ $31\frac{1}{4}\text{¢}$ 597 yd. @ $43\frac{3}{4}\text{¢}$ 785 yd. @ $56\frac{1}{4}\text{¢}$

14.

967 yd. @ $68\frac{3}{4}\text{¢}$ 749 yd. @ $81\frac{1}{4}\text{¢}$ 638 yd. @ $56\frac{1}{4}\text{¢}$ 497 yd. @ $43\frac{3}{4}\text{¢}$ 978 yd. @ $18\frac{3}{4}\text{¢}$

15.

645 yd. @ $93\frac{3}{4}\text{¢}$ 385 yd. @ $31\frac{1}{4}\text{¢}$ 756 yd. @ $68\frac{3}{4}\text{¢}$ 877 yd. @ $43\frac{3}{4}\text{¢}$ 567 yd. @ $18\frac{3}{4}\text{¢}$

16.

384 $\frac{3}{4}$ yd. @ $37\frac{1}{2}\text{¢}$ 564 $\frac{1}{2}$ yd. @ $31\frac{1}{4}\text{¢}$ 768 $\frac{1}{4}$ yd. @ $43\frac{3}{4}\text{¢}$ 543 $\frac{1}{2}$ yd. @ $18\frac{3}{4}\text{¢}$ 589 $\frac{3}{4}$ yd. @ $43\frac{3}{4}\text{¢}$

17.

645 $\frac{1}{2}$ yd. @ 25 ¢853 yd. @ $12\frac{1}{2}\text{¢}$ 649 $\frac{1}{2}$ yd. @ $12\frac{1}{2}\text{¢}$ 756 $\frac{3}{4}$ yd. @ $62\frac{1}{2}\text{¢}$ 957 $\frac{1}{4}$ yd. @ $56\frac{1}{4}\text{¢}$

18.

134 $\frac{1}{2}$ yd. @ $81\frac{1}{4}\text{¢}$ 423 $\frac{3}{4}$ yd. @ $56\frac{1}{4}\text{¢}$ 235 $\frac{1}{4}$ yd. @ 50 ¢337 $\frac{1}{4}$ yd. @ 75 ¢456 $\frac{1}{2}$ yd. @ $87\frac{1}{2}\text{¢}$

The following table is to be used in the same manner as the table on page 9 in transforming aliquant parts into aliquot parts.

TABLE OF HALVES, THIRDS, QUARTERS, SIXTHS, AND TWELFTHS

	HALVES	THIRDS	QUARTERS	SIXTHS	TWELFTHS
1					$.08\frac{1}{3}$
2				$.16\frac{2}{3}$	
3			.25		
4		$.33\frac{1}{3}$			
5					$.41\frac{2}{3}$
6	.50				
7					$.58\frac{1}{3}$
8		$.66\frac{2}{3}$			
9			.75		
10				$.83\frac{1}{3}$	
11					$.91\frac{2}{3}$
12	1.00	1.00	1.00	1.00	1.00

Study the table on the preceding page until each part is readily recognized as a fractional part of unity and as a fractional part of any other value in the table. To illustrate: $.33\frac{1}{3}$ is not only $\frac{1}{3}$ of 1.00, but it is also $\frac{2}{3}$ of .50; $\frac{4}{9}$ of .75; $\frac{4}{5}$ of $.41\frac{2}{3}$, etc. Also $.41\frac{2}{3}$ is not only $\frac{5}{12}$ of 1.00, but it is $\frac{5}{6}$ of .50; $\frac{1}{2}$ of $.83\frac{1}{3}$; $\frac{5}{4}$ of $.33\frac{1}{3}$, etc. These ratios and others of a similar nature are important for skillful work in computation.

In the following exercises make extensions mentally, as far as possible, and find the total value of each group:

1.

48 lb. @ $16\frac{2}{3}\text{¢}$
 54 lb. @ $66\frac{2}{3}\text{¢}$
 30 lb. @ $83\frac{1}{3}\text{¢}$
 54 lb. @ $33\frac{1}{3}\text{¢}$
 72 lb. @ $66\frac{2}{3}\text{¢}$

2.

66 yd. @ $33\frac{1}{3}\text{¢}$
 76 yd. @ 25 ¢
 76 yd. @ $12\frac{1}{2}\text{¢}$
 87 yd. @ $16\frac{2}{3}\text{¢}$
 93 yd. @ $66\frac{2}{3}\text{¢}$

3.

124 yd. @ $33\frac{1}{3}\text{¢}$
 342 yd. @ $66\frac{2}{3}\text{¢}$
 274 yd. @ $16\frac{2}{3}\text{¢}$
 195 yd. @ $83\frac{1}{3}\text{¢}$
 432 yd. @ $66\frac{2}{3}\text{¢}$

4.

64 lb. @ $6\frac{1}{4}\text{¢}$
 96 lb. @ $33\frac{1}{3}\text{¢}$
 88 lb. @ $37\frac{1}{2}\text{¢}$
 91 lb. @ $66\frac{2}{3}\text{¢}$
 98 lb. @ $16\frac{2}{3}\text{¢}$

5.

48 qt. @ $33\frac{1}{3}\text{¢}$
 72 qt. @ $16\frac{2}{3}\text{¢}$
 96 qt. @ $12\frac{1}{2}\text{¢}$
 60 qt. @ $6\frac{1}{4}\text{¢}$
 81 qt. @ $16\frac{2}{3}\text{¢}$

6.

120 yd. @ $37\frac{1}{2}\text{¢}$
 126 yd. @ $16\frac{2}{3}\text{¢}$
 132 yd. @ $83\frac{1}{3}\text{¢}$
 144 yd. @ $33\frac{1}{3}\text{¢}$
 324 yd. @ $83\frac{1}{3}\text{¢}$

7.

36 yd. @ $8\frac{1}{3}\text{¢}$
 48 yd. @ $16\frac{2}{3}\text{¢}$
 24 yd. @ $41\frac{2}{3}\text{¢}$
 60 yd. @ $58\frac{1}{3}\text{¢}$
 72 yd. @ $83\frac{1}{3}\text{¢}$

8.

66 yd. @ $33\frac{1}{3}\text{¢}$
 84 yd. @ $41\frac{2}{3}\text{¢}$
 120 yd. @ $91\frac{2}{3}\text{¢}$
 42 yd. @ $58\frac{1}{3}\text{¢}$
 54 yd. @ $16\frac{2}{3}\text{¢}$

9.

75 lb. @ $8\frac{1}{3}\text{¢}$
 88 lb. @ $16\frac{2}{3}\text{¢}$
 90 lb. @ $41\frac{2}{3}\text{¢}$
 96 lb. @ $58\frac{1}{3}\text{¢}$
 84 lb. @ $66\frac{2}{3}\text{¢}$

10.

24 yd. @ $58\frac{1}{3}\text{¢}$
 36 yd. @ $41\frac{2}{3}\text{¢}$
 60 yd. @ $91\frac{2}{3}\text{¢}$
 48 yd. @ $16\frac{2}{3}\text{¢}$
 72 yd. @ $33\frac{1}{3}\text{¢}$

11.

140 lb. @ $33\frac{1}{3}\text{¢}$
 160 lb. @ $91\frac{2}{3}\text{¢}$
 240 lb. @ $58\frac{1}{3}\text{¢}$
 360 lb. @ $41\frac{2}{3}\text{¢}$
 118 lb. @ $8\frac{1}{3}\text{¢}$

12.

172 lb. @ $41\frac{2}{3}\text{¢}$
 196 lb. @ $8\frac{1}{3}\text{¢}$
 234 lb. @ $16\frac{2}{3}\text{¢}$
 540 lb. @ $58\frac{1}{3}\text{¢}$
 660 lb. @ $91\frac{2}{3}\text{¢}$

In these exercises follow the illustrations given on page 10 and find the total value of each group:

13.

746 yd. @ $83\frac{1}{3}\text{¢}$
 465 yd. @ $41\frac{2}{3}\text{¢}$
 746 yd. @ $58\frac{1}{3}\text{¢}$
 867 yd. @ $91\frac{2}{3}\text{¢}$
 479 yd. @ $41\frac{2}{3}\text{¢}$

14.

769 yd. @ $66\frac{2}{3}\text{¢}$
 976 yd. @ $8\frac{1}{3}\text{¢}$
 645 yd. @ $58\frac{1}{3}\text{¢}$
 843 yd. @ $41\frac{2}{3}\text{¢}$
 768 yd. @ $91\frac{2}{3}\text{¢}$

15.

357 yd. @ $83\frac{1}{3}\text{¢}$
 579 yd. @ $8\frac{1}{3}\text{¢}$
 757 yd. @ $58\frac{1}{3}\text{¢}$
 646 yd. @ $66\frac{2}{3}\text{¢}$
 487 yd. @ $41\frac{2}{3}\text{¢}$

16.

$527\frac{1}{2}$ yd. @ $33\frac{1}{3}\text{¢}$
 $647\frac{3}{4}$ yd. @ $16\frac{2}{3}\text{¢}$
 $756\frac{1}{4}$ yd. @ $33\frac{1}{3}\text{¢}$
 $587\frac{1}{2}$ yd. @ $66\frac{2}{3}\text{¢}$
 $648\frac{3}{4}$ yd. @ $41\frac{2}{3}\text{¢}$

17.

$362\frac{1}{2}$ yd. @ $58\frac{1}{3}\text{¢}$
 $658\frac{1}{2}$ yd. @ $41\frac{2}{3}\text{¢}$
 $856\frac{1}{4}$ yd. @ $91\frac{2}{3}\text{¢}$
 $347\frac{3}{4}$ yd. @ $58\frac{1}{3}\text{¢}$
 $423\frac{1}{2}$ yd. @ $8\frac{1}{3}\text{¢}$

18.

$634\frac{1}{2}$ yd. @ $16\frac{2}{3}\text{¢}$
 $587\frac{1}{4}$ yd. @ $83\frac{1}{3}\text{¢}$
 $536\frac{1}{2}$ yd. @ $8\frac{1}{3}\text{¢}$
 $589\frac{3}{4}$ yd. @ $58\frac{1}{3}\text{¢}$
 $341\frac{1}{2}$ yd. @ $43\frac{3}{4}\text{¢}$

ALIUOT PARTS APPLIED TO INTEGERS

The aliquot part principle applies to integers as well as to decimal values.

Since the Arabic system of numbers is a decimal system, multiplications of a given number by $.01\frac{1}{4}$, $.12\frac{1}{2}$, $1\frac{1}{4}$, $12\frac{1}{2}$, 125, 1250, etc. are identical except for the position of the decimal point in the product. From the standpoint of aliquot parts, each of the foregoing suggested multipliers is an eighth.

ILLUSTRATION

Multiply $93\frac{1}{4}$ by $.01\frac{1}{4}$; by $.12\frac{1}{2}$; by $1\frac{1}{4}$; by $12\frac{1}{2}$; by 125, and by 1250.

$\frac{1}{8}$ of 9.325	= 1.165625	($.01\frac{1}{4}$ = $\frac{1}{8}$ of .1)
$\frac{1}{8}$ of 93.25	= 11.65625	($.12\frac{1}{2}$ = $\frac{1}{8}$ of 1.)
$\frac{1}{8}$ of 932.5	= 116.5625	($1\frac{1}{4}$ = $\frac{1}{8}$ of 10)
$\frac{1}{8}$ of 9325	= 1165.625	($12\frac{1}{2}$ = $\frac{1}{8}$ of 100)
$\frac{1}{8}$ of 93,250	= 11656.25	(125 = $\frac{1}{8}$ of 1000)
$\frac{1}{8}$ of 932,500	= 116562.5	(1250 = $\frac{1}{8}$ of 10,000)

The table on the following page shows a limited number of practical ratios like the foregoing:

TABLE SHOWING RATIO OF DECIMAL AND INTEGRAL ALIQUOT PARTS

	.1	1.	10.	100.	1000.	10,000.
$\frac{1}{16}$	.006 $\frac{1}{4}$	.06 $\frac{1}{4}$	.62 $\frac{1}{2}$	6 $\frac{1}{4}$	62 $\frac{1}{2}$	625
$\frac{3}{16}$	.018 $\frac{3}{4}$	.18 $\frac{3}{4}$	1 $\frac{7}{8}$	18 $\frac{3}{4}$	187 $\frac{1}{2}$	1875
$\frac{5}{16}$	*.03 $\frac{1}{8}$	.31 $\frac{1}{4}$	3 $\frac{1}{8}$	31 $\frac{1}{4}$	312 $\frac{1}{2}$	3125
$\frac{1}{12}$	.008 $\frac{1}{3}$	.08 $\frac{1}{3}$	.83 $\frac{1}{3}$	8 $\frac{1}{3}$	83 $\frac{1}{3}$	833 $\frac{1}{3}$
$\frac{5}{12}$	** .04 $\frac{1}{6}$	.41 $\frac{2}{3}$	4 $\frac{1}{6}$	41 $\frac{2}{3}$	416 $\frac{2}{3}$	4166 $\frac{2}{3}$
$\frac{1}{8}$	.01 $\frac{1}{4}$	.12 $\frac{1}{2}$	1 $\frac{1}{4}$	12 $\frac{1}{2}$	125	1250
$\frac{3}{8}$	.03 $\frac{3}{4}$	.37 $\frac{1}{2}$	3 $\frac{3}{4}$	37 $\frac{1}{2}$	375	3750
$\frac{1}{6}$	.01 $\frac{2}{3}$	.16 $\frac{2}{3}$	1 $\frac{2}{3}$	16 $\frac{2}{3}$	166 $\frac{2}{3}$	1666 $\frac{2}{3}$
$\frac{1}{4}$	.02 $\frac{1}{2}$	.25	2 $\frac{1}{2}$	25	250	2500
$\frac{1}{3}$	.03 $\frac{1}{3}$	.33 $\frac{1}{3}$	3 $\frac{1}{3}$	33 $\frac{1}{3}$	333 $\frac{1}{3}$	3333 $\frac{1}{3}$
$\frac{1}{2}$	.05	.50	5.	50	500	5000

* .03 $\frac{1}{8}$ is also $\frac{1}{32}$ of 1.

** .04 $\frac{1}{6}$ is also $\frac{1}{24}$ of 1.

ILLUSTRATIONS

1. Multiply 846 by 416 $\frac{2}{3}$.

$$a. \quad 846,000 = 1000 \times 846$$

$$b. \quad 423,000 = 500 \times 846 \left(\frac{1}{2} \text{ of } 1000 \times 846 \right)$$

$$c. \quad 70,500 = 83\frac{1}{3} \times 846 \left(\frac{1}{6} \text{ of } 500 \times 846 \right)$$

$$d. \quad 352,500 = 416\frac{2}{3} \times 846 \text{ (Difference between } b \text{ and } c)$$

2. Multiply 32 $\frac{1}{4}$ by 1666 $\frac{2}{3}$.

$$a. \quad 322,500 = 10,000 \times 32\frac{1}{4}$$

$$b. \quad 53,750 = 1666\frac{2}{3} \times 32\frac{1}{4} \left(\frac{1}{6} \text{ of } 10,000 \times 32\frac{1}{4} \right)$$

3. Multiply 56 $\frac{1}{8}$ by 31 $\frac{1}{4}$.

$$a. \quad 5612.5 = 100 \times 56\frac{1}{8}$$

$$b. \quad 1403.125 = 25 \times 56\frac{1}{8} \left(\frac{1}{4} \text{ of } 100 \times 56\frac{1}{8} \right)$$

$$c. \quad 350.78125 = 6\frac{1}{4} \times 56\frac{1}{8} \left(\frac{1}{4} \text{ of } 25 \times 56\frac{1}{8} \right)$$

$$d. \quad 1753.90625 = 31\frac{1}{4} \times 56\frac{1}{8} \text{ (Sum of } b \text{ and } c)$$

Using the method explained on the preceding page, make the following extensions and find the total for each group:

1.	2.	3.
426 yd. @ \$1.66 $\frac{2}{3}$	124 A. @ \$ 125	128 lb. @ \$.03 $\frac{1}{8}$
572 yd. @ \$3.33 $\frac{1}{3}$	86 Cows @ \$ 62 $\frac{1}{2}$	432 lb. @ \$.04 $\frac{1}{6}$
519 yd. @ \$2.50	34 Autos @ \$3750	576 lb. @ \$.03 $\frac{1}{3}$
737 yd. @ \$1.87 $\frac{1}{2}$	58 Boats @ \$3125	525 lb. @ \$.02 $\frac{1}{2}$
843 yd. @ \$.41 $\frac{2}{3}$	64 Bonds @ \$ 250	744 lb. @ \$.08 $\frac{1}{3}$
4.	5.	6.
427 $\frac{1}{4}$ $\times$ 12 $\frac{1}{2}$	91 $\frac{1}{2}$ $\times$.31 $\frac{1}{4}$	4.134 $\times$ 37 $\frac{1}{2}$
374 $\frac{1}{2}$ $\times$ 1875	91 $\frac{1}{2}$ $\times$ 312 $\frac{1}{2}$	16.24 $\times$ 625
547 $\frac{1}{2}$ $\times$ 333 $\frac{1}{3}$	87 $\frac{3}{8}$ $\times$.18 $\frac{3}{4}$	.0256 $\times$ 1875
321 $\frac{3}{4}$ $\times$ 5000	87 $\frac{3}{8}$ $\times$ 1 $\frac{7}{8}$	4484 $\times$.02 $\frac{1}{2}$
47.25 $\times$ 166 $\frac{2}{3}$	69 $\frac{3}{4}$ $\times$ 187 $\frac{1}{2}$	5652 $\times$.01 $\frac{1}{4}$

The principle of aliquot parts may be applied to “quantities” as well as to “prices.”

7.	8.	9.
1250 yd. @ \$1.36	250 bu. @ \$.96	37 $\frac{1}{2}$ lb. @ \$.32
3333 $\frac{1}{3}$ yd. @ \$2.25	187 $\frac{1}{2}$ bu. @ \$1.12	31 $\frac{1}{4}$ lb. @ \$1.44
3125 yd. @ \$1.84	625 bu. @ \$.84	250 lb. @ \$.03 $\frac{1}{2}$
4166 $\frac{2}{3}$ yd. @ \$1.92	1250 bu. @ \$.78	125 lb. @ .07 $\frac{1}{4}$
1875 yd. @ \$3.44	833 $\frac{1}{3}$ bu. @ \$1.14	312 $\frac{1}{2}$ lb. @ \$.13 $\frac{1}{2}$
2500 yd. @ \$1.34	166 $\frac{2}{3}$ bu. @ \$1.56	43 $\frac{3}{4}$ lb. @ \$.56
3750 yd. @ \$1.44	333 $\frac{1}{3}$ bu. @ \$1.65	68 $\frac{3}{4}$ lb. @ \$.14 $\frac{1}{2}$
5625 yd. @ \$2.32	375 bu. @ \$1.64	750 lb. @ \$1.36
4500 yd. @ \$1.88	62 $\frac{1}{2}$ bu. @ \$2.60	87 $\frac{1}{2}$ lb. @ \$.76

The aliquot and aliquant parts used in the preceding exercises are sometimes referred to as “direct aliquots” and “direct aliquants.”

There is a considerable number of other values closely related to many of the “direct” aliquots or aliquants, which for convenience are called “indirect aliquots” or “near aliquots.” A knowledge of these ratios is helpful in attaining skill in calculation.

A partial list of such values is contained in the following table:

INDIRECT OR NEAR ALIQUOT PARTS

VALUES	RATIOS
.07 $\frac{1}{2}$	.10 - $\frac{1}{4}$ of .10
.09	.10 - .01 ($\frac{1}{10}$ of .10)
.11	.10 + .01
.11 $\frac{1}{4}$	.10 + .01 $\frac{1}{4}$ ($\frac{1}{8}$ of .10)
.11 $\frac{1}{2}$	.12 $\frac{1}{2}$ - .01
.13 $\frac{1}{3}$	.10 + .03 $\frac{1}{3}$ ($\frac{1}{3}$ of .10)
.13 $\frac{1}{2}$	.12 $\frac{1}{2}$ + .01
.13 $\frac{3}{4}$	.12 $\frac{1}{2}$ + .01 $\frac{1}{4}$ ($\frac{1}{10}$ of .12 $\frac{1}{2}$)
.15	.10 + .05 ($\frac{1}{2}$ of .10)
.17 $\frac{1}{2}$	.12 $\frac{1}{2}$ + .05, or .20 - .02 $\frac{1}{2}$ ($\frac{1}{8}$ of .20)
.18	.20 - .02 ($\frac{1}{10}$ of .20)
.22 $\frac{1}{2}$	.20 + .02 $\frac{1}{2}$ ($\frac{1}{8}$ of .20) or .25 - .02 $\frac{1}{2}$ ($\frac{1}{10}$ of .25)
.23 $\frac{1}{3}$	.20 + .03 $\frac{1}{3}$ ($\frac{1}{6}$ of .20)
.23 $\frac{3}{4}$	.25 - .01 $\frac{1}{4}$ ($\frac{1}{20}$ of .25)
.24	.25 - .01
.26	.25 + .01
.26 $\frac{2}{3}$	.16 $\frac{2}{3}$ + .10 or .20 + .06 $\frac{2}{3}$ ($\frac{1}{3}$ of .20)
.27 $\frac{1}{2}$	.25 + .02 $\frac{1}{2}$ ($\frac{1}{10}$ of .25)
.33 $\frac{3}{4}$	.30 + .03 $\frac{3}{4}$ ($\frac{1}{8}$ of .30)
.35	.25 + .10

ILLUSTRATIONS

- Find the value of 572 yd. of cloth at 11 $\frac{1}{4}$ ¢ a yard.
 - \$57.20 = value at \$.10
 - 7.15 = value at .01 $\frac{1}{4}$ ($\frac{1}{8}$ of \$.10)
 - \$64.35 = value at \$.11 $\frac{1}{4}$ (Sum of *a* and *b*)
- Find the value of 84 $\frac{1}{2}$ lb. of meat at 28 $\frac{3}{4}$ ¢ a pound.
 - \$84.50 = value at \$1.
 - 21.125 = value at \$.25
 - 2.1125 = value at .02 $\frac{1}{2}$ ($\frac{1}{10}$ of \$.25)
 - 1.0562 = value at .01 $\frac{1}{4}$ ($\frac{1}{2}$ of $\frac{1}{10}$ of \$.25)
 - \$24.29 = value at \$.28 $\frac{3}{4}$ (Sum of *b*, *c*, and *d*)

Using the foregoing illustrations as suggestions, find the total value of each of the following groups :

1.	2.	3.
48 yd. @ $7\frac{1}{2}\text{¢}$	90 yd. @ 18 ¢	$7\frac{1}{2}$ yd. @ 36¢
56 yd. @ 9 ¢	96 yd. @ 22 ¢	$11\frac{1}{2}$ yd. @ 48¢
64 yd. @ $13\frac{1}{2}\text{¢}$	110 yd. @ $22\frac{1}{2}\text{¢}$	16 yd. @ 45¢
80 yd. @ $11\frac{1}{2}\text{¢}$	116 yd. @ $27\frac{1}{2}\text{¢}$	$17\frac{1}{2}$ yd. @ 62¢
88 yd. @ $17\frac{1}{2}\text{¢}$	124 yd. @ 35 ¢	18 yd. @ 65¢
4.	5.	6.
55 yd. @ 44¢	132 yd. @ 22 ¢	$219\frac{1}{2}$ yd. @ 33¢
55 yd. @ 36¢	141 yd. @ $22\frac{1}{2}\text{¢}$	$313\frac{1}{4}$ yd. @ 35¢
65 yd. @ 33¢	123 yd. @ $17\frac{1}{2}\text{¢}$	$317\frac{1}{2}$ yd. @ 44¢
51 yd. @ 27¢	163 yd. @ $27\frac{1}{2}\text{¢}$	$273\frac{1}{2}$ yd. @ 45¢
39 yd. @ 55¢	172 yd. @ 27 ¢	$124\frac{3}{4}$ yd. @ 55¢
7.	8.	9.
$428\frac{1}{2}$ yd. @ $26\frac{2}{3}\text{¢}$	$647\frac{1}{2}$ yd. @ 35 ¢	$513\frac{1}{4}$ yd. @ $13\frac{3}{4}\text{¢}$
$317\frac{1}{4}$ yd. @ $27\frac{1}{2}\text{¢}$	$839\frac{1}{4}$ yd. @ $33\frac{3}{4}\text{¢}$	$243\frac{1}{2}$ yd. @ 24 ¢
$584\frac{3}{4}$ yd. @ $17\frac{1}{2}\text{¢}$	$513\frac{3}{4}$ yd. @ $23\frac{3}{4}\text{¢}$	$372\frac{1}{2}$ yd. @ 26 ¢
$637\frac{1}{2}$ yd. @ $22\frac{1}{2}\text{¢}$	$417\frac{3}{4}$ yd. @ $13\frac{3}{4}\text{¢}$	$521\frac{1}{4}$ yd. @ $23\frac{1}{3}\text{¢}$
$143\frac{1}{2}$ yd. @ $13\frac{1}{3}\text{¢}$	$621\frac{1}{2}$ yd. @ $11\frac{1}{4}\text{¢}$	$472\frac{3}{4}$ yd. @ 15 ¢

REVIEW

Using the shortest method of solution in each case, find the total value of each of the following :

1.	2.	3.
76 yd. @ $37\frac{1}{2}\text{¢}$	78 lb. @ $18\frac{3}{4}\text{¢}$	87 lb. @ $6\frac{1}{4}\text{¢}$
92 yd. @ $31\frac{1}{4}\text{¢}$	93 lb. @ 25 ¢	94 lb. @ $8\frac{1}{3}\text{¢}$
83 yd. @ $43\frac{3}{4}\text{¢}$	82 lb. @ $33\frac{1}{3}\text{¢}$	104 lb. @ $12\frac{1}{2}\text{¢}$
78 yd. @ $41\frac{2}{3}\text{¢}$	13 lb. @ $31\frac{1}{4}\text{¢}$	231 lb. @ $16\frac{2}{3}\text{¢}$
107 yd. @ $6\frac{1}{4}\text{¢}$	9 lb. @ $37\frac{1}{2}\text{¢}$	321 lb. @ $18\frac{3}{4}\text{¢}$
113 yd. @ $62\frac{1}{2}\text{¢}$	27 lb. @ $41\frac{2}{3}\text{¢}$	88 lb. @ $93\frac{3}{4}\text{¢}$
114 yd. @ $83\frac{1}{3}\text{¢}$	32 lb. @ $43\frac{3}{4}\text{¢}$	136 lb. @ $91\frac{2}{3}\text{¢}$
131 yd. @ $16\frac{2}{3}\text{¢}$	84 lb. @ 50 ¢	316 lb. @ $83\frac{1}{3}\text{¢}$
125 yd. @ 25 ¢	79 lb. @ $56\frac{1}{4}\text{¢}$	28 lb. @ $81\frac{1}{4}\text{¢}$
143 yd. @ $6\frac{1}{4}\text{¢}$	97 lb. @ $58\frac{1}{3}\text{¢}$	7 lb. @ $68\frac{3}{4}\text{¢}$

4.	5.	6.
117 lb. @ $93\frac{3}{4}\text{¢}$	13 lb. @ $56\frac{1}{4}\text{¢}$	67 lb. @ $6\frac{1}{4}\text{¢}$
104 lb. @ $91\frac{2}{3}\text{¢}$	27 lb. @ 50 ¢	74 lb. @ $8\frac{1}{3}\text{¢}$
116 lb. @ $87\frac{1}{2}\text{¢}$	4 lb. @ $43\frac{3}{4}\text{¢}$	83 lb. @ $12\frac{1}{2}\text{¢}$
203 lb. @ $83\frac{1}{3}\text{¢}$	9 lb. @ $41\frac{2}{3}\text{¢}$	99 lb. @ $16\frac{2}{3}\text{¢}$
308 lb. @ $81\frac{1}{4}\text{¢}$	17 lb. @ $37\frac{1}{2}\text{¢}$	199 lb. @ $18\frac{3}{4}\text{¢}$
27 lb. @ 75 ¢	37 lb. @ $33\frac{1}{3}\text{¢}$	127 lb. @ 25 ¢
96 lb. @ $68\frac{3}{4}\text{¢}$	45 lb. @ $31\frac{1}{4}\text{¢}$	217 lb. @ $31\frac{1}{4}\text{¢}$
84 lb. @ $66\frac{2}{3}\text{¢}$	83 lb. @ 25 ¢	134 lb. @ $33\frac{1}{3}\text{¢}$
119 lb. @ $62\frac{1}{2}\text{¢}$	107 lb. @ $18\frac{3}{4}\text{¢}$	229 lb. @ $37\frac{1}{2}\text{¢}$
17 lb. @ $58\frac{1}{3}\text{¢}$	119 lb. @ $16\frac{2}{3}\text{¢}$	168 lb. @ $41\frac{2}{3}\text{¢}$
45 lb. @ $12\frac{1}{2}\text{¢}$	128 lb. @ $18\frac{3}{4}\text{¢}$	39 lb. @ $31\frac{1}{4}\text{¢}$
7.	8.	9.
456 $\frac{1}{2}$ yd. @ $18\frac{3}{4}\text{¢}$	543 $\frac{1}{2}$ yd. @ $68\frac{3}{4}\text{¢}$	68 $\frac{3}{4}$ yd. @ 17 ¢
931 $\frac{1}{4}$ yd. @ $31\frac{1}{4}\text{¢}$	743 $\frac{1}{4}$ yd. @ $56\frac{1}{4}\text{¢}$	81 $\frac{1}{4}$ yd. @ 34 ¢
722 $\frac{3}{4}$ yd. @ $41\frac{2}{3}\text{¢}$	198 $\frac{3}{4}$ yd. @ $43\frac{3}{4}\text{¢}$	93 $\frac{3}{4}$ yd. @ $28\frac{1}{2}\text{¢}$
927 $\frac{1}{2}$ yd. @ $56\frac{1}{4}\text{¢}$	217 $\frac{1}{2}$ yd. @ $58\frac{1}{3}\text{¢}$	58 $\frac{1}{3}$ yd. @ $47\frac{1}{4}\text{¢}$
324 $\frac{1}{4}$ yd. @ $43\frac{3}{4}\text{¢}$	727 $\frac{1}{2}$ yd. @ $68\frac{3}{4}\text{¢}$	56 $\frac{1}{4}$ yd. @ $56\frac{1}{2}\text{¢}$
143 $\frac{3}{4}$ yd. @ $58\frac{1}{3}\text{¢}$	372 $\frac{3}{4}$ yd. @ $81\frac{1}{4}\text{¢}$	31 $\frac{1}{4}$ yd. @ \$1.17
741 $\frac{1}{2}$ yd. @ $93\frac{3}{4}\text{¢}$	723 $\frac{3}{4}$ yd. @ $41\frac{2}{3}\text{¢}$	18 $\frac{3}{4}$ yd. @ \$1.73
864 $\frac{1}{2}$ yd. @ $81\frac{1}{4}\text{¢}$	278 $\frac{1}{4}$ yd. @ $18\frac{3}{4}\text{¢}$	41 $\frac{2}{3}$ yd. @ \$2.34
633 $\frac{3}{4}$ yd. @ $31\frac{1}{4}\text{¢}$	529 $\frac{1}{4}$ yd. @ $6\frac{1}{4}\text{¢}$	81 $\frac{1}{4}$ yd. @ \$1.94
742 $\frac{1}{2}$ yd. @ $91\frac{2}{3}\text{¢}$	378 $\frac{3}{4}$ yd. @ $93\frac{3}{4}\text{¢}$	68 $\frac{3}{4}$ yd. @ \$3.47
10.	11.	12.
24 lb. @ $2\frac{1}{2}\text{¢}$	123 lb. @ 10 ¢	84 lb. @ 15 ¢
24 lb. @ $3\frac{1}{3}\text{¢}$	113 lb. @ 11 ¢	92 lb. @ $17\frac{1}{2}\text{¢}$
48 lb. @ 5 ¢	120 lb. @ $11\frac{1}{4}\text{¢}$	64 lb. @ 22 ¢
44 lb. @ $7\frac{1}{2}\text{¢}$	140 lb. @ 12 ¢	72 lb. @ $23\frac{3}{4}\text{¢}$
56 lb. @ 9 ¢	150 lb. @ $13\frac{1}{3}\text{¢}$	44 lb. @ 24 ¢
54 lb. @ $26\frac{2}{3}\text{¢}$	37 lb. @ 11 ¢	140 lb. @ $31\frac{1}{4}\text{¢}$
88 lb. @ $27\frac{1}{2}\text{¢}$	28 lb. @ $13\frac{3}{4}\text{¢}$	150 lb. @ $37\frac{1}{2}\text{¢}$
144 lb. @ $31\frac{1}{4}\text{¢}$	44 lb. @ $18\frac{3}{4}\text{¢}$	160 lb. @ $43\frac{3}{4}\text{¢}$
160 lb. @ $33\frac{3}{4}\text{¢}$	48 lb. @ 22 ¢	180 lb. @ 45 ¢

In the table on the following page the more common aliquot and aliquant parts are arranged in order of value.

ALIQUOT PARTS OF 100 — COMPARATIVE PARTS (Continued)

	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{16}$	Cents
27											$\frac{8}{15}$		53 $\frac{1}{3}$
28								$\frac{5}{9}$					55 $\frac{5}{9}$
29												$\frac{9}{16}$	56 $\frac{1}{4}$
30						$\frac{4}{7}$							57 $\frac{7}{7}$
31										$\frac{7}{12}$			58 $\frac{1}{3}$
32				$\frac{3}{5}$									60
33							$\frac{5}{8}$						62 $\frac{1}{2}$
34		$\frac{2}{3}$											66 $\frac{2}{3}$
35												$\frac{11}{16}$	68 $\frac{3}{4}$
36									$\frac{7}{10}$				70
37						$\frac{5}{7}$							71 $\frac{3}{7}$
38											$\frac{11}{15}$		73 $\frac{1}{3}$
39			$\frac{3}{4}$										75
40								$\frac{7}{9}$					77 $\frac{7}{9}$
41				$\frac{4}{5}$									80
42												$\frac{13}{16}$	81 $\frac{1}{4}$
43					$\frac{5}{6}$								83 $\frac{1}{3}$
44						$\frac{6}{7}$							85 $\frac{5}{7}$
45											$\frac{13}{15}$		86 $\frac{2}{3}$
46							$\frac{7}{8}$						87 $\frac{1}{2}$
47								$\frac{8}{9}$					88 $\frac{8}{9}$
48									$\frac{9}{10}$				90
49										$\frac{11}{12}$			91 $\frac{2}{3}$
50											$\frac{14}{15}$		93 $\frac{1}{3}$
51												$\frac{15}{16}$	93 $\frac{3}{4}$
52	$\frac{2}{2}$	$\frac{3}{3}$	$\frac{4}{4}$	$\frac{5}{5}$	$\frac{6}{6}$	$\frac{7}{7}$	$\frac{8}{8}$	$\frac{9}{9}$	$\frac{10}{10}$	$\frac{12}{12}$	$\frac{15}{15}$	$\frac{16}{16}$	100

13.

24 A.	@ \$ 25
18 cows	@ \$125
72 sheep	@ \$ 7 $\frac{1}{2}$
16 plows	@ \$ 12 $\frac{1}{2}$
32 horses	@ \$125
160 bu.	@ \$ 1 $\frac{1}{4}$
330 bu.	@ \$ 1.66 $\frac{2}{3}$
48 wagons	@ \$125
18 engines	@ \$250
28 tables	@ \$ 62.50
24 guns	@ \$ 37.50
48 coats	@ \$ 87.50

14.

25 A.	@ \$96
33 $\frac{1}{3}$ bu.	@ \$ 1.08
166 $\frac{2}{3}$ bu.	@ \$.84
1250 bu.	@ \$.64
2500 bu.	@ \$.78
1600 bu.	@ \$ 1.25
16 $\frac{2}{3}$ bu.	@ \$ 1.44
3800 lb.	@ \$.12 $\frac{1}{2}$
1375 lb.	@ \$.56
1875 lb.	@ \$.68
4375 lb.	@ \$.32
6875 lb.	@ \$.64

SOME APPLICATIONS OF ALIUOT PARTS

The principle of aliquot parts is in no sense confined to aliquot parts of 100. A system of aliquot parts may be built on any number or value.

Finding the cost of articles by the 100, 1000, or ton.

ILLUSTRATIONS

1. Find the value of 1764 lb. of feed at \$2.75 a hundredweight.

- a. $\frac{176}{40} = \text{value at } \$10 \text{ per cwt.}$
 b. $\frac{44}{10} = \text{value at } \$ 2.50 \text{ per cwt. } (\frac{1}{4} \text{ of } \$10)$
 c. $\frac{44}{10} = \text{value at } .25 \text{ per cwt. } (\frac{1}{10} \text{ of } \$2.50)$
 d. $\frac{48}{51} = \text{value at } \$ 2.75 \text{ per cwt. (Sum of } b \text{ and } c)$

At \$1 a hundredweight the value would be \$17.64. At \$10 a hundredweight the value would be 10 times \$17.64 or \$176.40, etc.

2. Find the value of 28,490 ft. of lumber at \$33.75 a thousand.

- a. $\frac{284}{90} = \text{value at } \$10 \text{ per M.}$
 b. $\frac{854}{70} = \text{value at } \$30 \text{ per M. (3 times } \$10)$
 c. $\frac{106}{84} = \text{value at } 3.75 \text{ per M. } (\frac{1}{8} \text{ of } \$30)$
 d. $\frac{961}{54} = \text{value at } \$33.75 \text{ per M. (Sum of } b \text{ and } c)$

At \$1 per M., the value would be \$28.49. At \$10 per M., the value would be 10 times \$28.49, or \$284.90, etc.

3. At \$23.75 a ton, find the value of a load of hay weighing 3682 lb.

a. $\$36\overline{)82} = \text{value at } \$20 \text{ a ton}$

b. $4\overline{)6025} = \text{value at } 2.50 \text{ a ton } (\frac{1}{8} \text{ of } \$20)$

c. $2\overline{)3012} = \text{value at } 1.25 \text{ a ton } (\frac{1}{2} \text{ of } \$2.50)$

d. $\$43\overline{)72} = \text{value at } \$23.75 \text{ a ton (Sum of } a, b, \text{ and } c)$

At \$20 a ton the price would be \$.01 a pound. At \$.01 a pound, the value of 3682 lb. would be \$36.82. Hence \$36.82 is the value at \$20 a ton.

4. Find the value of 13,425 bricks at \$11.45 per M.

$$\begin{array}{r} 13.425 \\ 11.45 \\ \hline 67125 \\ 53700 \\ 13425 \\ 13425 \\ \hline 153.71625 \end{array}$$

The use of aliquot parts in this exercise is not advantageous.

\$153.72, value

5. Find the value of 18,490 lb. of coal at \$12 a ton.

$$\begin{array}{r} 18.490 \\ 6 \\ \hline 110.94 \end{array}$$

Aliquot parts are of no advantage here. Instead of reducing the quantity to tons (by dividing by 2000) and multiplying the result by 12, divide the price by 2, which gives \$6, the price per 1000 lb.

\$110.94, value

6. Find the value of 1975 fence posts at \$7.75 per C.

$$\begin{array}{r} 19.75 \\ 8 \\ \hline 158.00 \\ 4.94 \text{ } (\frac{1}{4} \text{ of } \$19.75) \\ \hline 153.06 \end{array}$$

\$7.75 is $\$1$ less than \$8. Hence multiply by 8, and deduct the value at $\$1$ per C.

\$153.06, value

NOTE. All multiplications may be made by the use of aliquot parts. The use of aliquot parts, however, is not always advisable. The student should decide when the use of aliquot parts is an advantage, and should calculate accordingly. To determine whether the use of aliquot parts is profitable in a given calculation requires a ready analysis of numbers and a quick perception of their relation one to another.

In the following exercises analyze the price or the quantity and determine in advance the method of calculation:

1. 220 lb. bran @ \$1.35 per cwt.; @ \$1.40 per cwt.
2. 350 lb. feed @ 2.50 per cwt.; @ 2.25 per cwt.
3. 675 lb. nails @ 5.50 per cwt.; @ 5.75 per cwt.
4. 875 lb. salt @ .80 per cwt.; @ .88 per cwt.
5. 950 posts @ 6.25 per C.; @ 6.75 per C.
6. 1,100 posts @ 7.25 per C.; @ 7.70 per C.
7. 1,250 posts @ 8.00 per C.; @ 7.20 per C.
8. 1,550 lb. lead @ 7.50 per cwt.; @ $6.37\frac{1}{2}$ per cwt.
9. 38,440 lb. pig iron @ 1.50 per cwt.; @ $1.62\frac{1}{2}$ per cwt.
10. 24,680 lb. sugar @ 5.50 per cwt.; @ 5.75 per cwt.
11. 1,875 ft. lumber @ 35.00 per M.; @ 45.00 per M.
12. 1,450 ft. lumber @ 25.00 per M.; @ 22.00 per M.
13. 23,360 bricks @ 18.75 per M.; @ 17.50 per M.
14. 14,500 shingles @ $5.62\frac{1}{2}$ per M.; @ 5.25 per M.
15. 346 ft. lumber @ 33.00 per M.; @ 31.25 per M.
16. 23,465 ft. lumber @ 37.50 per M.; @ 33.75 per M.
17. 47,860 bricks @ 11.25 per M.; @ 12.50 per M.
18. 12,375 bricks @ 13.75 per M.; @ 15.00 per M.
19. 7,850 shingles @ 6.75 per M.; @ 7.50 per M.
20. 319 ft. lumber @ 45.00 per M.; @ 55.00 per M.
21. 2,660 lb. hay @ 18.00 per T.; @ 22.50 per T.
22. 3,474 lb. hay @ 26.00 per T.; @ 24.00 per T.
23. 3,096 lb. straw @ 14.50 per T.; @ 13.75 per T.
24. 7,276 lb. straw @ 13.75 per T.; @ 12.50 per T.
25. 1,680 lb. feed @ 24.60 per T.; @ 27.90 per T.
26. 4,288 lb. feed @ 26.00 per T.; @ 27.00 per T.
27. 13,890 lb. coal @ 13.75 per T.; @ 12.50 per T.
28. 47,976 lb. coal @ 13.50 per T.; @ 11.25 per T.
29. 1,890 lb. coal @ 9.50 per T.; @ 8.80 per T.
30. 560 lb. coal @ 12.50 per T.; @ 13.50 per T.

Finding the cost of commodities by the bushel.

Many grains and some vegetables are bought and sold at wholesale by the bushel.

By measure of capacity a stricken bushel occupies 2150.42 cu. in. of space. A heaped bushel of vegetables occupies 2747.71 cu. in.

The bushel of commerce, however, is a "weight" bushel.

The following table shows the weight of a bushel of some of the most common grains and vegetables. These are the weights adopted by Congress for customs purposes. The weights adopted by some of the states vary slightly from some of the weights given here.

WEIGHT OF A BUSHEL OF CERTAIN GRAINS AND VEGETABLES

COMMODITY	WEIGHT IN POUNDS	COMMODITY	WEIGHT IN POUNDS
Barley	48	Oats	32
Beans (white)	60	Onions	57
Buckwheat	48	Peas	60
Clover Seed	60	Potatoes (Irish)	60
Corn (in ear)	70	Rye	56
Corn (shelled)	56	Timothy Seed	45
Corn meal	48	Wheat	60

ILLUSTRATIONS

1. Find the value of a carload of barley weighing 38,476 lb. at \$.76 a bushel.

a. $\$384\overline{76} = \text{value at } \$.48 \text{ a bushel}$

b. $192\overline{38} = \text{value at } .24 \text{ a bushel } (\frac{1}{2} \text{ of } \$.48)$

c. $32\overline{06} = \text{value at } .04 \text{ a bushel } (\frac{1}{6} \text{ of } \$.24)$

d. $\$609\overline{20} = \text{value at } \$.76 \text{ a bushel (Sum of } a, b, \text{ and } c)$

Assume a price for a bushel equivalent to \$.01 a pound. Since barley weighs 48 lb. to the bushel, at \$.01 a pound a bushel would be worth \$.48. If the price were \$.01 a pound (that is, \$.48 a bushel) the value of 38,476 lb. would be \$384.76. Since the price is \$.76 instead of \$.48, the value at \$.76 a bushel is found by adding to \$384.76 such aliquot parts of \$384.76 as are necessary to "build up" to the value at \$.76 a bushel. Inspection shows that the values at \$.24 and \$.04 are required to be added to the value at \$.48 to arrive at the correct result.

2. Find the value of 16,725 lb. of Irish potatoes at \$.55 a bushel.

a. $\$167\overline{)25} = \text{value at } \$.60 \text{ a bushel}$

b. $\$13\overline{)94} = \text{value at } .05 \text{ a bushel } (\frac{1}{20} \text{ of } \$.60)$

c. $\$153\overline{)31} = \text{value at } \$.55 \text{ a bushel } (\text{Subtract } b \text{ from } a)$

At \$.01 a pound potatoes are worth \$.60 a bushel, and 16,725 lb. are worth \$167.25. \$.55 is \$.05 or $\frac{1}{20}$ of \$.60 less than \$.60. Hence deduct $\frac{1}{20}$ of \$167.25 from itself and the result is the value at \$.55.

Find the total value of each group :

1.

89,764 lb. oats	at \$.56 a bushel
69,843 lb. rye	at \$.95 a bushel
79,844 lb. barley	at \$.88 a bushel
74,898 lb. wheat	at \$1.30 a bushel
41,792 lb. buckwheat	at \$.70 a bushel

2.

47,824 lb. oats	at \$.82 a bushel
39,868 lb. corn	at \$.88 a bushel
54,972 lb. barley	at \$.95 a bushel
3,842 lb. timothy seed	at \$5.25 a bushel
63,924 lb. wheat	at \$1.25 a bushel

3.

3,878 lb. clover seed	at \$15.50 a bushel
58,076 lb. beans	at \$ 5.10 a bushel
83,744 lb. corn meal	at \$ 1.56 a bushel
21,432 lb. onions	at \$ 1.42 $\frac{1}{2}$ a bushel
41,260 lb. buckwheat	at \$.72 a bushel
82,648 lb. timothy seed	at \$ 6.75 a bushel

4.

45,618 lb. oats	at \$.62 a bushel
53,536 lb. rye	at \$.87 a bushel
75,356 lb. barley	at \$.92 a bushel
36,972 lb. Irish potatoes	at \$.87 a bushel
54,896 lb. beans	at \$5.70 a bushel
6,390 lb. timothy seed	at \$6.30 a bushel

SIMPLE INTEREST

Simple interest at any rate per cent is most readily calculated by the application of the principle of aliquot parts.

INTEREST AT 6%

At 6%, the interest on \$1 for 1 year is \$.06.

At 6%, the interest on \$1 for 2 mo. (60 da.) is \$.01.

At 6%, the interest on \$1 for 6 da. is \$.001.

Since the interest on \$1 for 6 days is \$.001, the interest on \$1570 for 6 da. is 1570 times \$.001 or \$1.570; also, since the interest on \$1 for 60 days is \$.01, the interest on \$1570 for 60 da. is 1570 times \$.01, or \$15.70.

That is, to find the interest on *any* sum of money at 6% for 6 days, point off 3 places in the principal sum; and for 60 days, point off 2 places in the principal sum.

State the interest at 6% on each of the following for (a) 6 da.; (b) 60 da.:

1. \$950	5. \$150	9. \$5000	13. \$1000
2. \$1125	6. \$90	10. \$2875	14. \$1300
3. \$1250	7. \$115.50	11. \$12,000	15. \$4307
4. \$1300	8. \$880	12. \$4570	16. \$150,000

The aliquot parts of 6 are 3, 2, and 1.

The multiples of 6 are 12, 18, 24, 30, 36, 42, 48, 54, etc.

The aliquot parts of 60 are

30 ($\frac{1}{2}$ of 60)	5 ($\frac{1}{12}$ of 60)
20 ($\frac{1}{3}$ of 60)	4 ($\frac{1}{15}$ of 60)
15 ($\frac{1}{4}$ of 60)	3 ($\frac{1}{20}$ of 60) ($\frac{1}{2}$ of 6)
12 ($\frac{1}{5}$ of 60)	2 ($\frac{1}{30}$ of 60) ($\frac{1}{3}$ of 6)
10 ($\frac{1}{6}$ of 60)	1 ($\frac{1}{60}$ of 60) ($\frac{1}{6}$ of 6)
6 ($\frac{1}{10}$ of 60)	

ILLUSTRATIONS

Find the interest at 6% on:

1. \$960 for 18 da.	\$ 960 = int. for 6 da.
	\$288 = int. for 18 da. (3×6 da.)

Find the interest at 6% on:

2. \$1150 for 27 da.

\$1	150 = int. for 6 da.
4	60 = int. for 24 da. (4×6 da.)
5	75 = int. for 3 da. ($\frac{1}{2}$ of 6 da.)
\$5	18 = int. for 27 da.
3. \$1475 for 38 da.

\$1	475 = int. for 6 da.
8	85 = int. for 36 da. (6×6 da.)
4	91 = int. for 2 da. ($\frac{1}{3}$ of 6 da.)
\$9	34 = int. for 38 da.
4. \$1467.50 for 47 da.

\$ 1	467.50 = int. for 6 da.
11	740 0 = int. for 48 da. (8×6 da.)
	244 5 = int. for 1 da. ($\frac{1}{6}$ of 6 da.)
\$11	495 5
\$11	50 = int. for 47 da.
5. \$1754.75 for 73 da.

\$17	54.75 = int. for 60 da.
3	50 95 = int. for 12 da. ($\frac{1}{5}$ of 60 da.)
	29 24 = int. for 1 da. ($\frac{1}{12}$ of 12 da.)
\$21	34 94
\$21	35 = int. for 73 da.
6. \$4432.80 for 119 da.

\$44	32.80 = int. for 60 da.
88	65 6 = int. for 120 da. (2×60 da.)
	73 88 = int. for 1 da. ($\frac{1}{6}$ of 6 da.)
\$87	91 72
\$87	92 = int. for 119 da.

Use 6 days as a basis in reckoning interest for less than 60 days, except for the exact aliquot parts of 60, such as 30 days, 20 days, etc. Use *exact* aliquot parts or multiples of 6 days and 60 days in "building up" or "down" for any given number of days.

Find the interest at 6% on:

1.	2.	3.
\$828 for 45 da.	\$736 for 105 da.	\$482.75 for 12 da.
\$936 for 15 da.	\$888 for 110 da.	\$321.50 for 18 da.
\$832 for 75 da.	\$960 for 115 da.	\$428.48 for 15 da.
\$648 for 70 da.	\$822 for 150 da.	\$320.10 for 20 da.

4.	5.	6.
\$597.30 for 36 da.	\$731.45 for 13 da.	\$673.87 for 21 da.
\$312.47 for 40 da.	\$806.40 for 14 da.	\$832.49 for 22 da.
\$218.38 for 42 da.	\$318.25 for 11 da.	\$738.27 for 25 da.
\$712.80 for 6 da.	\$208.14 for 10 da.	\$387.39 for 29 da.
\$811.75 for 10 da.	\$327.38 for 16 da.	\$873.93 for 5 da.
\$928.30 for 30 da.	\$445.50 for 19 da.	\$783.79 for 3 da.

7.	8.	9.
\$720.30 for 25 da.	\$475 for 26 da.	\$425 for 38 da.
\$628.50 for 44 da.	\$938 for 33 da.	\$576 for 67 da.
\$731.20 for 14 da.	\$840 for 17 da.	\$545 for 88 da.
\$906.75 for 9 da.	\$729 for 19 da.	\$750 for 112 da.
\$887.63 for 12 da.	\$325 for 39 da.	\$800 for 99 da.
\$674.70 for 13 da.	\$480 for 47 da.	\$235 for 81 da.

COMPUTING TIME

Computations of time, for interest purposes, have to do with counting forward or backward from a fixed date, and with finding the time between two dates.

Time may be expressed in days, months, months and days, or in years, months, and days.

“Time expressed in days” means the exact number of days specified. 40 days after Aug. 27 is Oct. 6; 40 days before Aug. 27 is July 18.

The term “month” means a calendar month, that is, it is the time from a given day in one month to the same day in the next succeeding month. For interest purposes, a month is 30 days.

From $\left\{ \begin{array}{l} \text{June 10 to July 10} \\ \text{Aug. 15 to Sept. 15} \\ \text{Feb. 4 to Mar. 4} \end{array} \right\}$ is in each case 1 month.

An exception to the preceding statement occurs in those cases in which there is no “same day” in the next succeeding month.

From $\left\{ \begin{array}{l} \text{Aug. 31 to Sept. 30} \\ \text{Oct. 31 to Nov. 30} \\ \text{Jan. 31 to Feb. 28 (29)} \end{array} \right\}$ is in each case 1 month.

Hence in counting forward or backward a given number of months, from a fixed date, the given period of time ends on the same day of the month as that on which it began, when such a day exists; otherwise it ends on the last day of the month.

ILLUSTRATIONS

2 mo. after Apr. 13 is June 13
 3 mo. after Aug. 31 is Nov. 30
 4 mo. after Oct. 31 is Feb. 28
 3 mo. before Dec. 15 is Sept. 15
 2 mo. before April 30 is Feb. 28

In counting forward months and days one must know whether compound time or legal time is intended. In **compound time** every month is regarded as having 30 days. In **legal time** calendar months are counted for the whole months and exact days for any portion of a month.

3 mo. 25 da. forward from April 15 gives :

(a) *By Compound Time*

4 - 15

3 - 25

8 - 10, that is, Aug. 10

(b) *By Legal Time*

April 15 + 3 mo. = July 15

25 days after July 15 = Aug. 9

3 mo. 25 days before April 15 gives:

(a) *By Compound Time*

(1923) 4 - 15

3 - 25

(1922) 12 - 20, that is, Dec. 20

(b) *By Legal Time*

April 15 - 3 mo. = Jan. 15

25 days back from Jan. 15 =

Dec. 21

There is no fixed custom of finding the time between two dates. For short periods of time the tendency is toward exact days; for longer times, especially for periods greater than a year, the time is expressed as months and days, or as years, months, and days. The result is called "compound time." For periods less than a year, another method, sometimes called "legal time," or "banker's time," is used. The months are reckoned as twelfths of a year, and the days as 365ths of a year. An illustration will make the several plans clear.

ILLUSTRATION

Find the time from Mar. 24 to June 17.

(a) *By Exact Time*

There are 7 days remaining in March
 30 days in April
 31 days in May
 and 17 days in June to be counted.
 85 days, exact time.

(b) *By Compound Time*

6	17	2 mo. 23 days expressed as
3	24	days would be 83 days.
2	23	

(c) *By Legal or Banker's Time*

It is 2 mo. from Mar. 24 to May 24.

It is 24 days from May 24 to June 17.

Hence the time is 2 mo. 24 days.

For interest purposes the time is 2 mo. ($\frac{1}{6}$ of a year) and $\frac{24}{365}$ of a year.*

Interest for exact days is computed either on a "360-day" basis or on a "365-day" basis. The calculation of interest on a "360-day" basis has been illustrated on pages 27-29.

Interest on a "365-day" basis, called **exact interest** or **accurate interest**, is reckoned on the theory that each day is $\frac{1}{365}$ of a year. If the time is 85 days, the rate 5%, and the principal \$1600, the solution is:

$$\frac{.05 \times 85 \times \$1600}{365} = \$18.63.$$

Owing to a discrepancy of 5 days between the two "bases" the difference between the interest on a "360-day" basis and the interest on a "365-day" basis is $\frac{1}{72}$ ($\frac{5}{360}$) of the exact interest or $\frac{1}{72}$ ($\frac{5}{365}$) of the "ordinary" interest.

That is, exact interest is $\frac{1}{72}$ less than ordinary interest.

* In practice interest calculations are on a 360-day basis, owing to the greater ease of calculation and the very slight discrepancy in the interest.

Interest for years, months, and days**ILLUSTRATIONS**

1. Find the interest at 6% on \$1975.50 for 1 yr. 11 mo. 9 da.

$$\begin{array}{r|l}
 \$ 1975.50 & = \text{int. for } 2 \text{ mo.} \\
 21730.50 & = \text{int. for } 22 \text{ mo. } (11 \times 2 \text{ mo.}) \\
 987.75 & = \text{int. for } 1 \text{ mo. } (\frac{1}{2} \text{ of } 2 \text{ mo.}) \\
 197.55 & = \text{int. for } 6 \text{ da. } (\frac{1}{10} \text{ of } 2 \text{ mo.}) \\
 98.78 & = \text{int. for } 3 \text{ da. } (\frac{1}{10} \text{ of } 1 \text{ mo.}) \\
 \hline
 \$23014.58 & \\
 \$23015 & = \text{int. for } 1 \text{ yr. 11 mo. 9 da.}
 \end{array}$$

Point off 2 places for 2 mo., the same as for 60 days. Reduce the years and months to months. Multiply \$19.755 by 11 to find the interest for 22 months; divide \$19.755 by 2 to find the interest for 1 month; divide \$19.755 by 10 to find the interest for 6 days; and divide \$9.8755 to find the interest for 3 days. The total is the interest for 1 yr. 11 mo. 9 da.

2. Find the interest on \$1900 at 6% from June 17 to Sept. 10, legal time.

The time is 2 mo. 24 da.

$$\begin{array}{r|l}
 \$1900 & = \text{int. for } 2 \text{ mo.} \\
 760 & = \text{int. for } 24 \text{ da. } (4 \times 6 \text{ da.}) \\
 \hline
 \$2660 & = \text{int. for } 2 \text{ mo. 24 da.}
 \end{array}$$

Find the accurate interest on:

1. \$4564 from Aug. 11, 1923 to May 15, 1924, at 5%
2. \$3870 from June 1, 1923 to Dec. 30, 1923, at 6%
3. \$4330 from Jan. 3, 1923 to Oct. 14, 1923, at 9%
4. \$5786 from Feb. 23, 1923 to Jan. 4, 1924, at 6%
5. \$3342 from May 16, 1923 to Aug. 1, 1923, at 10%
6. \$5540 from Oct. 21, 1923 to Mar. 1, 1924, at 12%
7. \$48.75 from Jan. 5, 1921 to Aug. 4, 1925, at 6%
8. \$125.50 from Dec. 3, 1922 to Nov. 15, 1924, at 5%
9. \$456.73 from Oct. 18, 1923 to Jan. 31, 1924, at 4%
10. \$592.15 from June 11, 1923 to Dec. 7, 1923, at 6%

Find the interest and the amount at 6%, (a) for compound time, (b) for exact time, and (c) for banker's or legal time * on each of the following :

1. \$798.40 from Feb. 28, 1923 to Sept. 13, 1923
2. \$1260 from July 1, 1922 to Mar. 14, 1923
3. \$1375 from May 10, 1923 to Feb. 10, 1924
4. \$1760 from June 1, 1924 to Dec. 30, 1924
5. \$1840 from Oct. 17, 1922 to July 10, 1923
6. \$6600 from May 15, 1923 to Feb. 4, 1924
7. \$7500 from June 9, 1924 to Oct. 1, 1924
8. \$8000 from Mar. 1, 1923 to Dec. 10, 1923
9. \$12,000 from Feb. 14, 1922 to Oct. 1, 1923
10. \$1000 from Aug. 13, 1922 to June 20, 1923
11. \$3600 from Mar. 4, 1922 to Oct. 17, 1923
12. \$4200 from June 30, 1923 to Jan. 1, 1924
13. \$426.74 from May 8, 1923 to Dec. 4, 1924
14. \$587.50 from Mar. 4, 1923 to Dec. 25, 1923
15. \$736.40 from Aug. 17, 1923 to July 1, 1924
- 16-20. Find the interest and the amount at 5% and at 7% on problems 1-5 inclusive.
- 21-25. Find the interest and the amount at $5\frac{1}{2}\%$ and at $6\frac{1}{2}\%$ on problems 6-10 inclusive.
- 26-30. Find the interest and the amount at $7\frac{1}{2}\%$ and 4% on problems 11-15 inclusive.

At 6% interest it takes \$1 one year to earn \$.06.

Therefore it takes $\frac{1}{6}$ of a year, or 60 da., for \$1 to earn \$.01.

Similarly, at 5% it takes \$1 one year to earn \$.05.

Hence at 5%, it takes $\frac{1}{5}$ of a year, or 72 da., for \$1 to earn \$.01.

Also, at 4%, it takes $\frac{1}{4}$ of a year, or 90 da., for \$1 to earn \$.01, etc.

If the rate of interest is 5%, pointing off 2 places in the principal gives the interest for 72 days.

If the rate of interest is 4%, pointing off 2 places in the principal gives the interest for 90 days, and so on.

* Use 360-day basis for the days.

ILLUSTRATIONS

Find the interest at 5% on :

1. \$1372 for 57 da. $\begin{array}{r|l} \$13 & 72 = \text{int. for 72 da.} \\ 6 & 86 = \text{int. for 36 da. } (\frac{1}{2} \text{ of 72 da.}) \\ 3 & 43 = \text{int. for 18 da. } (\frac{1}{2} \text{ of 36 da.}) \\ \hline & 57 \ 17 = \text{int. for 3 da. } (\frac{1}{6} \text{ of 18 da.}) \\ \hline \$10 & 86 = \text{int. for 57 da.} \end{array}$
2. \$2438.75 for 113 da. $\begin{array}{r|l} \$24 & 38.75 = \text{int. for 72 da.} \\ 12 & 19 \ 38 = \text{int. for 36 da. } (\frac{1}{2} \text{ of 72 da.}) \\ 1 & 35 \ 48 = \text{int. for 4 da. } (\frac{1}{9} \text{ of 36 da.}) \\ \hline & 33 \ 87 = \text{int. for 1 da. } (\frac{1}{4} \text{ of 4 da.}) \\ \hline \$38 & 27 \ 48 \\ \$38 & 27 = \text{int. for 113 da.} \end{array}$

Find the interest at 4% on :

3. \$ 3283.60 for 77 da. $\begin{array}{r|l} \$32 & 83.60 = \text{int. for 90 da.} \\ 16 & 41 \ 8 = \text{int. for 45 da. } (\frac{1}{2} \text{ of 90 da.}) \\ 10 & 94 \ 53 = \text{int. for 30 da. } (\frac{1}{3} \text{ of 90 da.}) \\ \hline & 72 \ 97 = \text{int. for 2 da. } (\frac{1}{15} \text{ of 30 da.}) \\ \hline \$28 & 09 \ 30 \\ \$28 & 09 = \text{int. for 77 da.} \end{array}$
4. \$4724.60 for 130 da. $\begin{array}{r|l} \$47 & 24.60 = \text{int. for 90 da.} \\ 15 & 74 \ 87 = \text{int. for 30 da. } (\frac{1}{3} \text{ of 90 da.}) \\ \hline 5 & 24 \ 96 = \text{int. for 10 da. } (\frac{1}{3} \text{ of 30 da.}) \\ \hline \$68 & 24 \ 43 \\ \$68 & 24 = \text{int. for 130 da.} \end{array}$

Determine the "day basis" for $4\frac{1}{2}\%$ and $7\frac{1}{2}\%$; and calculate the interest on the following: (Show total for each group.)

At 4%

1. \$7236.50 for 46 da.
 \$6247.85 for 75 da.
 \$5278.92 for 33 da.
 \$4782.45 for 53 da.
 \$3684.25 for 66 da.

At $4\frac{1}{2}\%$

2. \$3372 for 58 da.
 \$6325 for 68 da.
 \$4785 for 75 da.
 \$3695 for 36 da.
 \$2385 for 43 da.

At 5%

3. \$4728.50 for 112 da.
 \$6435.75 for 87 da.
 \$3784.50 for 13 da.
 \$4784.90 for 19 da.
 \$6375.50 for 25 da.
 \$2472.75 for 37 da.
 \$5842.50 for 24 da.
 \$6321.60 for 32 da.

At $7\frac{1}{2}\%$

4. \$1145 for 56 da.
 \$1450 for 67 da.
 \$2500 for 14 da.
 \$3950 for 11 da.
 \$4825 for 21 da.
 \$6475 for 86 da.
 \$3175 for 77 da.
 \$4896 for 83 da.
5. \$4725 for 73 da. at 2%
 \$7350 for 118 da. at 2%
 \$5650 for 137 da. at 3%
 \$6875 for 151 da. at 3%
 \$7675 for 98 da. at 8%
 \$4590 for 111 da. at 8%
 \$7385 for 123 da. at $7\frac{1}{2}\%$
 \$9576 for 155 da. at 6%
 \$8920 for 117 da. at 5%
6. \$972.50 for 91 da. at 9%
 \$879.25 for 17 da. at 9%
 \$754.75 for 83 da. at 10%
 \$831.40 for 113 da. at 10%
 \$998.50 for 101 da. at 12%
 \$892.60 for 133 da. at 12%
 \$583.50 for 81 da. at 4%
 \$697.50 for 168 da. at $4\frac{1}{2}\%$
 \$731.60 for 88 da. at 9%

For odd fractional rates

In practice, rates vary from those given in the preceding exercises. Experience shows that the simplest and easiest method of calculation to be used with odd fractional rates is first to calculate the interest at one of the foregoing rates, and then to adjust the result to correspond to the desired rate.

ILLUSTRATIONS

1. Find the interest on \$4275.60 for 48 da. at $4\frac{3}{8}\%$.
- a. $\$4275.60 = \text{int. for 9 da. at } 4\%$
 b. $21378 = \text{int. for 45 da. at } 4\% (5 \times 9 \text{ da.})$
 c. $14252 = \text{int. for 3 da. at } 4\% (\frac{1}{3} \text{ of } 9 \text{ da.})$
 d. $228032 = \text{int. for 48 da. at } 4\% (\text{Sum of } b \text{ and } c)$
 e. $14252 = \text{int. for 48 da. at } \frac{1}{4}\% (\frac{1}{16} \text{ of } 4\%)$
 f. $7126 = \text{int. for 48 da. at } \frac{1}{8}\% (\frac{1}{2} \text{ of } \frac{1}{4}\%)$
 g. $\$249410 = \text{int. for 48 da. at } 4\frac{3}{8}\% (\text{Sum of } d, e, \text{ and } f)$

3. \$4200 for 47 da.	$\$47 = \text{int. for 600 da.}$ $\$3290 = \text{int. for 4200 da.}$
4. \$4700 for 111 da.	$\$111 = \text{int. for 600 da.}$ $888 = \text{int. for 4800 da.}$ $185 = \text{int. for 100 da.}$ $\$8695 = \text{int. for 4700 da.}$

By interchanging principal and time, find the interest at 6% on :

- | | |
|------------------------|--------------------------|
| 1. \$6000 for 29 da. | 19. \$60 for 142 da. |
| 2. \$3000 for 47 da. | 20. \$90 for 101 da. |
| 3. \$600 for 83 da. | 21. \$150 for 140 da. |
| 4. \$1200 for 19 da. | 22. \$1500 for 64 da. |
| 5. \$1500 for 37 da. | 23. \$2100 for 44 da. |
| 6. \$2000 for 81 da. | 24. \$2700 for 14 da. |
| 7. \$12,000 for 43 da. | 25. \$6600 for 31 da. |
| 8. \$9000 for 53 da. | 26. \$7200 for 25 da. |
| 9. \$8000 for 79 da. | 27. \$7800 for 17 da. |
| 10. \$1000 for 33 da. | 28. \$8400 for 13 da. |
| 11. \$180 for 55 da. | 29. \$540 for 38 da. |
| 12. \$120 for 74 da. | 30. \$630 for 41 da. |
| 13. \$300 for 71 da. | 31. \$72 for 35 da. |
| 14. \$480 for 92 da. | 32. \$30 for 80 da. |
| 15. \$360 for 103 da. | 33. \$240 for 70 da. |
| 16. \$200 for 110 da. | 34. \$15 for 100 da. |
| 17. \$900 for 111 da. | 35. \$15,000 for 160 da. |
| 18. \$600 for 132 da. | 36. \$12,600 for 131 da. |

INTEREST ON DAILY BALANCES

Banks usually allow interest on a customer's deposits if they average over a given minimum amount determined by the bank. The interest is reckoned monthly or quarterly on the depositor's daily balances and credited to the depositor's account.

Allowing interest at $2\frac{1}{2}\%$, find the amount of interest to be credited to the following account July 1.

THE CHEMICAL NATIONAL BANK IN ACCOUNT WITH A. LANG
DEPOSITS, WITHDRAWALS, AND DAILY BALANCES FOR JUNE, 1923

DEPOSITS		CHECKS	BALANCES	DAY NUMBERS	PRINCIPAL AT INT. FOR 1 DAY
June 1			15,500	3	\$ 465(00)
4	6500		22,000	1	220
5	1500	4275	19,225	5	960
10		1527.50	17,697.50	6	1062
16	8750	2000	24,447.50	5	1220
21	3400		27,847.50	3	834
24	1522.50	3425	25,945	4	1036
28	1275		27,220	2	544
30		5760	21,460	1	215
					6) \$655 6
					3) 109 266
					4)36 422
					9 105
					\$45 527

The day numbers indicate the number of days a given balance remained unchanged. The balance, \$15,500, on hand at the beginning of the month, was unchanged for 3 days, hence interest was allowed on \$15,500 for 3 days, or what is the same thing, on \$46,500 for 1 day.

On June 4 a deposit of \$6500 was made which increased the bank balance to \$22,000. This amount remained unchanged for 1 day, hence interest was allowed on \$22,000 for 1 day.

On July 5 the balance was decreased to \$19,225. Since no more deposits or withdrawals were made until the 10th of the month, this balance was on deposit 5 days. It is the custom of many banks to allow interest only on the nearest whole \$100 of the deposit, hence in multiplying by the day number, 5, only the 192 (hundreds) is multiplied, giving 960 (hundreds), etc.

The several amounts on which interest is to be allowed for 1 day are added, and the interest at $2\frac{1}{2}\%$ is calculated on the total for 1 day.

Two places have been omitted in the several addends, hence pointing off 1 place in the total gives the interest for 6 days at 6%. (See page 27.) Dividing \$655.6 by 6 gives the interest for 1 day at 6%. Dividing \$109.266 by 3 gives the interest for 1 day at 2%. Dividing \$36.422 by 4 gives the interest at $\frac{1}{2}\%$. Adding the interest at 2% and at $\frac{1}{2}\%$ gives the interest at $2\frac{1}{2}\%$.*

* In case of large daily balances, bankers frequently reckon the interest on a 365-day basis, and, in leap year, on a 366-day basis.

Another solution similar to that for finding interest on daily balances is the one often used by brokers who borrow, on demand, for their clients large sums of money for speculative purposes. This method differs from the daily balance method in that the rate of interest is subject to frequent change.

ILLUSTRATION

A speculator borrowed on demand through a broker and returned funds as follows. Find the interest due Nov. 1.

DATE	BORROWED	RETURNED	INTEREST RATE
Oct. 1	\$25,000		$4\frac{1}{4}\%$
7	\$15,000		$4\frac{1}{8}\%$
11		\$10,000	
13	\$16,000		$5\frac{1}{4}\%$
18		\$20,000	$5\frac{1}{2}\%$
21		\$10,000	
25	\$20,000		$5\frac{3}{4}\%$
28		\$15,000	6 %

Interest on

Interest on

$\$25,000$ for 6 da. at $4\frac{1}{4}\%$ = \$ 637,500 for 1 da. at 1%
 $\$40,000$ for 4 da. at $4\frac{1}{8}\%$ = \$ 660,000 for 1 da. at 1%
 $\$30,000$ for 2 da. at $4\frac{1}{8}\%$ = \$ 247,500 for 1 da. at 1%
 $\$46,000$ for 5 da. at $5\frac{1}{4}\%$ = \$1,207,500 for 1 da. at 1%
 $\$26,000$ for 3 da. at $5\frac{1}{2}\%$ = \$ 429,000 for 1 da. at 1%
 $\$16,000$ for 4 da. at $5\frac{1}{2}\%$ = \$ 352,000 for 1 da. at 1%
 $\$36,000$ for 3 da. at $5\frac{3}{4}\%$ = \$ 621,000 for 1 da. at 1%
 $\$21,000$ for 4 da. at 6 % = \$ 504,000 for 1 da. at 1%
 $\$4,658,500$ for 1 da. at 1%

6) $\$4658.500$ = int. for 6 da. at 6%

$\underline{6)776.4166}$ = int. for 1 da. at 6%

$\$129.40$ = int. for 1 da. at 1%

Since the amounts, days, and rates are all varying values, it is necessary to reduce each item to a uniform time and rate. *Unit time* and *unit rate* may be found by multiplying the amount of the loan, the number of days, and the rate (used as an integer) together. The first loan of \$25,000 was made on Oct. 1, at $4\frac{1}{4}\%$. No further loan was made until Oct. 7. Hence interest is to be charged on \$25,000 for 6 days at $4\frac{1}{4}\%$. ($6 \times 4\frac{1}{4} \times \$25,000 = \$637,500$) This is equivalent to the interest on \$637,500 for 1 day at 1%. On

Oct. 7 a further loan of \$15,000 was made, which increased the total loan to \$40,000; the rate of interest changed to $4\frac{1}{8}\%$. No further loans were made, nor was any return payment made, until Oct. 11. Hence interest is to be charged on \$40,000 for 4 days at $4\frac{1}{8}\%$, which is equivalent to interest on \$660,000 for 1 day at 1%. ($4 \times 4\frac{1}{8} \times \$40,000 = \$660,000$)

On Oct. 11, \$10,000 was returned. That left only \$30,000 on which interest is to be charged for 2 days. There was no change in the rate, hence the former rate of $4\frac{1}{8}\%$ applies. The interest on \$30,000 for 2 days at $4\frac{1}{8}\%$ equals the interest on \$247,500 for 1 day at 1%, etc.

After all the amounts are reduced to unit time and unit rate, the several products are added. The total is \$4,658,500, on which interest is to be reckoned for 1 day at 1%.

Pointing off 3 places gives the interest for 6 days at 6%. Dividing the result, \$4658.50, by 6 gives the interest for 1 day at 6%. Dividing the last result, \$776.4166, by 6 gives the interest for 1 day at 1%. The result, \$129.40, is the amount of interest due Nov. 1.

1. Find the amount of interest due on the first of September on the following account:

DATES	BORROWED	RETURNED	INTEREST RATE
Aug. 4	\$20,000		$5\frac{1}{4}\%$
10	\$30,000		$6\frac{1}{2}\%$
12			$6\frac{3}{4}\%$
16		\$25,000	
19	\$15,000		$5\frac{3}{4}\%$
20			6 %
22		\$20,000	$6\frac{1}{4}\%$
26	\$30,000		$6\frac{1}{2}\%$
28			7 %

2. Deposits were made and checks were drawn as follows by a depositor. Find his daily balances and the amount of interest at 2% to be credited to his account Dec. 1.

DATES	DEPOSITS	CHECKS	BALANCE
Nov. 1			\$15,475.60
4	\$3475	\$1500; \$275; \$540	
9		\$500; \$375.50	
12	\$5284		
15	\$1000	\$1375; \$1100; \$15.25	
17		\$950; \$1632.50	
19	\$1300		
23	\$1650		
26		\$575; \$860.50; \$1321.27	
27	\$2500		
30		\$2000; \$943.72	

III. DEVELOPING SKILL AND FACILITY IN HANDLING NUMBERS

In a recent investigation made in twenty-five different kinds of business it was learned that of the time spent on the four operations of addition, subtraction, multiplication, and division, more than one half was spent on addition, and more than two fifths on multiplication.

ADDITION

There are 45 primary addition combinations of two figures each. They must be known perfectly in order that one may be rapid in addition. The following exercise has been found very effective in developing accuracy and skill in addition:

54,972
73,865
128,837
202,702
331,539
534,241
865,780
1,400,021
2,265,801
3,665,822

9,523,580

Start with any two numbers. Suppose you take 54,972 and 73,865; add them and obtain 128,837 as the result. Next add 73,865 and 128,837. The result is 202,702. Continue adding the last two numbers until a column of 10 or more numbers is formed. Then add the whole column.

This device provides the necessary fundamental drill in the primary combinations and also provides practice in columnar addition.

Another helpful device is to add two or more figures at a time; that is, to add groups of figures. This plan follows logically the preceding exercise. One must recognize at sight the sum of two or more digits and treat it as a single digit. A helpful device in developing skill is to have several columns of numbers ruled lightly as shown on page 42.

47	83	76
92	85	78
56	49	83
72	57	86
32	45	95
85	76	74
52	38	74
92	56	74
57	93	27
54	78	92
98	78	76
49	23	89

Add both vertically and horizontally. *Think* the sum of each pair of digits and add it as if it were a single digit, naming results only. Thus, in adding the first column from the bottom up, say only "24, 32, 41, 50, 64." Check the result by adding from the top down, saying "23, 32, 40, 49, 64," and so on.

There is no "short cut" in addition. Proficiency is attained only through practice.

MULTIPLICATION

Multiplication permits of several devices to lessen the time and labor of performing the operation. The suggestions which follow are some of the most practical short methods.

For multiplication by aliquot parts see pages 9-26.

ILLUSTRATIONS

1. Write the product of 11×45 .

PRINCIPLE

495

45

45

495

Simply write the sum of the digits 4 and 5 between the digits.

2. Write the product of $11 \times 74,839$.

PRINCIPLE

823,229

74839

74839

823,229

Note that the units' figure is the first figure of the product. The units' and the tens' figures added give the tens' figure of the product (with 1 to carry). The tens' and the hundreds' figures added, plus carrying figure, give the hundreds' figure of the product, and so on.

3. Multiply 476 by 31.

$$\begin{array}{r} 476 \\ 1428 \\ \hline 14,756 \end{array} \quad 31$$

Write the multiplier to one side, instead of under the multiplicand. To save time, do not "multiply" by 1, but by 3 only. Write the result as illustrated.

4. Multiply 274 by 317.

$$\begin{array}{r} 274 \\ 1918 \\ 822 \\ \hline 86,858 \end{array} \quad 317$$

The "1" of the multiplier is really 10. Do not "multiply" by it, but by 7 and by 3 (hundred). Write results as illustrated and add.

5. Multiply 5432 by 324.

$$\begin{array}{r} 5432 \\ 324 \\ \hline 21\ 728 \\ 1\ 738\ 24 \\ \hline 1,759,968 \end{array}$$

The multiplier is $320 + 4$. 320 is 80 times 4. Multiply by 4 as usual. Multiply the result, 21,728, by 80, that is, multiply by 8 and write the product as illustrated.

This plan (when usable) saves multiplying by a third digit, and lessens the number of figures to be added.

6. Multiply 6387 by 735.

$$\begin{array}{r} 6387 \\ 735 \\ \hline 4\ 470\ 9 \\ 223\ 545 \\ \hline 4,694,445 \end{array}$$

The multiplier is $700 + 35$. 35 is $.05 \times 700$ (5×7). Multiply first by 7 (hundred) placing the product as illustrated. Multiply the result, 44709 (00) by .05 (that is by 5) and write the result as shown. (One third of the time and labor is eliminated.)

7. Write the square of 63.

$$(60 \times 66) + 3^2 = 3969$$

$$63 = 60 + 3$$

$$\begin{array}{r} 60 + 3 \\ 60 + 3 \\ \hline (3 \times 60) + 3^2 \\ 60^2 + (3 \times 60) \\ \hline 60^2 + (6 \times 60) + 3^2 = (66 \times 60) + 3^2 \end{array}$$

$$63 \text{ is } 3 \text{ more than } 60$$

$$63 + 3 = 66$$

$$66 \times 60 = 3960$$

$$\begin{array}{r} 3 \times 3 = 9 \\ \hline 3969 \end{array}$$

This illustration is based on the algebraic principle of finding the square of the sum of two quantities.

8. Write the square of 78.

$$\begin{array}{rcl}
 (76 \times 80) + 2^2 & = & 6084 \\
 78 & = & 80 - 2 \\
 \begin{array}{r}
 80 - 2 \\
 - (2 \times 80) + 2^2 \\
 \hline
 80^2 - (2 \times 80) \\
 \hline
 80^2 - (4 \times 80) + 2^2 = (76 \times 80) + 2^2
 \end{array}
 \end{array}
 \qquad
 \begin{array}{rcl}
 78 \text{ is } 2 \text{ less than } 80 \\
 78 - 2 & = & 76 \\
 76 \times 80 & = & 6080 \\
 2 \times 2 & = & \underline{4} \\
 & & 6084
 \end{array}$$

This illustration is based on the algebraic principle of finding the square of the difference of two quantities.

The plan illustrated in examples 7 and 8 may be used in squaring any number of two digits.

“ Cross ” multiplication

ILLUSTRATIONS

1. Multiply 47 by 36.

$$\begin{array}{rcl}
 & \text{STEPS} & \\
 \begin{array}{r}
 47 \\
 36 \\
 \hline
 1692
 \end{array}
 & \begin{array}{l}
 6 \times 7 = 42 \\
 (6 \times 4) + 4 \text{ (carried)} + 3 \times 7 = 49 \\
 (3 \times 4) + 4 \text{ (carried)} = 16.
 \end{array}
 \end{array}$$

The purpose is to determine the final result without writing the partial products. The multiplication is performed in the usual way, except that the two cross products, 6×4 and 3×7 , and the carrying figure, 4, are added without writing any intermediate results.

Practice alone will develop skill.

Some calculators carry the foregoing principle into use with numbers of three digits.

2. Multiply 523 by 467.

$$\begin{array}{rcl}
 & \text{STEPS} & \\
 \begin{array}{r}
 523 \\
 467 \\
 \hline
 244,241
 \end{array}
 & \begin{array}{l}
 7 \times 3 = 21 \\
 (7 \times 2) + 2 \text{ (carried)} + (6 \times 3) = 34 \\
 (7 \times 5) + 3 \text{ (carried)} + (4 \times 3) + (6 \times 2) = 62 \\
 (6 \times 5) + 6 \text{ (carried)} + (4 \times 2) = 44 \\
 (4 \times 5) + 4 \text{ (carried)} = 24
 \end{array}
 \end{array}$$

Keeping the following principles in mind helps one to remember the order of procedure in this exercise:

Units times units gives units.

Units times tens gives } tens.
Tens times units gives }

Units times hundreds gives } hundreds.
Hundred times units gives }
Tens times tens gives }

Tens times hundreds gives } thousands.
Hundreds times tens gives }

Hundreds times hundreds gives ten thousands.

3. Multiply 42.347624 by 32.14326 correct to the nearest hundredth.

$$\begin{array}{r} 42.347\ 624 \\ 62\ 341.23 \\ \hline \end{array}$$

1st step. Write the multiplier in the inverse order of digits in such position that the units' figure of the multiplier is two places further to the right of the decimal point in the multiplicand than required in the final result. The units' figure of the multiplier, 2, is written under 6, the fourth decimal place in the multiplicand.

$$\begin{array}{r} 42.347\ 624 \\ 62\ 341.23 \\ \hline 127\ 042\ 87\ * \end{array}$$

2d step. Commence multiplying by the right-hand digit of the inverted multiplier. Begin the multiplication with the digit directly above the "multiplier" digit you are using.

$$\begin{array}{r} 42.347\ 624 \\ 62\ 341.23 \\ \hline 127\ 042\ 87 \end{array}$$

3d step. Proceed with each digit of the multiplier the same as with the first digit, beginning with the digit in the multiplicand directly over the multiplying digit. Place the first, or right-hand, figure of each partial product under the right-hand figure of the preceding partial product, because they are of the same decimal order.

$$\begin{array}{r} 8\ 469\ 52 \\ 423\ 47 \\ 169\ 39 \\ 12\ 70 \end{array}$$

4th step. Add the several partial products and point off as many places in the result as units' digit of the multiplier is to the right of the decimal point in the multiplicand.

$$\begin{array}{r} 84 \\ 25 \\ \hline 1361.19\ 04 \end{array}$$

This method of multiplying decimals does away with finding 8 or 10 decimal places in the product when only 3 or 4 are needed.

* 7 is obtained by carrying 1 from 3×4 . This is done to keep the fourth decimal place correct in the several partial products.

DIVISION

Division permits of very few short methods. Only three devices for lessening time and labor will be noted here.

ILLUSTRATIONS

1. Divide 538.75 by 42, expressing the quotient to the nearest hundredth.

$$42 = 6 \times 7$$

$$\begin{array}{r} 6 \overline{)538.75} \\ 7 \overline{)89.7916} \\ 12.827 = 12.83 \end{array}$$

If the divisor is a composite number, that is, one that has exact integral factors, divide the dividend by one of the factors, and then divide the quotient obtained by another factor, and so on until all the factors have been used.

If an *exact* result, instead of a result to the nearest second or third decimal place, is desired, proceed as in the following illustration.

2. Divide $372\frac{1}{3}$ by 56.

$$56 = 7 \times 8$$

$$\begin{array}{r} 7 \overline{)372\frac{1}{3}} \\ 8 \overline{)53\frac{4}{21}} \\ 6\frac{10}{168} \end{array}$$

Divide by the factors of the divisor as before and retain the exact fractional result each time.

3. Divide \$4500 by 347.89232 correct to the nearest cent.

$$\begin{array}{r} \$1 . \\ 347.89232 \overline{) \$4500.00} \end{array} \begin{array}{l} 000.0000 \\ | \end{array}$$

1st step. Write the divisor and the dividend in the usual positions and annex zeros to make as many decimal places in the dividend as there are in the divisor.

2d step. Place decimal points after the annexed zeros as illustrated. The upper decimal point is for the quotient.

3d step. Annex 4 more zeros to the right of these decimal points (4 zeros because the quotient must be determined to the nearest cent, 2 places, and 2 more places are allowed for "safety"). That is, annex, after the decimal point, 2 zeros more than the number of decimal places required in the completed quotient.

4th step. Determine first quotient figure and place it as illustrated.

5th step. Before multiplying the divisor by the quotient figure, count the number of figures or digits of the dividend to the right of the first quotient figure. In this case there are 5 such figures.

6th step. Beginning after the first figure of the dividend, count off 5 places and draw a vertical line as in the illustration. No calculation figures are needed to the right of the line, hence cancel the right-hand two figures in the divisor.

7th step. Multiply the remaining figures of the divisor by 1, the quotient figure, and place the result like this:

$$\begin{array}{r}
 \$1 . \\
 347.89232) \$4500.00 | 000.0000 \\
 \underline{3478 \ 92} \\
 \\
 \$12.935 = \$12.94
 \end{array}$$

$$\begin{array}{r}
 347.89232) \$4500.00 | 000.0000 \\
 \underline{3478 \ 92} \\
 1021 \ 08 \\
 \underline{695 \ 78} \\
 325 \ 30 \\
 \underline{313 \ 10} \\
 12 \ 20 \\
 \underline{10 \ 43} \\
 1 \ 77 \\
 \underline{1 \ 73}
 \end{array}$$

Subtract as usual.

8th step. Instead of "bringing down" a figure, cancel one more figure in the divisor. The next quotient figure is 2.

From this point on alternate steps 7 and 8 until the complete quotient is obtained.

Study the following illustrations carefully.

4. Find to the nearest hundredth of a mill the cost per unit if 215,875 units cost \$487.64.

$$\begin{array}{r}
 \$.002259 = 2.26 \text{ mills} \\
 215875) \$487.64 | 0000 \\
 \underline{431 \ 75} \\
 55 \ 89 \\
 \underline{43 \ 17} \\
 12 \ 72 \\
 \underline{10 \ 79} \\
 1 \ 93 \\
 \underline{1 \ 93}
 \end{array}$$

The fact that the divisor is an integer makes no difference with the working of the principle.

5. Divide \$21,560.50 by 435.4268 to the nearest mill.

$$\$49.5157 = \$49.516$$

$$\begin{array}{r}
 435.4268 \overline{) \$21,560.50000000} \\
 \underline{17 \ 417 \ 072} \\
 4 \ 143 \ 428 \\
 \underline{3 \ 918 \ 841} \\
 224 \ 587 \\
 \underline{217 \ 713} \\
 6 \ 874 \\
 \underline{4 \ 354} \\
 2 \ 520 \\
 \underline{2 \ 177} \\
 343 \\
 \underline{304}
 \end{array}$$

If the first figure of the divisor is greater than the first figure of the dividend, begin counting after the second figure instead of after the first figure, as directed in step 6, on page 47.

Sometimes it is necessary to divide a considerable number of values by the same divisor. In such cases time may be saved and errors eliminated by the following plan :

Assume that it is desired to use the number 47,855 many times as a divisor. For every quotient figure obtained, 47,855 must be multiplied and the product subtracted from the dividend. Before proceeding with the division, set up a list of all possible products, thus :

1. 47,855
2. 95,710
3. 143,565
4. 191,420
5. 239,275
6. 287,130
7. 334,985
8. 382,840
9. 430,695
10. 478,550

To insure accuracy in this list of products double the first number to obtain the second. Add the first and second to obtain the third. Add the first and third to make the fourth, and so on.

Find the tenth number merely to check the accuracy of the other numbers.

Practice Exercise

Extend, as illustrated on page 41, each of the following to a column of ten numbers and find the sum of the entire column. The final total should be found in one and one half minutes or less.

$$\begin{array}{r} 1. \\ 42,376 \\ \underline{22,431} \end{array}$$

$$\begin{array}{r} 2. \\ 31,346 \\ \underline{27,891} \end{array}$$

$$\begin{array}{r} 3. \\ 56,325 \\ \underline{67,226} \end{array}$$

$$\begin{array}{r} 4. \\ 34,387 \\ \underline{29,567} \end{array}$$

$$\begin{array}{r} 5. \\ 43,231 \\ \underline{62,476} \end{array}$$

$$\begin{array}{r} 6. \\ 54,724 \\ \underline{31,826} \end{array}$$

$$\begin{array}{r} 7. \\ 47,389 \\ \underline{43,826} \end{array}$$

$$\begin{array}{r} 8. \\ 32,468 \\ \underline{51,327} \end{array}$$

$$\begin{array}{r} 9. \\ 41,684 \\ \underline{82,392} \end{array}$$

$$\begin{array}{r} 10. \\ 46,822 \\ \underline{32,373} \end{array}$$

$$\begin{array}{r} 11. \\ 43,576 \\ \underline{21,492} \end{array}$$

$$\begin{array}{r} 12. \\ 48,327 \\ \underline{72,564} \end{array}$$

$$\begin{array}{r} 13. \\ 53,721 \\ \underline{68,496} \end{array}$$

$$\begin{array}{r} 14. \\ 82,367 \\ \underline{98,924} \end{array}$$

$$\begin{array}{r} 15. \\ 37,817 \\ \underline{95,326} \end{array}$$

Drill on the following, adding by groups of two figures:

$$\begin{array}{r} 1. \\ 25 \overline{86} \overline{79} \\ 56 \overline{47} \overline{58} \\ 68 \overline{69} \overline{70} \\ 85 \overline{36} \overline{47} \\ 59 \overline{68} \overline{75} \\ 96 \overline{47} \overline{58} \\ 66 \overline{47} \overline{53} \\ 67 \overline{58} \overline{69} \\ 77 \overline{87} \overline{56} \\ 75 \overline{43} \overline{62} \end{array}$$

$$\begin{array}{r} 2. \\ 36 \overline{47} \overline{58} \\ 64 \overline{65} \overline{78} \\ 46 \overline{57} \overline{48} \\ 67 \overline{56} \overline{48} \\ 76 \overline{57} \overline{48} \\ 61 \overline{28} \overline{36} \\ 13 \overline{54} \overline{62} \\ 34 \overline{25} \overline{36} \\ 46 \overline{57} \overline{48} \\ 62 \overline{73} \overline{64} \end{array}$$

$$\begin{array}{r} 3. \\ 45 \overline{86} \overline{97} \\ 57 \overline{64} \overline{53} \\ 77 \overline{68} \overline{59} \\ 77 \overline{26} \overline{35} \\ 27 \overline{48} \overline{56} \\ 71 \overline{46} \overline{57} \\ 17 \overline{56} \overline{47} \\ 76 \overline{47} \overline{58} \\ 66 \overline{75} \overline{48} \\ 64 \overline{85} \overline{77} \end{array}$$

$$\begin{array}{r} 4. \\ 56 \overline{45} \overline{37} \\ 66 \overline{54} \overline{73} \\ 61 \overline{28} \overline{95} \\ 17 \overline{56} \overline{43} \\ 78 \overline{27} \overline{36} \\ 87 \overline{16} \overline{89} \\ 77 \overline{58} \overline{64} \\ 78 \overline{67} \overline{59} \\ 88 \overline{75} \overline{69} \\ 47 \overline{89} \overline{68} \end{array}$$

$$\begin{array}{r} 5. \\ 64 \overline{75} \overline{65} \\ 45 \overline{74} \overline{83} \\ 51 \overline{27} \overline{37} \\ 19 \overline{48} \overline{57} \\ 93 \overline{14} \overline{15} \\ 39 \overline{12} \overline{34} \\ 32 \overline{83} \overline{74} \\ 21 \overline{49} \overline{58} \\ 12 \overline{93} \overline{14} \\ 29 \overline{21} \overline{34} \end{array}$$

Add by groups of two figures:

6.	7.	8.	9.	10.
17 36 45	29 20 38	16 47 54	34 35 26	46 45 37
76 47 58	93 64 52	69 28 37	41 37 46	68 37 46
66 47 35	32 83 74	98 37 46	16 45 34	89 78 69
69 28 37	28 92 37	87 26 35	62 83 74	98 67 98
94 75 86	86 98 56	76 25 34	24 86 97	87 56 47
44 75 86	65 84 93	68 27 36	48 37 46	71 48 57
24 95 86	55 84 93	84 85 76	85 36 47	16 35 47
34 85 76	52 13 94	49 68 57	55 26 34	65 26 42
14 85 96	23 74 65	98 47 56	57 48 56	55 36 43
84 15 96	39 51 69	87 36 45	75 34 21	55 34 25

Practice on the following until the choice of a short method becomes natural and habitual. Total the answers in each group.

1.	2.	3.
41 × 527	146 × 3284	11 × 632
31 × 643	517 × 4682	11 × 589
17 × 489	641 × 5324	11 × 9246
81 × 938	814 × 7246	11 × 64,035
61 × 594	193 × 8924	11 × 700,472
16 × 743	581 × 9079	11 × 952,400

4.	5.	6.	7.
428 × 5234	749 × 5872	47 × 47	56 × 37
648 × 3245	856 × 4327	56 × 56	63 × 28
545 × 1234	279 × 3469	63 × 63	58 × 67
369 × 2336	654 × 4235	72 × 72	77 × 83
427 × 5465	819 × 4321	34 × 34	94 × 82
486 × 6285	729 × 2468	89 × 89	46 × 76

8.	9.	10.	11.
456 × 638	57 × 347	728 × 6072	476.55 ÷ 48
478 × 259	632 × 364	864 × 5937	342.67 ÷ 27
725 × 836	71 × 5483	842 × 8642	4863 ÷ 36
837 × 296	125 × 7288	981 × 6232	7472 ÷ 54
517 × 678	517 × 439	69 × 69	594.35 ÷ 42
278 × 527	11 × 8976	96 × 96	821.58 ÷ 63

12.

- $54.78624 \times 31.597642$ to nearest .01
 $536.546786 \times 4.86755432$ to nearest .01
 $5.64537 \times .14635342$ to nearest .01
 $234.54324 \times 31.534263$ to nearest .001
 $.4364537 \times .13234658$ to nearest .0001
 $14862.78 \times .0147263$ to nearest .01

13.

- $26.32452 \times .00587$ to nearest .0001
 $8241 \times .001234$ to nearest .001
 637×7427
 816×4526
 83×83
 250×4384

14.

- $\sqrt{\$1,000,000} \div 11.5017741$ to nearest cent
 $\$5,500,000 \div 3462.2534654$ to nearest cent
 $\$15,000 \div 375.431456$ to nearest cent
 $\$52,500 \div 645.6354765$ to nearest mill
 $\$56,750 \div 34342.53433$ to nearest mill
 $\$734.57 \div 2.768346$ to nearest mill

15.

- $5726 \div 81$
 $9432 \div 72$
 $17255 \div 85$
 $42.645 \div 56$
 $842.63 \div 112$
 $716.25 \div 125$

16.

- | | |
|-------------------|-----------------------------------|
| 4789.26 | } $\div 347.85$
to nearest .01 |
| 38472.50 | |
| 50847.65 | |
| 29784.35 | |
| 95483.50 | |
| $\sqrt{87407.08}$ | |

17.

- | | |
|--------------------------|-----------|
| Find, to the nearest | 547.25 |
| tenth per cent, what per | 16,472.31 |
| cent each of the follow- | 9,387.65 |
| ing numbers is of their | 96.24 |
| total: | 1,328.97 |

IV. GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE

GREATEST COMMON DIVISOR

The greatest common divisor of two or more numbers is the largest number that will exactly divide each of them.

Greatest common divisor by inspection

In many cases the G. C. D. of two or more numbers can be determined by inspection by observing the following principles :

PRINCIPLES. 1. The G. C. D. of two or more numbers cannot be greater than the smallest number in the list.

2. The G. C. D. of two or more numbers cannot be greater than the smallest difference between any two numbers in the list.

3. The G. C. D. of two or more numbers is either (a) the smallest number, (b) the smallest difference, or (c) a factor of one of the numbers.

ILLUSTRATIONS

1. Find the G. C. D. of 24, 72, 120, and 168.

By inspection it is seen that the smallest number, 24, will divide each of the several numbers, hence it is the G. C. D.

2. Find the G. C. D. of 70, 168, 224, and 252.

The smallest number is 70. The smallest difference is 28 ($252 - 224$). By inspection 28 is seen not to be a divisor of 70. $28 = 2 \times 14$. Since 28 is not a common divisor, either 2 or 14 must be eliminated. Since the greatest common divisor is sought, eliminate the 2. 14 is seen to be a divisor of each of the numbers. Hence 14 is the G. C. D.

If the G. C. D. cannot be determined by inspection, proceed as follows :

ILLUSTRATION

Find the G. C. D. of 188, 470, 705, and 940.

188	470	705	940	Divide all the other numbers by the smallest number, writing the remainders as shown in the solution. Thus, $470 \div 188$ has a remainder of 94, etc. Also "bring down" the "divisor." Divide again by the smallest number. By inspection, you now see that 47 is the G. C. D.
188	94	141	0	
0	94	47	0	
47 is the G. C. D.				

Find, by inspection when possible, the G. C. D. of the following:

- | | |
|------------------|---------------------------|
| 1. 912 and 1121 | 6. 961, 1271, and 372 |
| 2. 792 and 1016 | 7. 2167, 2561, and 985 |
| 3. 387 and 559 | 8. 48, 72, 120, and 288 |
| 4. 85 and 119 | 9. 6859, 11,191, and 3610 |
| 5. 2813 and 3589 | 10. 90, 108, 144, and 111 |

LEAST COMMON MULTIPLE

The least common mutiple of two or more numbers is the smallest number that will exactly contain each of them.

Least common multiple by inspection

In the majority of practical cases, the L. C. M. can be found by inspection by observing the following principles.

PRINCIPLES. 1. The L. C. M. of two or more numbers cannot be smaller than the largest number in the list.

2. The L. C. M. of two or more numbers is either (a) the largest number, or (b) a multiple of the largest number.

ILLUSTRATIONS

1. Find the L. C. M. of 9, 16, and 24.

24, the largest number, is not the L. C. M. Hence try 48, 72, 96, etc., until you find a number that is a common multiple. By trial you find it to be 144.

If the L. C. M. is not readily discovered by inspection, proceed as follows :

2. Find the L. C. M. of 6, 8, 12, 15, 20, 24, 30, and 36.

$$\begin{array}{r} 36 \overline{) 6 \quad 8 \quad 12 \quad 15 \quad 20 \quad 24 \quad 30 \quad 36} \\ \underline{ 5 \quad 2 \quad 5 \quad 1} \end{array}$$

$$36 \times 2 \times 5 = 360$$

Write all the numbers in one line. Cancel all the numbers that are contained in any of the other numbers. Select one of the remaining numbers as a divisor. (Use as divisor a number which has a factor common to itself and to one or more of the other numbers.) Use 36 in this case. The G. C. D. of 36 and 20 is 4. 20 divided by 4 is 5. Write 5. The G. C. D. of 36 and 24 is 12. 24 divided by 12 is 2. Write 2. Continue in the same manner through the list of numbers. You now obtain a new line of numbers. Proceed as at first. Cancel all numbers contained in some other number in the line. Divide as before, if necessary, and continue the process until the remaining numbers have no common factor except unity. The L. C. M. is the product of all the divisors and the remaining numbers.

Find, by inspection when possible, the L. C. M. of :

1. 2, 4, 6, 12, 15

6. 13, 15, 25, 65, 75

2. 3, 6, 9, 18, 30

7. 24, 36, 48, 42, 120

3. 5, 12, 15, 30, 40

8. 6, 20, 48, 100, 144

4. 8, 12, 24, 60, 84

9. 9, 16, 64, 80, 128

5. 15, 25, 36, 75, 100

10. 36, 45, 72, 105, 150

V. FRACTIONS

Fractions are discussed in this text chiefly for the purpose of showing some simple, easy methods of handling them.

The decimal equivalent of the everyday common business fractions should be memorized. Those equivalents are indicated in the discussion of Aliquot Parts, pages 9-21.

In reducing several fractions to equivalent fractions having a common denominator, apply the principles for finding the least common multiple of several numbers, as stated on page 53.

ILLUSTRATION

Reduce to their least common denominator, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{12}$, and $\frac{7}{16}$.

The L. C. D. cannot be less than 16.

The L. C. D. is not 16.

The L. C. D. is a multiple of 16.

$2 \times 16 = 32$. 32 is not the L. C. D.

$3 \times 16 = 48$. 48 is the L. C. D.

Therefore

$$\frac{1}{2}, \frac{2}{3}, \frac{5}{12}, \frac{7}{16} = \frac{24}{48}, \frac{32}{48}, \frac{20}{48}, \frac{21}{48}$$

Reduce to their least common denominator:

1. $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$

2. $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}, \frac{5}{12}$

3. $\frac{1}{4}, \frac{5}{8}, \frac{7}{12}, \frac{1}{9}$

4. $\frac{5}{8}, \frac{3}{4}, \frac{3}{16}, \frac{5}{12}, \frac{1}{2}$

5. $\frac{7}{16}, \frac{3}{4}, \frac{9}{10}, \frac{3}{20}, \frac{7}{8}$

ADDITION AND SUBTRACTION

Before fractions can be added or subtracted, they must have a common denominator. Many fractions may be added and subtracted mentally, that is, by inspection.

ILLUSTRATIONS

1. Add
- $\frac{1}{3}$
- and
- $\frac{1}{7}$
- .

Written Process

$$\frac{1}{3} + \frac{1}{7} = \frac{7}{21} + \frac{3}{21} = \frac{10}{21}$$

Inspection Process

$$1. \quad 7 + 3 = 10$$

$$2. \quad 7 \times 3 = 21$$

Therefore, $\frac{10}{21}$

The common denominator is 21. The new numerators are 7 and 3 respectively. The total is $\frac{10}{21}$.

Note in the process at the left that the new numerators are the same as the old denominators.

Therefore, when two fractions whose numerators are 1 are to be added, add the denominators for the new numerator, and multiply them together for the new denominator.

2. Add
- $\frac{2}{7}$
- and
- $\frac{2}{9}$
- .

Written Process

$$\frac{2}{7} + \frac{2}{9} = \frac{18}{63} + \frac{14}{63} = \frac{32}{63}$$

Inspection Process

$$(7 + 9) \times 2 = 32$$

$$7 \times 9 = 63$$

Therefore, $\frac{32}{63}$

In the written process, note that the new numerators are twice as large as the old denominators. Hence to find the sum of two fractions whose numerators are the same, add the old denominators and multiply the sum by one of the numerators for the new numerator; multiply the old denominators together for the new denominator.

3. Add
- $\frac{1}{4}$
- ,
- $\frac{1}{9}$
- , and
- $\frac{5}{12}$
- .

Written Process

$$\begin{aligned} \frac{1}{4} + \frac{1}{9} + \frac{5}{12} &= \frac{9}{36} + \frac{4}{36} + \frac{15}{36} \\ &= \frac{28}{36} = \frac{7}{9} \end{aligned}$$

Inspection Process

$$\frac{1}{4} + \frac{1}{9} = \frac{13}{36}$$

$$\frac{5}{12} = \frac{15}{36}$$

$$\frac{13}{36} + \frac{15}{36} = \frac{28}{36} = \frac{7}{9}$$

By the inspection method no writing is needed. It is entirely a mental operation.

In adding common and decimal fractions both must be reduced to the same common denominator before adding.

The student should drill carefully and thoroughly on additions like the following, till he can add simple fractions as readily as whole numbers.

4. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{9}{16}$.

To add these fractions, the student should "think" all operations, naming only the results. ("Think" the part in italics.) *The least common denominator is 16; the first numerator is 8; the second is 12, 20; the third is 10; 30, 39; $\frac{39}{16} = 2\frac{7}{16}$.*

5. Add $15\frac{1}{3}$, 18.234 , $12\frac{3}{4}$, and $9.156\frac{1}{4}$.

$$\begin{array}{rcl} 15\frac{1}{3} & = & 15.333\frac{1}{3} \\ 18.234 & = & 18.234 \\ 12\frac{3}{4} & = & 12.75 \\ 9.156\frac{1}{4} & = & 9.156\frac{1}{4} \\ & & \hline & & 55.473\frac{7}{12} \end{array}$$

If the decimals do not permit of ready reduction to common fractions, change the common fractions to decimals. Reduce each decimal to the same number of places, unless, as in the case of the $\frac{3}{4}$, which in decimal form extends to only two places, the remaining place, or places, would, if written, be zeros.

6. From $\frac{1}{3}$ take $\frac{1}{5}$.

Written Process

$$\frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = \frac{2}{15}$$

Inspection Process

$$5 - 3 = 2$$

$$5 \times 3 = 15$$

Therefore, $\frac{2}{15}$

In the written process, note that the new numerators are the old denominators in inverse order. Hence take the difference between the old denominators for the new numerator, and the product of the denominators for the new denominator.

7. From $\frac{2}{3}$ take $\frac{2}{7}$.

Written Process

$$\frac{2}{3} - \frac{2}{7} = \frac{14}{21} - \frac{6}{21} = \frac{8}{21}$$

Inspection Process

$$(7 - 3) \times 2 = 8$$

$$7 \times 3 = 21$$

Therefore, $\frac{8}{21}$

Drill Exercises

Add the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{12}$	$\frac{1}{11}$
$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{2}$

Add the following:

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{7}$
$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{9}$
$\frac{5}{6}$	$\frac{9}{20}$	$\frac{5}{6}$	$\frac{3}{14}$	$\frac{2}{6}$	$\frac{11}{12}$	$\frac{5}{27}$	$\frac{4}{15}$	$\frac{8}{45}$	$\frac{5}{21}$

Add the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{2}{7}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{7}$	$\frac{4}{5}$	$\frac{4}{9}$
$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{8}$	$\frac{2}{11}$	$\frac{2}{11}$	$\frac{3}{5}$	$\frac{3}{16}$	$\frac{3}{8}$	$\frac{4}{7}$	$\frac{4}{11}$
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{5}{8}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{6}{7}$	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{7}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
$\frac{5}{9}$	$\frac{5}{7}$	$\frac{6}{8}$	$\frac{6}{10}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{7}$
				$\frac{8}{21}$	$\frac{4}{15}$	$\frac{1}{21}$	$\frac{11}{15}$	$\frac{7}{20}$	$\frac{8}{35}$

Add the following:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{7}{9}$	$\frac{9}{12}$
$\frac{3}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{7}{8}$
$\frac{5}{12}$	$\frac{2}{3}$	$\frac{9}{16}$	$\frac{1}{10}$	$\frac{7}{20}$	$\frac{7}{12}$	$\frac{5}{6}$	$\frac{1}{2}$	$\frac{5}{18}$	$\frac{3}{8}$
						$\frac{1}{2}$	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{2}{3}$
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{7}{12}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{5}{8}$	$\frac{5}{16}$	$\frac{9}{10}$	$\frac{15}{32}$	$\frac{5}{18}$	$\frac{9}{16}$
$\frac{1}{2}$	$\frac{5}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{2}{3}$
$\frac{5}{6}$	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{9}{32}$	$\frac{5}{6}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{3}{4}$	$\frac{5}{12}$	$\frac{1}{6}$

Add (mentally, when possible):

- $\frac{2}{3} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8}$
- $5\frac{4}{5} + \frac{1}{3} + \frac{1}{2} + \frac{7}{10}$
- $\frac{5}{8} + \frac{9}{16} + .25 + .37\frac{1}{2}$
- $16\frac{3}{4} + 17\frac{2}{3} + 24\frac{5}{8} + 56.5$
- $47\frac{3}{5} + 96.2 + 143\frac{9}{10} + 54.25$
- $122.125\frac{1}{3} + 63\frac{1}{2} + 88.0625 + 15\frac{1}{6}$
- $49\frac{3}{4} + 72\frac{5}{8} + 99\frac{11}{16} + \frac{1}{2}$
- $51\frac{1}{4} + 64\frac{1}{2} + 48\frac{3}{4} + 54 + 50\frac{3}{4}$
- $32\frac{3}{16} + 29\frac{3}{8} + 19\frac{3}{4} + 54$
- $78.125 + 31.5 + 83.333\frac{1}{3} + .66\frac{2}{3}$

Subtract (mentally):

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{2}{7}$	$\frac{2}{5}$	$\frac{2}{7}$

Find (mentally, when possible) the value of:

- | | | |
|--|---|--|
| 1. $\frac{1}{2} + \frac{1}{3} - \frac{1}{6}$ | 13. $\frac{3}{4} + \frac{1}{3} - \frac{7}{12}$ | 25. $\frac{7}{8} - \frac{1}{2} + \frac{3}{4} - \frac{3}{8} + \frac{5}{16}$ |
| 2. $\frac{1}{3} + \frac{1}{4} - \frac{5}{12}$ | 14. $\frac{3}{5} + \frac{1}{2} - \frac{9}{10}$ | 26. $\frac{9}{16} - \frac{1}{2} + \frac{7}{8} - \frac{3}{16} + \frac{1}{4}$ |
| 3. $\frac{1}{3} + \frac{1}{5} - \frac{7}{15}$ | 15. $\frac{3}{4} + \frac{3}{8} - \frac{1}{2}$ | 27. $\frac{5}{6} + \frac{1}{3} - \frac{1}{2} - \frac{1}{6} + \frac{2}{3}$ |
| 4. $\frac{2}{3} + \frac{2}{7} - \frac{5}{21}$ | 16. $\frac{5}{6} + \frac{1}{2} - \frac{2}{3}$ | 28. $\frac{4}{9} + \frac{2}{3} - \frac{5}{6} + \frac{1}{2} - \frac{1}{3}$ |
| 5. $\frac{3}{4} + \frac{3}{5} - \frac{7}{20}$ | 17. $\frac{3}{8} + \frac{2}{3} - \frac{5}{12}$ | 29. $\frac{5}{8} + \frac{1}{4} + \frac{1}{2} - \frac{3}{8} - \frac{1}{2}$ |
| 6. $\frac{2}{7} + \frac{2}{9} - \frac{1}{3}$ | 18. $\frac{4}{5} + \frac{1}{3} - \frac{11}{15}$ | 30. $\frac{15}{16} - \frac{1}{2} - \frac{1}{4} + \frac{3}{8} + \frac{3}{4}$ |
| 7. $\frac{1}{3} + \frac{3}{8} - \frac{1}{12}$ | 19. $\frac{5}{8} + \frac{3}{4} - \frac{7}{8}$ | 31. $\frac{9}{10} - \frac{4}{5} + \frac{1}{2} + \frac{3}{4} - \frac{17}{20}$ |
| 8. $\frac{1}{4} + \frac{1}{5} - \frac{3}{10}$ | 20. $\frac{3}{5} + \frac{3}{7} - \frac{9}{35}$ | 32. $\frac{8}{9} - \frac{1}{6} - \frac{1}{3} + \frac{2}{18} - \frac{1}{4}$ |
| 9. $\frac{1}{8} + \frac{1}{9} - \frac{5}{24}$ | 21. $\frac{3}{5} + \frac{3}{8} - \frac{1}{2}$ | 33. $\frac{11}{12} - \frac{3}{4} + \frac{1}{6} + \frac{1}{3} - \frac{1}{2}$ |
| 10. $\frac{1}{4} + \frac{1}{7} - \frac{5}{14}$ | 22. $\frac{4}{7} + \frac{4}{11} - \frac{5}{7}$ | 34. $\frac{11}{16} + \frac{1}{4} - \frac{5}{8} + \frac{1}{2} - \frac{3}{4}$ |
| 11. $\frac{1}{2} + \frac{2}{3} - \frac{5}{6}$ | 23. $\frac{5}{8} + \frac{1}{2} - \frac{3}{4}$ | 35. $\frac{3}{4} - \frac{1}{2} + \frac{3}{8} + \frac{5}{16} - \frac{1}{4}$ |
| 12. $\frac{2}{3} + \frac{1}{4} - \frac{5}{12}$ | 24. $\frac{2}{3} + \frac{3}{4} - \frac{1}{2}$ | 36. $\frac{7}{8} + \frac{3}{4} - \frac{1}{2} - \frac{5}{16} + \frac{3}{16}$ |

MULTIPLICATION

ILLUSTRATIONS

1. Multiply $\frac{5}{8}$ by 4.

$$4 \times \frac{5}{8} = \frac{20}{8} = \frac{5}{2}$$

$$4 \times \frac{5}{8} = \frac{5}{2}$$

Both solutions are given for the purpose of illustration.

Theoretically the first solution is correct. 4 times 5 eighths is 20 eighths, just as 4 times 5 dollars is 20 dollars. In practice, the second solution is preferred in all cases in which the denominator is divisible by the integer.

2. Multiply $282\frac{3}{7}$ by 32.

$$\begin{array}{r} 282\frac{3}{7} \\ 32 \\ \hline 13\frac{5}{7} \\ 564 \\ \hline 846 \\ 9037\frac{5}{7} \end{array}$$

Do not change $282\frac{3}{7}$ to an improper fraction, but multiply "directly" by 32. $32 \times \frac{3}{7}$ should be a mental operation. Thus, $32 \times \frac{3}{7} = \frac{96}{7} = 13\frac{5}{7}$.

3. Multiply $57\frac{2}{3}$ by $27\frac{1}{2}$.

$$\begin{array}{r} 57\frac{2}{3} \\ 27\frac{1}{2} \\ \hline 1\frac{1}{3} \\ 28\frac{1}{2} \\ 18 \\ \hline 1539 \\ 1585\frac{5}{6} \end{array}$$

1. $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$
2. $\frac{1}{2}$ of 57 = $28\frac{1}{2}$
3. $27 \times \frac{2}{3} = 18$
4. $27 \times 57 = 1539$

This solution is known as the "four-step process."

The "four steps" are written out for illustration. They are not to be written for everyday work. Each "step" is a quick, easy, mental operation. Use cross multiplication in multiplying the integers. The addition of the fractions is a mental operation, as explained on page 56.

4. Multiply $43\frac{3}{4}$ by $21\frac{3}{4}$.

$$\begin{array}{r} 43\frac{3}{4} \\ 21\frac{3}{4} \\ \hline 48\frac{9}{16} \\ 903 \\ \hline 951\frac{9}{16} \end{array}$$

Thus,

1. $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$
2. $\left\{ \begin{array}{l} \frac{3}{4} \times 43 \\ \frac{3}{4} \times 21 \end{array} \right\} = \frac{3}{4} \text{ of } 64 = 48$
3. $21 \times 43 = 903$

In finding the product of two mixed numbers whose fractions are alike, the "four steps" may be reduced to three.

That is, for the second step, when the two fractions are alike, add the two integers and multiply their sum by one of the fractions.

5. Find the value of $47\frac{1}{4}$ yd. of lace at $19\frac{1}{4}$ ¢ a yard.

In exercises of this kind, requiring an answer in dollars and cents, a careful estimate will enable one to eliminate the fractions in the partial products. To write the fractions in full would give a result like the following:

$$\begin{array}{r} \$.19\frac{1}{4} \\ 47\frac{1}{4} \\ \hline 17 \\ 8\ 93 \\ \hline \$9.10 \end{array}$$

$$\begin{array}{r} \frac{1}{16} \\ 16\frac{1}{2} \\ \hline 893 \end{array}$$

From the standpoint of dollars and cents this is equivalent to writing 17 as illustrated. A little forethought of this kind will eliminate many unnecessary figures.

6. Multiply $6\frac{1}{2}$ by $6\frac{1}{2}$.

Long Method Short Method

$$\begin{array}{r} 6\frac{1}{2} \\ 6\frac{1}{2} \\ 3\frac{1}{4} \\ 3 \\ 36 \\ \hline 42\frac{1}{4} \end{array}$$

$$\begin{array}{r} 6\frac{1}{2} \\ 6\frac{1}{2} \\ \hline 42\frac{1}{4} \end{array}$$

In the *long method* note that the two cross products together equal 6. The multiplication of 6 by 6 plus the two cross products equals 7×6 .

Hence, the *short method*, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{4}$.
 $7 \times 6 = 42$.

This short method may be used in all cases in which the integers are alike and the sum of the fractions is 1.

7. Multiply $8\frac{1}{4}$ by $8\frac{3}{4}$.

Since the sum of the fractions is 1, and the integers are alike, the result may be written thus:

$$\begin{array}{r} 8\frac{1}{4} \\ 8\frac{3}{4} \\ \hline 72\frac{3}{16} \end{array}$$

$$\begin{array}{l} \frac{3}{4} \text{ of } \frac{1}{4} = \frac{3}{16} \\ 9 \times 8 = 72 \end{array}$$

8. Multiply $5.33\frac{1}{3}$ by $5.66\frac{2}{3}$.

$$\begin{array}{r} 5.33\frac{1}{3} \\ 5.66\frac{2}{3} \\ \hline 30.2222\frac{2}{9} = 30\frac{2}{9} \end{array}$$

Think $5\frac{2}{3}$ times $5\frac{1}{3}$
 $\frac{2}{3}$ of $\frac{1}{3} = \frac{2}{9} = .2222\frac{2}{9}$
 $6 \times 5 = 30$

9. Multiply 875 by 825.

$$\begin{array}{r} 875 \\ 825 \\ \hline 721,875 \end{array}$$

Imagine decimal points after each 8. Think $8\frac{1}{4} \times 8\frac{3}{4}$

$$\frac{1}{4} \text{ of } \frac{3}{4} = \frac{3}{16} = .1875$$

$$9 \times 8 = 72$$

Hence 721,875, without any decimal point.

Drill Exercises

Multiply :

- | | | |
|--------------------------------|--|--|
| 1. $16\frac{3}{5} \times 28$ | 11. $47\frac{7}{9} \times 86$ | 21. $56\frac{4}{9} \times 9\frac{3}{4}$ |
| 2. $18\frac{3}{4} \times 115$ | 12. $69\frac{3}{17} \times 85$ | 22. $4\frac{1}{2} \times 9\frac{3}{8}$ |
| 3. $14\frac{5}{12} \times 93$ | 13. $67 \times 4.13\frac{3}{8}$ | 23. $76\frac{3}{4} \times 12\frac{1}{2}$ |
| 4. $15\frac{6}{7} \times 99$ | 14. $87 \times .312\frac{1}{3}$ | 24. $89\frac{4}{5} \times 17\frac{1}{2}$ |
| 5. $24\frac{3}{14} \times 50$ | 15. $34 \times 4.37\frac{5}{6}$ | 25. $93\frac{3}{5} \times 27\frac{3}{4}$ |
| 6. $89\frac{1}{15} \times 160$ | 16. $15\frac{3}{8} \times 16\frac{1}{4}$ | 26. $87\frac{3}{8} \times 29\frac{7}{9}$ |
| 7. $39\frac{3}{11} \times 138$ | 17. $18\frac{3}{4} \times 26\frac{1}{2}$ | 27. $91\frac{1}{2} \times 41\frac{1}{3}$ |
| 8. $58\frac{4}{7} \times 44$ | 18. $27\frac{5}{6} \times 34\frac{3}{5}$ | 28. $47\frac{2}{3} \times 38\frac{5}{6}$ |
| 9. $91\frac{1}{16} \times 110$ | 19. $37\frac{3}{7} \times 29\frac{4}{7}$ | 29. $93\frac{3}{4} \times 86\frac{2}{3}$ |
| 10. $38\frac{3}{16} \times 92$ | 20. $38\frac{3}{4} \times 34\frac{1}{4}$ | 30. $74\frac{3}{4} \times 19\frac{1}{5}$ |

Fractions, when not expressed in full, are to be considered as fourths. Thus, $\$.07^3 = \$.07\frac{3}{4}$; $\$.05^2 = \$.05\frac{2}{4}$, that is, $\$.05\frac{1}{2}$, etc.

Write results by inspection :

- | | | |
|----------------------------|----------------------------|----------------------------|
| 1. 27^3 yd. at $\$.07^2$ | 2. 35^1 yd. at $\$.07^1$ | 3. 64^1 yd. at $\$.08^2$ |
| 31^1 yd. at $.08^3$ | 43^3 yd. at $.08^1$ | 76^2 yd. at $.07^3$ |
| 45^2 yd. at $.09^1$ | 72^1 yd. at $.05^2$ | 48^3 yd. at $.08^2$ |
| 56^3 yd. at $.07^3$ | 53^3 yd. at $.06^2$ | 39^2 yd. at $.04^3$ |
| 38^2 yd. at $.06^1$ | 58^2 yd. at $.04^3$ | 17^2 yd. at $.12^3$ |
| 51^2 yd. at $.07^3$ | 63^1 yd. at $.07^1$ | 19^3 yd. at $.13^3$ |
| 4. 72^3 yd. at $\$.10^1$ | 5. 21^3 yd. at $\$.07^2$ | 6. 44^1 yd. at $\$.08^1$ |
| 42^2 yd. at $.09^2$ | 34^3 yd. at $.06^3$ | 29^3 yd. at $.07^1$ |
| 63^1 yd. at $.07^2$ | 56^2 yd. at $.08^2$ | 64^2 yd. at $.07^3$ |
| 16^3 yd. at $.15^2$ | 47^1 yd. at $.09^1$ | 29^3 yd. at $.05^2$ |
| 22^2 yd. at $.18^3$ | 53^3 yd. at $.07^3$ | 12^2 yd. at $.26^1$ |
| 31^3 yd. at $.23^2$ | 29^1 yd. at $.07^1$ | 14^3 yd. at $.35^2$ |
| 7. 59^2 yd. at $\$.05^2$ | 8. 26^1 yd. at $\$.07^2$ | 9. 32^2 yd. at $\$.06^2$ |
| 39^3 yd. at $.09^3$ | 32^3 yd. at $.07^1$ | 54^1 yd. at $.07^1$ |
| 45^1 yd. at $.06^2$ | 49^2 yd. at $.08^3$ | 69^3 yd. at $.09^3$ |
| 73^2 yd. at $.08^2$ | 38^2 yd. at $.22^2$ | 57^1 yd. at $.07^1$ |
| 67^3 yd. at $.07^2$ | 59^3 yd. at $.33^2$ | 48^2 yd. at $.06^2$ |
| 83^1 yd. at $.09^1$ | 47^2 yd. at $.17^2$ | 27^3 yd. at $.05^3$ |

Multiply (mentally):

- | | | |
|-------------------------------------|---------------------------------------|--|
| 1. $8\frac{1}{2}$ by $8\frac{1}{2}$ | 6. $1\frac{1}{2}$ by $1\frac{1}{2}$ | 11. $5\frac{1}{2}$ by $5\frac{1}{2}$ |
| 2. $7\frac{1}{2}$ by $7\frac{1}{2}$ | 7. $10\frac{1}{2}$ by $10\frac{1}{2}$ | 12. 6.5 by 6.5 |
| 3. 3.5 by 3.5 | 8. 11.5 by 11.5 | 13. $9\frac{1}{2}$ by $9\frac{1}{2}$ |
| 4. 4.5 by 4.5 | 9. $13\frac{1}{2}$ by $13\frac{1}{2}$ | 14. $15\frac{1}{2}$ by $15\frac{1}{2}$ |
| 5. $2\frac{1}{2}$ by $2\frac{1}{2}$ | 10. 12.5 by 12.5 | 15. $20\frac{1}{2}$ by $20\frac{1}{2}$ |

Multiply (mentally):

- | | | |
|--|---|---|
| 1. $6\frac{1}{2} \times 10\frac{1}{2}$ | 6. $7\frac{1}{2} \times 9\frac{1}{2}$ | 11. $14\frac{1}{4} \times 8\frac{1}{4}$ |
| 2. $8\frac{1}{3} \times 7\frac{1}{3}$ | 7. 11.25×9.25 | 12. $5\frac{3}{5} \times 7\frac{3}{5}$ |
| 3. $5\frac{3}{4} \times 11\frac{3}{4}$ | 8. $7\frac{2}{3} \times 8\frac{2}{3}$ | 13. $12\frac{3}{4} \times 9\frac{3}{4}$ |
| 4. 8.25×4.25 | 9. $10\frac{1}{5} \times 15\frac{1}{5}$ | 14. $14\frac{1}{3} \times 7\frac{1}{3}$ |
| 5. $6\frac{1}{3} \times 9\frac{1}{3}$ | 10. 11.5×8.5 | 15. $15\frac{3}{7} \times 6\frac{3}{7}$ |

Make extensions by inspection and find the total for each group:

- | | | |
|---|---|-------------------------------|
| 1. | 2. | 3. |
| 65 crates at \$6.50 | $10\frac{1}{2}$ lb. at \$.75 | $12\frac{1}{2}$ A. at \$125 |
| 85 crates at 7.50 | $5\frac{1}{2}$ lb. at .65 | $15\frac{1}{2}$ A. at 155 |
| $4\frac{1}{2}$ bu. at .75 | $7\frac{1}{2}$ lb. at .07 $\frac{1}{2}$ | $16\frac{1}{2}$ A. at 85 |
| $9\frac{1}{2}$ bu. at .85 | $3\frac{1}{2}$ lb. at .15 $\frac{1}{2}$ | $20\frac{1}{2}$ A. at 75 |
| 4. | 5. | 6. |
| $5\frac{1}{4}$ yd. at \$.07 $\frac{1}{4}$ | $8\frac{3}{4}$ lb. at \$.08 $\frac{3}{4}$ | $15\frac{1}{2}$ bu. at \$1.50 |
| $9\frac{1}{4}$ yd. at .18 $\frac{1}{4}$ | $9\frac{3}{4}$ lb. at .11 $\frac{3}{4}$ | 350 lb. at \$1.50 per cwt. |
| $7\frac{1}{4}$ yd. at .09 $\frac{1}{4}$ | $5\frac{3}{4}$ lb. at 3.75 | 2500 ft. at \$18.50 per M. |
| $13\frac{1}{4}$ yd. at .07 $\frac{1}{4}$ | $12\frac{3}{4}$ lb. at 4.75 | 4500 ft. at \$ 6.50 per M. |

Write by inspection the results of the following indicated operations:

- | | | |
|---|---|--|
| 1. $7\frac{1}{3} \times 8\frac{2}{3}$ | 6. 11.4×11.6 | 11. $9.62\frac{1}{2} \times 9.37\frac{1}{2}$ |
| 2. $9\frac{1}{4} \times 9\frac{3}{4}$ | 7. $12\frac{3}{8} \times 12\frac{5}{8}$ | 12. $433\frac{1}{3} \times 466\frac{2}{3}$ |
| 3. $12\frac{1}{4} \times 12\frac{3}{4}$ | 8. 14.25×14.75 | 13. 975×925 |
| 4. $5\frac{1}{6} \times 5\frac{5}{6}$ | 9. $10.33\frac{1}{3} \times 10.66\frac{2}{3}$ | 14. 1325×1375 |
| 5. 6.3×6.7 | 10. $4\frac{3}{7} \times 4\frac{4}{7}$ | 15. $766\frac{2}{3} \times 733\frac{1}{3}$ |

DIVISION

Division is the reverse of multiplication. Hence the reverse operation is necessary.

ILLUSTRATIONS

1. Divide $\frac{8}{9}$ by 4.

$$\frac{8}{9} \div 4 = \frac{2}{9}$$

8 ninths divided by 4 gives 2 ninths, just as 8 dollars divided by 4 gives 2 dollars.

2. Divide $\frac{7}{8}$ by 4.

$$\frac{7}{8} \div 4 = \frac{7}{32}$$

Since the numerator, 7, is not exactly divisible by 4, the denominator is multiplied. That is, the size of each part (eighth) is diminished.

Compare these two operations with multiplication of a fraction by an integer, page 59.

3. Divide $86\frac{1}{4}$ by 7.

$$\begin{array}{r} 7 \overline{)86\frac{1}{4}} \\ \underline{12\frac{9}{8}} \end{array}$$

Divide "directly" by 7. Change the remainder to an improper fraction and divide as in the preceding exercise.

$$\begin{aligned} 2\frac{1}{4} &= \frac{9}{4} \\ \frac{9}{4} \div 7 &= \frac{9}{28} \end{aligned}$$

4. Divide 45 by $\frac{3}{5}$.

$$45 \div \frac{3}{5} = 75$$

Do not "invert the divisor and proceed as in multiplication" in simple exercises of this kind. Perform the *opposite* operation from multiplication.

$$45 \div 3 = 15$$

$$15 \times 5 = 75$$

5. Divide 55 by $\frac{3}{5}$.

$$55 \div \frac{3}{5} = 91\frac{2}{3}$$

If the numerator of the divisor is not exactly contained in the dividend, first multiply the dividend by the denominator of the divisor, and divide the result by the numerator, thus:

$$5 \times 55 = 275$$

$$275 \div 3 = 91\frac{2}{3}$$

Note the exact steps in the preceding exercises, and compare the operation with that of multiplication, page 59.

6. Divide 4782 by $\frac{5}{6}$.

5)4782

$$\begin{array}{r} 956\frac{2}{5} \\ \underline{5738\frac{2}{5}} \end{array}$$

If the divisor lacks only one fractional unit of being equal to unity, divide by the numerator of the divisor and add the result to the dividend.

This plan is equivalent to multiplying the quotient $956\frac{2}{5}$ by 6. 4782 is 5 times $956\frac{2}{5}$, hence adding $956\frac{2}{5}$ to 4782 gives a result equal to 6 times $956\frac{2}{5}$.

7. Divide 547 by $22\frac{3}{4}$.

$22\frac{3}{4}$) 547

$$\begin{array}{r} 4 \quad \quad 4 \\ \hline 24 \quad 24\frac{4}{91} \\ 91)2188 \\ \underline{182} \\ 368 \\ \underline{364} \\ \frac{4}{91} \end{array}$$

First, clear of fractions by multiplying both divisor and dividend by the denominator of the fraction in the divisor. Second, divide as in simple numbers.

8. Divide $183\frac{2}{3}$ by $27\frac{1}{4}$.

$27\frac{1}{4}$) $183\frac{2}{3}$

$$\begin{array}{r} 12 \quad \quad 12 \\ \hline 6 \quad 6\frac{24}{327} \\ 327)2204 \\ \underline{1962} \\ \frac{242}{327} \end{array}$$

First, clear of fractions by multiplying both divisor and dividend by the least common denominator of the two fractions. Second, divide as in simple numbers.

9. Divide 42.55 by 5.

$$\begin{array}{r} 5)42.55 \\ \hline 8.51 \end{array}$$

Divide as in simple numbers.

10. Divide 8.5 by .3125.

$$\begin{array}{r} 27.2 \\ \times 3125 \overline{)85000.0} \\ 6250 \\ \underline{22500} \\ 21875 \\ \underline{625} \\ 625 \end{array}$$

Move the decimal point in the divisor to the right of its right-hand digit. Move the decimal point in the dividend the *same* number of places to the right. Annex zeros when necessary. Divide as in simple numbers.

Drill Exercises

Find the value of:

- | | | |
|---------------------------|----------------------------|-------------------------------|
| 1. $\frac{24}{31} \div 6$ | 11. $\frac{31}{32} \div 6$ | 21. $18\frac{3}{4} \div 8$ |
| 2. $.3125 \div 25$ | 12. $\frac{43}{8} \div 7$ | 22. $26\frac{7}{8} \div 4$ |
| 3. $.738 \div 9$ | 13. $.125 \div 50$ | 23. $47\frac{5}{9} \div 2$ |
| 4. $.001 \div 100$ | 14. $.0125 \div 500$ | 24. $167\frac{8}{9} \div 21$ |
| 5. $.00128 \div 16$ | 15. $.8448 \div 8$ | 25. $568\frac{3}{7} \div 43$ |
| 6. $\frac{19}{21} \div 4$ | 16. $3\frac{3}{4} \div 5$ | 26. $828\frac{3}{11} \div 69$ |
| 7. $\frac{15}{26} \div 5$ | 17. $6\frac{2}{3} \div 10$ | 27. $939\frac{4}{15} \div 22$ |
| 8. $\frac{11}{4} \div 5$ | 18. $8\frac{3}{4} \div 7$ | 28. $896\frac{4}{5} \div 15$ |
| 9. $\frac{25}{6} \div 5$ | 19. $3\frac{2}{3} \div 7$ | 29. $948\frac{5}{8} \div 12$ |
| 10. $\frac{19}{8} \div 3$ | 20. $9\frac{4}{5} \div 14$ | 30. $776\frac{7}{9} \div 9$ |

Find the value of:

- | | | |
|---|---|--|
| 1. $16 \div \frac{5}{8}$ | 19. $864\frac{5}{8} \div 21\frac{1}{2}$ | 37. $21\frac{1}{3} \div 3\frac{1}{8}$ |
| 2. $28 \div \frac{4}{7}$ | 20. $438\frac{6}{7} \div 219\frac{3}{7}$ | 38. $56\frac{3}{4} \div 7\frac{1}{3}$ |
| 3. $36 \div \frac{12}{17}$ | 21. $644\frac{3}{8} \div 16\frac{1}{2}$ | 39. $92\frac{1}{2} \div 6\frac{2}{3}$ |
| 4. $89 \div \frac{2}{3}$ | 22. $849\frac{5}{9} \div 13\frac{2}{3}$ | 40. $84\frac{5}{8} \div 9\frac{1}{4}$ |
| 5. $56 \div \frac{6}{7}$ | 23. $674\frac{5}{9} \div 11\frac{1}{3}$ | 41. $71\frac{3}{4} \div 11\frac{1}{2}$ |
| 6. $38 \div \frac{19}{20}$ | 24. $84.96\frac{1}{2} \div 15\frac{1}{4}$ | 42. $63\frac{7}{8} \div 13\frac{2}{3}$ |
| 7. $42 \div \frac{16}{11}$ | 25. $60.83\frac{1}{3} \div 12\frac{1}{6}$ | 43. $73\frac{5}{8} \div 19\frac{1}{4}$ |
| 8. $32 \div \frac{16}{17}$ | 26. $.0482 \div .0002$ | 44. $123\frac{5}{16} \div 17\frac{5}{8}$ |
| 9. $48 \div \frac{9}{10}$ | 27. $59.68\frac{1}{2} \div .0005$ | 45. $231\frac{3}{7} \div 86\frac{1}{2}$ |
| 10. $64 \div \frac{7}{8}$ | 28. $11.25 \div .003\frac{3}{4}$ | 46. $475 \div 14.32796$ |
| 11. $978 \div 85\frac{1}{8}$ | 29. $131 \div .008\frac{1}{3}$ | 47. $689 \div 273.5487$ |
| 12. $125 \div 1\frac{2}{3}$ | 30. $45.75 \div .001\frac{2}{3}$ | 48. $934 \div 146.1407$ |
| 13. $250 \div 1.66\frac{2}{3}$ | 31. $31.25 \div 006\frac{1}{4}$ | 49. $1270 \div 56.38276$ |
| 14. $375 \div 3.33\frac{1}{3}$ | 32. $.583\frac{1}{3} \div 70$ | 50. $.0475 \div .025$ |
| 15. $964\frac{5}{8} \div 16\frac{1}{4}$ | 33. $500 \div 4.16\frac{2}{3}$ | 51. $.0078 \div 1950$ |
| 16. $963\frac{8}{9} \div 8\frac{2}{3}$ | 34. $16.875 \div 1\frac{1}{4}$ | 52. $1950 \div .0078$ |
| 17. $458\frac{3}{4} \div 19\frac{1}{2}$ | 35. $247.50 \div .06\frac{2}{3}$ | 53. $327\frac{1}{3} \div 76\frac{1}{2}$ |
| 18. $732\frac{3}{8} \div 19\frac{3}{4}$ | 36. $38.75 \div 3\frac{3}{4}$ | 54. $934\frac{1}{8} \div 56\frac{1}{4}$ |

VI. PAY ROLLS

The 44-hour week (8 hr. daily from Monday to Friday inclusive, and 4 hr. on Saturday) is common in many industries.

The scale of wages is based on an hourly or a daily rate. The day is thought of as beginning at 8 A.M. and ending at 5 P.M., with 1 hr. for luncheon, except on Saturday, when the day ends at 12 M.

Hours of labor between 8 A.M. and 5 P.M. (or between any other regularly prescribed hours beginning and ending the day of labor) are *regular time*. Hours of labor after 5 P.M. (or other prescribed closing hour) are generally considered *overtime*.

Overtime is usually paid for at higher rate than regular time. The extra wage may be a quarter or a half greater than, or even double, the regular wage. It is expressed as time and a half ($1\frac{1}{2} \times 60¢ = 90¢$), etc.

The following illustration will show how an employee's weekly wages are determined on the basis of a 44-hour week; rate 60¢ an hour, with time and a half allowed for overtime.

	M.	T.	W.	Th.	F.	S.	HOURS		RATE	WAGES		
							Reg. Time	Over-time		Reg. Time	Over-time	Total
Anson, John	8	9	10	5	7	4	40	3	60¢	24.00	2.70	26.70

Regular time and overtime are reckoned separately. The regular time for Tuesday and Wednesday is 8 hr. for each day. Hence the total regular time is 8 hr. + 8 hr. + 8 hr. + 5 hr. + 7 hr. + 4 hr., or 40 hr. The overtime is 1 hr. on Tuesday and 2 hr. on Wednesday, or a total of 3 hr. The rate (wage per hour) being 60¢, the regular wages amount to $40 \times 60¢$, or \$24.00. Since the regular rate is 60¢, the overtime rate is 90¢ an hour. The overtime wages amount to $3 \times 90¢$, or \$2.70. The total wages are \$26.70.

In many kinds of business the money due an employee for wages is put into a "pay envelope," and handed to him at the close of the week's work. This method necessitates, if there are many employees, having ready a large amount of money and the right kinds of bills and coins to put into the envelopes. The number and kinds of bills and coins required are determined by means of a "currency breakup"

or change sheet. For the wages in the preceding problem, together with the wages of several other employees, the currency breakup would appear as follows :

No.	Amt.	\$20	\$10	\$5	\$2	\$1	50¢	25¢	10¢	5¢	1¢
*1.	\$26.70	1		1		1	1		2		
2.	27.18	1		1	1				1	1	3
3.	31.23	1	1			1			2		3
4.	29.87	1		1	2		1	1	1		2
5.	34.69	1	1		2		1		1	1	4
	149.67	5	2	3	5	2	3	1	7	2	12
		100	20	15	10	2	1.50	.25	.70	.10	.12

When the correct amount of change required has thus been determined, the result is copied on a *currency memorandum*, and sent to the bank with a pay-roll check. In cashing the check, the paying teller provides the exact amount of change called for. The pay-roll clerk is then able to fill the pay envelopes without delay.

*Frequently the names as well as the numbers of the clerks are given, as shown in the form on page 69.

THE EXCHANGE BANK		
CURRENCY MEMORANDUM		
NEW YORK, <i>Sept. 9, 1923</i>		
DEPOSITOR <i>Walter Mahon</i>		
	DOLLARS	Cts.
Bills \$ 1		2
" 2		10
" 5		15
" 10		20
" 20		100
" 50		
" 100		
Coin: Pennies		12
" Nickels		10
" Dimes		70
" Quarters		25
" Halves		1 50
" Gold		
Total		149 67

No. 963

New York, Sept. 9, 1923

EXCHANGE BANK

Pay to the order of A. B. Brooks for Pay Roll \$149 $\frac{67}{100}$

One hundred forty-nine and $\frac{67}{100}$ ----- Dollars.

PAYABLE THROUGH THE NEW YORK CLEARING HOUSE

Walter Mahon

1. The following time sheet is based on a 44-hour week. Allow time and a half for overtime. Using the form shown on page 67 find the total wages for each employee. Prepare the currency breakup, a currency memorandum, and write the pay-roll check.

No.	M.	T.	W.	Th.	F.	S.	HOURS		RATE	WAGES		
							Regular Time	Over-time		Regular Time	Over-time	Total
1	8	7	7 $\frac{1}{2}$	6	9	4			\$.60			
2	7	8 $\frac{1}{2}$	6	9	9	5			.60			
3	9	10	7	6	7 $\frac{1}{2}$	6 $\frac{1}{2}$			.64			
4	8	8	7	8	8	6			.62			
5	9	7	6	4 $\frac{1}{2}$	8	5			.50			
6	10	8	7	9	8	4			.60			
7	9	7	8	6	9	4			.50			
8	8 $\frac{1}{2}$	7	6	9 $\frac{1}{2}$	8 $\frac{1}{2}$	7 $\frac{1}{2}$			.64			

2. In the following time sheet the rate is for an 8-hour day, with time and a quarter for overtime. Prepare the pay roll, currency memorandum, and check, using the foregoing forms.

No.	NAMES OF EMPLOYEES	M.	T.	W.	Th.	F.	S.	RATE
1	Addison, James	8	10	9	11	2	4	\$6.40
2	Buell, Henry	9	6	9	8 $\frac{1}{2}$	9	4	6.00
3	Crane, John	10	9 $\frac{1}{2}$	8	7	8	3	6.00
4	Crawford, William	9	9	8	6	10	4	6.00
5	Jones, Albert	10	8	9 $\frac{1}{2}$	9	0	4	6.00
6	Landis, Frank	8	4	9	8	10	4	5.00

3. Find total time “punched” on the following time card, and the wages for the week. Count entire time from time “in” to time “out.”

FORM No. 3

No. / 18

NAME William MacCormack

TOTAL TIME Hr. RATE, 55¢

TOTAL WAGES \$

WEEK ENDING January 7 19

IN	OUT	IN	OUT	IN	OUT
7.50	11.30	12.20	5.30		
		12.30	5.30	6.45	11.30
7.45	11.30	12.45	5.45		
7.55	12.15			6.10	11.15
		12.00	5.40	6.30	11.20
8.00	12.20	12.50	6.10		
8.30	12.15	1.30	6.10		

4. A gang of carpenters working under the 8-hour law, with a maximum day of 10 hours, and one half extra pay for overtime, worked one week as follows:

TIME SHEET—WEEK ENDING MARCH 18, 1922

NAMES	M.	T.	W.	Th.	F.	S.	TOTAL TIME	RATE PER DAY
Bigelow, A. E. (foreman)	8	8½	8	9	8	10		\$7.50
Arnold, A. W.	8	8	8	8	8	9		7.00
Babcock, C. D.	8	8	8	9	8	8		7.00
Comstock, L. O.	8	8	8	8½	8	9		7.00
Evans, M. R.	8	8	8	8½	8	8		7.00
Geraghty, P. O.	8	8½	8	8½	8	9		7.00
Morrow, G. W.	8	8½	8	8	8	8		7.00
Peters, A. B.	8	8½	8	8¼	8	8		6.00
Rogers, H. F.	8	8¼	8	8	8	9		6.00
Simler, H.	8	8	8	8	8	8		6.00
Watson, F. G.	8	8½	8	9	8	10		6.00
Whiteley, I.	8	8	8	8	8	10		6.00

Prepare a pay roll for the gang.

5. The following time card shows the time of an employee for one week. Regular time is from 8 A.M. to 12 M. and from 1 P.M. to 5 P.M., except on Saturday, when regular time is from 8 A.M. to 12 M. Time before 8 A.M. and 1 P.M. is not counted, as that time is wasted in reaching place of work after ringing up. Lateness up to 15 minutes counts as 15 minutes lost time. Lateness of from 16 minutes to 30 minutes counts as 30 minutes lost time, etc. Ringing out before 12 M. or before 5 P.M. is charged as lost time in quarter hours, that is, 1-15 minutes counts as 15 minutes, 16-30 minutes, as 30 minutes, etc. Overtime is not allowed unless overtime worked is a full 15, 30, 45 minutes, etc., and then only in whole quarter hours.

If the day rate is \$5.50 with time and a half for overtime, except on Saturday P.M. when overtime counts double, find the wages for the week.

	A.M.		P.M.	
	In	Out	In	Out
M.	7:56	12:02	12:55	5:06
T.	8:04	12:03	12:56	5:04
W.	7:57	12:04	1:05	5:40
Th.	8:20	12:16	1:18	4:48
F.	8:50	11:50	1:35	5:03
S.	7:49	12:05	12:55	3:35

PIECEWORK — DIFFERENTIAL RATE PLAN

Instead of working by the day or the hour, some employees are paid by the piece. In that case the wages vary in proportion to the amount of work done. To find the amount of daily or weekly wages we simply multiply the price per piece by the number of pieces of work completed.

In some factories the piecework plan is modified so that a rapid or skilled worker receives a higher price per piece than a slower or unskilled worker receives. It may have been found from experience that the completion of from 32 to 35 pieces is a standard or average day's work, and that the manufacturer can afford to pay 15¢ apiece for the labor. To provide a slow worker with tools and machinery, heat, light, and factory space in which to work, costs

just as much as it does to provide for a fast worker. Assume that the overhead expense averages \$1 a day for each employee. Then if one worker completes 25 articles a day, the overhead expense per article is 4¢; if he completes 33 articles a day, the overhead is 3¢ each; and if he completes 40 articles a day, the overhead is but 2½¢ each.

Therefore, to equalize cost of production per article, a "differential rate" is paid. A slow worker receives less, and a fast worker receives more, per piece.

1. Using the following schedule of rates, find the daily wages and the total wages for the week, for each employee:

PIECEWORK PAY ROLL

NAMES	NUMBER OF PIECES					
	M.	T.	W.	Th.	F.	S.*
Brown, John	33	31	34	28	36	20
Carey, Leon	28	30	27	32	33	18
Dunne, Theo.	24	26	29	27	30	15
Evers, Mark	34	36	40	38	39	21

SCHEDULE OF RATES

PIECES PER DAY	PRICE PER PIECE
20 to 23	13½¢
24 to 27	14¢
28 to 31	14½¢
32 to 35 (Standard)	15¢
36 to 39	15½¢
40 to 43	16¢
44 to 47	16½¢

*Four hours only. Use the rate for twice the number of pieces finished.

VII. HOW TO SOLVE PROBLEMS

Before a problem can be solved it must be interpreted. To interpret a problem requires a knowledge of the conditions set forth in the problem, a knowledge of the business customs or procedure surrounding it, together with a knowledge of the Fundamental Principles of Arithmetic.

Principles of Notation and Numeration

1. Each removal of any figure (except zero) one place to the left toward or from the decimal point multiplies its value by ten.
2. Each removal of any figure (except zero) one place to the right toward or from the decimal point divides its value by ten.
3. Each zero annexed to an integer multiplies its value by ten.
4. Each zero prefixed to a decimal divides its value by ten.

Principles of Addition

5. Only like numbers or parts of like numbers can be added.
6. The sum is composed of the same kind of units as are the addends.

Principles of Subtraction

7. Only like numbers or parts of like numbers can be subtracted.
8. The remainder is composed of the same kind of units as are the minuend and the subtrahend.
9. The minuend is equal to the sum of the subtrahend and the remainder.

Principles of Multiplication

10. The multiplicand may be either an abstract or a concrete number.
11. The multiplier is an abstract number.
12. The multiplicand and the product are like numbers.

13. The multiplier and the multiplicand may be interchanged without affecting the value of the product. (This principle is called the *commutative law* of multiplication.)

14. The product of two factors divided by either of them will give the other one. (If there are more than two factors in any product, the product divided by any one of the factors will give a quotient equal to the product of all the other factors.)

Principles of Division

15. The dividend is equal to the product of the divisor and the quotient (plus the remainder, if any).

16. The dividend and the remainder are like numbers.

17. Multiplying the divisor divides the quotient by the same number.

18. Dividing the divisor multiplies the quotient by the same number.

19. Multiplying the dividend multiplies the quotient by the same number.

20. Dividing the dividend divides the quotient by the same number.

21. Multiplying or dividing both divisor and dividend by the same number does not change the value of the quotient.

NOTE. In the number of practical applications, Principles 14 and 21 are the most important.

Very many problems are reducible to one or more of three type forms. These three type forms may be stated thus:

1. *To find a part of a number.*
2. *To find a number when a given part of it is known.*
3. *To find what part one number is of another.*

Each of these type forms may be expressed in equation form, thus:

1. $\frac{3}{8}$ of \$56 = ?

2. $\frac{5}{6}$ of ? = \$25

3. ? $\times$ (that is, what part of) \$72 = \$48

Each such equation consists of two (or more) factors and a product. The factors are always together on one side of the sign of equality. The product always stands alone on the opposite side of the equality sign.

The solution of these equations is a very simple matter. It follows, then, that if a problem can be stated in one or more of the preceding typical equation forms, its solution will be readily accomplished. Correct interpretation, therefore, will enable one to reduce a given problem to its appropriate type form. Problem statement is important.

EXAMPLES

1. A man's income for a given year was \$16,000. His expenses were \$12,800. What part of his income did he save?

Note the question: What part of his income did he save?

$$? \text{ of } \$16,000 = \$3200$$

$$\$3200 \div \$16,000 = \frac{1}{5}, \text{ part saved (Prin. 14, page 74)}$$

2. A druggist's sales in 1923 were \$32,500, which was $\frac{1}{4}$ more than in 1922. Determine the sales in 1922.

$$\frac{1}{4} \text{ of sales in 1922} = \text{sales in 1922}$$

$$\frac{1}{4} \text{ of sales in 1922} = \text{increase}$$

$$\frac{5}{4} \text{ of sales in 1922} = \text{sales in 1923}$$

$$\frac{5}{4} \text{ of ?} = \$32,500$$

$$\$32,500 \div \frac{5}{4} = \$26,000, \text{ sales in 1922}$$

Principles 5 and 14 govern in this problem. The basis of comparison is the sales for 1922, which represents unity, as indicated by the equation

$$\frac{1}{4} \text{ of sales in 1922} = \text{sales in 1922.}$$

The increase in sales was $\frac{1}{4}$ of sales in 1922. The sales in 1923 were equal to the sum of the sales in 1922 and the increase, hence the expression

$$\frac{5}{4} \text{ of sales in 1922} = \text{sales in 1923.}$$

Substituting \$32,500 for "sales in 1923" and a question mark for the unknown value gives the equation

$$\frac{5}{4} \text{ of ?} = \$32,500.$$

Dividing the product by the known factor gives the value of the unknown factor.

3. If green coffee costs $13\frac{1}{2}\text{¢}$ a pound and it shrinks $\frac{1}{6}$ of its weight when roasted, find the cost of a sufficient quantity of green coffee to make 1 pound of roasted coffee.

$\frac{5}{6}$ of a pound costs $13\frac{1}{2}\text{¢}$

$\frac{1}{6}$ of a pound = shrinkage in weight

$\frac{5}{6}$ of a pound costs $13\frac{1}{2}\text{¢}$

$\frac{5}{6}$ of ? = $13\frac{1}{2}\text{¢}$

$13\frac{1}{2}\text{¢} \div \frac{5}{6} = 16.2\text{¢}$, cost of green coffee to make 1 lb. of
roasted coffee (Prin. 14, page 74)

A loss of $\frac{1}{6}$ of the weight of the coffee does not mean an increase of $\frac{1}{6}$ in the cost. The cost of a pound of roasted coffee is $\frac{1}{5}$ greater than the cost of a pound of green coffee.

4. In two years a merchant's profits were \$34,000. The first year's profits were $\frac{1}{5}$ less than the second year's profits. Find the profits for each year.

$\frac{3}{5}$ of second year's profits = second year's profits

$\frac{3}{5}$ of second year's profits = first year's profits

$\frac{1}{5}$ of second year's profits = both years' profits

$\frac{1}{5}$ of ? = \$34,000

$\$34,000 \div \frac{1}{5} = \$170,000$, second year's profits (Prin. 14, page 74)

$\frac{3}{5}$ of \$170,000 = \$102,000, first year's profits

In this problem note that there are two values, one of which is compared with the other. One of the values must represent unity, as shown in the solution.

Problems

✓ 1. The profits of a given business for 1923 exceeded the profits for 1922 by $\frac{9}{40}$. The profits for the two years were \$4838.93. Find the profits for each year.

✓ 2. A hatter marked his straw hats down $\frac{1}{3}$ to clear up his stock. He then made a profit of $\frac{1}{6}$ of the selling price of a hat that cost him \$1.25. At what price did he sell the hats before marking them down?

3. On goods sold at \$3.65 a profit of $\frac{1}{4}$ of the cost was made. Find the cost.

4. The gross cost of a manufactured article was \$5.40. Labor and other manufacturing costs were $\frac{1}{4}$ greater than the cost of the raw material. Find the cost of the raw material.

5. Find the cost of an article that sells for \$90 if $\frac{1}{9}$ of the cost is gained.

6. An article that cost \$18.60 was sold so as to make a gross selling profit of $\frac{1}{4}$ of the sales. For how much was it sold?

7. If woolen cloth shrinks $\frac{1}{20}$ of its length when sponged, what should be the selling price of sponged cloth that cost \$2.85 a yard before it was sponged, if a gross selling profit of $\frac{1}{4}$ of the cost is made?

8. Two men have together \$1875. One has $2\frac{1}{3}$ times as much as the other. How much has each?

9. A merchant has a 1-ton truck, a 3-ton truck, and a 5-ton truck. A car load of merchandise has to be transferred from the car to his warehouse. With the 1-ton truck it can be removed in 12 hours; with the 3-ton truck, in 8 hours; and with the 5-ton truck, in 6 hours. In what time can it be removed if all three trucks are used?

10. A contractor agrees to perform a piece of work requiring 28 men in 30 days (exclusive of Sundays). After working 15 days the men strike and are idle 8 days. How many additional men must be employed to complete the work in the contract time?

11. A suit, an overcoat, and a hat together cost \$120. The hat cost $\frac{1}{7}$ as much as the overcoat, and the suit as much as the hat and overcoat combined. Find the cost of each.

12. A contract calls for the completion of a piece of work in 45 days. It requires the labor of 48 men to carry out the contract. During the first 40 days 8 of the men are ill and unable to work more than an average of 5 days each. How many additional men must be employed to comply with the terms of the contract?

VIII. PERCENTAGE

The term *percentage* is used to include those subjects in business mathematics in which the ratio of values is expressed in terms per cent. The expression *per cent* is of Latin origin and signifies hundredths. The per cent sign (%) is generally used in place of the words "per cent."

Percentage involves no new principles of mathematics. It is, therefore, merely a continuation, or extension, of the work in fractions. The term *per cent* is used as a convenience. Thus, it is more convenient to say and write $6\frac{1}{2}\%$ than to say or write either $.06\frac{1}{2}$ or $\frac{13}{200}$, the decimal and common fraction equivalents of $6\frac{1}{2}\%$.

Either of the foregoing forms is likely to occur in problem work and one should readily recognize either form and be able to convert it into any desired form.

ILLUSTRATIONS

$$\begin{aligned}\frac{1}{40} &= .025 = 2\frac{1}{2}\% \\ \frac{1}{8} &= .125 = 12\frac{1}{2}\% \\ \frac{3}{16} &= .1875 = 18\frac{3}{4}\% \\ \frac{7}{12} &= .5833\frac{1}{3} = 58\frac{1}{3}\%\end{aligned}$$

Write each of the following fractions in both its decimal and its per cent form :

- | | | | |
|------------------|------------------|-------------------|---------------------|
| 1. $\frac{1}{4}$ | 4. $\frac{5}{8}$ | 7. $\frac{1}{60}$ | 10. $\frac{7}{16}$ |
| 2. $\frac{1}{3}$ | 5. $\frac{2}{7}$ | 8. $\frac{1}{80}$ | 11. $\frac{2}{15}$ |
| 3. $\frac{3}{8}$ | 6. $\frac{1}{9}$ | 9. $\frac{3}{20}$ | 12. $\frac{11}{16}$ |

Write each of the following per cents in both its decimal and its common fraction form :

- | | | | |
|----------------------|----------------------|-------------|--------------|
| 13. $\frac{1}{8}\%$ | 16. $.5\%$ | 19. 125% | 22. 3.5% |
| 14. $\frac{3}{10}\%$ | 17. $\frac{1}{2}\%$ | 20. 75% | 23. 1.25% |
| 15. $\frac{1}{20}\%$ | 18. $1\frac{1}{4}\%$ | 21. $.75\%$ | 24. 8% |

In practice, it is generally advantageous to use the common fraction equivalents of those rates that are convenient aliquot and aliquant parts of 100%. The principles of aliquot parts and their applications as explained in pages 9 to 40 apply just as well in percentage as in reckoning costs, etc.

Drill Exercises

Performing all operations mentally, find :

- | | | |
|------------------------------|-----------------------------|------------------------------|
| 1. 21% of 200 | 6. 125% of 960 | 11. $\frac{3}{4}\%$ of 84 |
| 2. 35% of 500 | 7. 80% of 1200 | 12. $1\frac{1}{2}\%$ of 64 |
| 3. $12\frac{1}{2}\%$ of 640 | 8. 75% of 1500 | 13. 150% of 630 |
| 4. 26% of 700 | 9. $2\frac{1}{2}\%$ of 1600 | 14. $3\frac{1}{3}\%$ of 7230 |
| 5. $87\frac{1}{2}\%$ of 1200 | 10. $1\frac{1}{4}\%$ of 888 | 15. $\frac{1}{10}\%$ of 8400 |

SHORT METHODS

EXAMPLE

Find 28% of \$7500.

$$\frac{3}{4} \text{ of } \$2800 = \$2100$$

By the commutative law of multiplication, the product of 28 times 75 equals the product of 75 times 28. Therefore 28% of \$7500 equals 75% of \$2800. $75\% = \frac{3}{4}$, and $\frac{3}{4}$ of \$2800 = \$2100.

In like manner find, mentally, the value of each of the following :

- | | | |
|-------------------------------|-------------------|-----------------------------------|
| 1. 16% of \$1250 | 6. 180% of \$750 | 11. 39% of \$666 $\frac{2}{3}$ |
| 2. 18% of \$333 $\frac{1}{3}$ | 7. 32% of \$375 | 12. 42% of \$166.66 $\frac{2}{3}$ |
| 3. 26% of \$2500 | 8. 36% of \$75 | 13. 132% of \$2500 |
| 4. 44% of \$2500 | 9. 240% of \$2500 | 14. 12% of \$25 |
| 5. 56% of \$5000 | 10. 320% of \$625 | 15. 48% of \$375 |

EXAMPLES

1. A man's income is \$1800 a year. If he saves \$630 a year, what per cent of his income does he save?

$$\begin{aligned} ? \% \text{ of } \$1800 &= \$630 \\ \$630 \div \$1800 &= .35 = 35\% \end{aligned}$$

The problem is of the third type form, as shown in the first equation. Dividing the percentage, \$630, by the base, \$1800, gives the rate, 35%.

2. \$1500 is what per cent greater than \$1200?

\$1500 - \$1200 = \$300, amount which \$1500 is greater than \$1200

\$300 = ? % of \$1200

$$\$300 \div \$1200 = \frac{300}{1200} = \frac{1}{4} = 25\%$$

First find how much \$1500 is greater than \$1200. The problem then becomes, "\$300 is what per cent of \$1200?" which, as shown in the solution, is 25%.

Reduce each problem (either mentally or in writing) to its proper type form. Its solution will then be entirely clear to you.

Use the fractional form of division, when possible; reduce the fraction to its lowest terms mentally; and place the result equal to its equivalent rate per cent.

Solve, mentally when possible, each of the following:

What per cent of

- | | | |
|----------------|--------------------|----------------------|
| 1. 12 is 6? | 5. 25 is 30? | 9. 2 bu. is 3 pk.? |
| 2. 18 is 12? | 6. 14 is 21? | 10. \$24 is \$2.40? |
| 3. 200 is 150? | 7. 450 is 225? | 11. \$1.75 is \$.75? |
| 4. 72 is 56? | 8. 4 yd. is 6 ft.? | 12. \$15 is \$.90? |

What per cent greater than

- | | |
|-----------------|------------------------------------|
| 13. 54 is 63? | 17. \$3000 is \$4200? |
| 14. 300 is 390? | 18. 3 pk. is 1 bu.? |
| 15. 560 is 630? | ✓ 19. 1 lb. Troy is 1 lb. avoird.? |
| 16. 270 is 300? | 20. 1 oz. avoird. is 1 oz. Troy? |

What per cent less than

- | | |
|---------------------|--------------------------------------|
| 21. 20 is 18? | 25. 1 oz. Troy is 1 silver dollar? |
| 22. 30 is 20? | ✓ 26. A long ton is a short ton? |
| 23. 50 is 40? | 27. $\frac{1}{2}$ is $\frac{1}{3}$? |
| 24. 1 bu. is 3 pk.? | 28. $\frac{1}{3}$ is $\frac{1}{4}$? |

EXAMPLES

1. The English pound sterling weighs 123.2744 grains. It is $91\frac{2}{3}\%$ fine (that is, pure gold). How many grains of fine gold does it contain?

$$91\frac{2}{3}\% \text{ of } 123.2744 \text{ gr.} = 113.0016 \text{ gr.}$$

Calculation

$$\begin{array}{r}
 91\frac{2}{3}\% = \frac{11}{12} \quad \text{Note the use of aliquot parts. Since the pound is} \\
 12)123.2744 \quad \frac{11}{12} \text{ fine, it is } \frac{1}{12} \text{ alloy. Hence deduct } \frac{1}{12} \text{ of total weight} \\
 \underline{10.2728} \quad \text{to find the weight of fine gold.} \\
 113.0016
 \end{array}$$

2. The nautical unit of length is the knot of 6080 ft. A fathom is .1% of a knot. A fathom is what per cent greater than 6 ft.?

$$.1\% \text{ of } 6080 \text{ ft.} = 6.08 \text{ ft., 1 fathom}$$

$$6.08 \text{ ft.} - 6 \text{ ft.} = .08 \text{ ft., difference}$$

$$? \% \text{ of } 6 \text{ ft.} = .08 \text{ ft.}$$

$$.08 \text{ ft.} \div 6 \text{ ft.} = .01\frac{1}{3} = 1\frac{1}{3}\% \text{ greater than } 6 \text{ ft.}$$

Having found the difference between 6 ft. and .1 % of 6080 ft., the question is "What per cent of 6 ft. is .08 ft.?" Solving the equation gives $1\frac{1}{3}\%$.

3. In a recent year 68,274,000 lb. of imported salt was used in the United States. This amount was .48% of the total consumption of salt in that year. Find how many tons of domestic salt were used in the same year.

$$.48\% \text{ of total salt} = 68,274,000 \text{ lb.}$$

$$68,274,000 \text{ lb.} \div .0048 = 14,223,750,000 \text{ lb., total}$$

$$14,223,750,000 \text{ lb.} - 68,274,000 \text{ lb.} = 14,155,476,000 \text{ lb. of domestic salt}$$

$$14,155,476,000 \text{ lb.} \div 2000 \text{ lb.} = 7,077,738, \text{ number of tons of domestic salt used}$$

In the preceding three solutions are illustrated the three type forms of problems. In practical work it is not desirable to *write* out the typical equation form as shown in these solutions.

A mental conception of the equation is helpful in making accurate solutions.

Problems

1. In a recent year there were employed in the United States in manufactures and mines 7,036,337 wage earners. Of this number 18.8% worked in establishments employing from 101 to 250 wage earners; 15.3% in establishments employing from 251 to 500 wage earners; 13.2% in establishments employing from 501 to 1000 wage

earners; and 17.8% in establishments employing over 1000 wage earners. How many wage earners were employed in each of the four classes of establishments?

✓ 2. Since 1849 the production of butter in the United States has increased from 157,000 tons to 1,120,000 tons a year. In the same period of time the population of the United States has increased from 23,000,000 to 105,000,000. Compare the percentage of increase, showing the relation by a bar graph.

3. During a recent fiscal year there passed through the Panama Canal freight as follows: from Atlantic to Pacific, 2,705,091 tons, and from Pacific to Atlantic, 4,172,558 tons. What per cent of the total passed each way?

✓ 4. American exports for a given year carried in American vessels amounted to 2596 million dollars, and in foreign vessels, to 4494 million dollars. What per cent was carried in American vessels?

5. The Department of Agriculture reported 251,000 fewer sheep in 1919 than in 1918, or a decrease of .514%. How many sheep were reported each year?

6. 21.3% of all wage earners of the United States employed in manufacturing industries were employed in textile manufactures in a recent year. The number engaged in textile manufactures was 1,498,664. Find the total number of wage earners in all manufacturing industries.

7. A manufacturer's output one year was sold for \$450,000, which was at a profit of 25% of the cost. By the use of improved methods and machinery the following year, he was able to increase his production 25%, at an additional cost of only 5%. What per cent of discount could he afford to give in order to make the same rate of profit as in the preceding year?

8. A manufactured article was sold for \$16.80. The raw material in it cost \$4.20; labor cost \$7.50; and overhead expense was \$.90. If the cost of raw material decreased 15%, the cost of labor decreased

10%, and the overhead remained the same, at what price should the article be sold to maintain the same rate of profit as before?

✓ 9. A merchant's sales for October were \$28,460.50, for November, \$35,827.43, and for December, \$46,921.07. If the sales for these three months were 35% of the sales for the entire year, find the average monthly sales for the remainder of the year.

✓ 10. Of a consignment of eggs 6% were broken. For the remainder \$45.12 was received at 32¢ a dozen. How many dozen eggs were in the consignment?

✓ 11. A druggist's sales increased 15% the second year, 20% the third year, and 25% the fourth year. If the fourth year's sales amounted to \$29,023.13, how much were the sales the first year?

12. A merchant's sales increased $16\frac{2}{3}\%$ the second year, 20% the third year, 25% the fourth year, and $14\frac{2}{7}\%$ the fifth year. The sales for the five years amounted to \$213,244.25. How much were the sales the first year?

TRADE DISCOUNT

In many kinds of business it is the custom of manufacturers and merchants to have fixed prices for their several kinds of merchandise. This fixed price is called the **list** or **catalogue price**. The list price is higher than the market price. To adjust the list price to meet market conditions, discounts are allowed the purchaser. Such discounts are called **trade**, or **commercial**, discounts.

An increase in the market price results in a smaller discount, while a decline in the market price increases the discount. If an article were listed at \$48, and the market or selling price were \$36, the article would then be sold at \$48 less 25%. If the market price should decline to \$30, an additional discount of $16\frac{2}{3}\%$ would be allowed. The article would then be sold at \$48 less 25% and $16\frac{2}{3}\%$.

When two or more discounts are deducted from a given list price they form a discount series. They are, however, generally referred to as a discount. It is to be noted in a discount series that each discount rate is deducted separately — that is, the several discount rates cannot be added together. The second discount is to be deducted

from what is left after the first discount is deducted; the third is deducted from what is left after the second is taken off; and so on.

EXAMPLES

1. The list price of an article is \$60. The discount is 25%, 20%, and 10%. Find the net price.

4	\$60	
	15	= 25% discount
5	45	= balance after deducting 1st discount
	9	= 20% discount
10	36	= balance after deducting 2d discount
	3.60	= 10% discount
	<u>\$32.40</u>	= net price

The order in which the discounts are deducted is immaterial. It is sometimes advantageous to deduct the second or third discount first. Compare the following two solutions:

2. Find the net cost of goods listed at \$40 if sold subject to a discount of $33\frac{1}{3}\%$, 25%, and 20%.

(a)

3	\$40	
	13.33	= $33\frac{1}{3}\%$ discount
4	26.67	
	6.67	= 25% discount
5	20.00	
	4.00	= 20% discount
	<u>\$16.00</u>	= net cost

(b)

4	\$40	
	10	= 25% discount
3	30	
	10	= $33\frac{1}{3}\%$ discount
5	20	
	4	= 20% discount
	<u>\$16</u>	= net cost

With the prevalence of calculating machines a table of net cost rate factors or of single discounts equal to a series is of importance.

A **net cost rate factor** is the ratio of the net cost to the list price. Assume a discount series of 20%, $12\frac{1}{2}\%$, and 10%.

$$\begin{array}{ccc} 100\% & 100\% & 100\% \\ \hline 20\% & 12\frac{1}{2}\% & 10\% \\ \hline 80\% \times 87\frac{1}{2}\% \times 90\% & = & 63\% \text{ N. C. R. F.} \end{array}$$

$$\text{Or, } \frac{8}{10} \times \frac{7}{8} \times \frac{9}{10} = \frac{63}{100} = 63\% \text{ N. C. R. F.}$$

Deduct each rate separately from 100%, and multiply the several remainders together. The product is the net cost rate factor. That is, the net cost is 63% of the list price.

	100%
	20% = 20% of 100%
8	80%
	10% = 12½% of 80%
10	70%
	7% = 10% of 70%
	63% = N. C. R. F.

The net cost rate factor may also be found by deducting the several rates successively from 100% as illustrated.

The single discount equal to a given series is the difference between 100% and the net cost rate factor. In the preceding series the equivalent single discount is 100% - 63%, or 37%.

The office worker should have a table of net cost rate factors or single discounts covering the most common series of discounts. Such tables are arranged on the following plan:

RATES %	10	12½	16⅔	20	25	33⅓
Net		.87500				
2½	.87750					
5						
5, 2½				.74100		
10						
10, 5					.64125	
20						
20, 5			.63333			
20, 10						
20, 10, 5						.45600

A discount series is composed of the rate at the head of a given column and the rate or rates at the left of a given line. Thus, the rate 16⅔% at the top of the third column and the rates 20% and 5% at the left, form a series of 16⅔%, 20%, and 5%, whose net cost rate factor is .63333, or 63.333%.

Verify the several net cost rate factors given and complete the table. (Give results in nearest fifth decimal place.)

Drill Exercise

Find the net price of each of the following :

LIST PRICE	DISCOUNT	LIST PRICE	DISCOUNT	LIST PRICE	DISCOUNT
1. \$45	$1\frac{1}{5}\%$	6. \$60	20 % and $16\frac{2}{3}\%$	11. \$240	20 % and $12\frac{1}{2}\%$
2. 72	$8\frac{1}{3}\%$	7. 75	20 % and $33\frac{1}{3}\%$	12. 540	50 % and 20 %
3. 50	4 %	8. 80	25 % and 10 %	13. 840	$37\frac{1}{2}\%$ and 20 %
4. 60	3 %	9. 120	$33\frac{1}{3}\%$ and 25 %	14. 960	40 % and $16\frac{2}{3}\%$
5. 75	8 %	10. 150	20 % and 10 %	15. 1200	50 % and 25 %

EXAMPLES

1. Note the application of trade discount in the following bill :

1 Thermometer		1.45	
2 Altitude Gauges	1.75	3.50	
12 Positive Key Air Valves	.15	1.80	
	Net		6.75
2 6-gal. Galv. Expansion tanks	13.50	27.00	
6 $1\frac{1}{4}$ " Union Elbows	4.75	28.50	
		55.50	
	$33\frac{1}{3}/20/5\%$		28.12
6 2" Water Rad. Valves	9.50	57.00	
4 $1\frac{1}{2}$ " Water Rad. Valves	7.50	30.00	
		87.00	
	25/20/10%		46.98
			81.85

If one uses a table of net cost rate factors, the net amount is found rather than the discount. Thus, in the second group of items in the preceding exercise, the net cost rate factor corresponding to the discount $33\frac{1}{3}\%$, 20%, and 5% is .50667. 50.667% of \$55.50 gives \$28.12, the net cost of those two items. The discount is not shown, unless for some particular reason it is desired to show it.

2. An article is to be sold for \$33.75. At what price should it be listed so as to allow a discount of $16\frac{2}{3}\%$ and 10%?

The net cost rate factor corresponding to $16\frac{2}{3}\%$ and 10% is 75%.

Hence 75% of list price = \$33.75

$\$33.75 \div .75 = \$45$, list price

Problems

Make out a bill for each of the following purchases :

✓ 1. A. C. Holt, Cincinnati, Ohio, bought of the Ames Hardware Company, Rochester, N. H., Sept. 1, 1923, on account 60 da., 3% 10 da. :

24 doz. C. S. Axes	@ \$10
12 doz. Hand Saws	@ \$14
Less 20%	
8 doz. Wrenches	@ \$12
Less 25% and 10%	

2. H. L. Williams, Paterson, N. J., bought of Warden Holt, Chicago, Ill., Dec. 14, 1923 on account 30 da., 1% 10 da. :

8 Wilton Rugs, 8' 3" × 10' 6"	@ \$35
6 Velvet Rugs 9' × 12'	@ \$45
210 yd. Brussels Carpet	@ \$ 1.25

A discount of $16\frac{2}{3}\%$ and 10% was allowed on the entire bill. Find the amount due Dec. 24, 1923.

3. B. S. Pelham and Company, Wheeling, W. Va., bought of Woodward Grocery Company, Norfolk, Va., Dec. 1, 1923, on account 90 da., 2% 30 da., 5% 10 da. :

20 mats Java Coffee, 1500 lb.	@ 25¢
30 bags Maracaibo Coffee, 3750 lb.	@ 20¢
30 bags Rio Coffee, 3750 lb.	@ 15¢

A discount of 15% and 10% was allowed on the above.

25 half-chests Japan Tea, 1875 lb.	@ 25¢
30 half-chests Oolong Tea, 1500 lb.	@ 45¢
25 half-chests Eng. Breakfast Tea, 1500 lb.	@ 30¢

A discount of 20% and 10% was allowed on the tea. Find the net amount of the bill if it was paid on Dec. 11, 1923.

4. A. J. Felton, New York, bought of Enterprise Shoe Company, Boston, Mass., Sept. 18, 1923, on account 30 da., 5% cash :

100 cases Patent Calf, 3600 pr.	@ \$2.50
80 cases Tan Calf, 2880 pr.	@ 2.25
60 cases Kangaroo, 2160 pr.	@ 2.25
120 cases French Kid, 4320 pr.	@ 3.50

If a discount of 25% and $12\frac{1}{2}\%$ was allowed on the entire bill, find the amount required to pay the bill at once. Receipt the bill.

5. At what price shall each of the items in the following bill be sold to gain 25%?

Boston, Mass., Aug. 2, 1923

Mr. James Ryan,
Pittsfield, Mass.

Bought of ANSEN, FISKE AND COMPANY

TERMS: NET 60 DAYS

6 Roll-top Desks	\$45.00	\$270	
8 Flat-top Desks	25.00	200	
12 China Closets	37.50	450	
9 Sideboards	62.50	562 50	
12 Dining Tables	25.00	300	
12 Sets Dining Chairs	20.00	240	
		\$2022 50	
Less 25% and 20%		809	\$1213 50

SUGGESTION. Find at what per cent of the list price the goods must be sold to make the required profit.

6. Find the net amount of the following items, and determine a selling price mark for each, if sold to gain $33\frac{1}{3}\%$.

2 doz. Smoothing Planes @ \$10.50, less $14\frac{2}{3}\%$
 4 doz. Screwdrivers @ \$5.40, less 25%
 200 Carriage Bolts, each $\frac{3}{8}'' \times 2''$, @ \$5.60
 $\frac{3}{8}'' \times 2\frac{1}{2}''$, @ \$5.80
 $\frac{1}{2}'' \times 4\frac{1}{2}''$, @ \$6.25
 Less $37\frac{1}{2}\%$ and 20%
 400 Machine Bolts, $\frac{3}{4}'' \times 6''$, @ \$8.40
 Less 50% and 10%

7. A sewing machine catalogued at \$40 is sold to a jobber subject to a discount of 25% and 20%. Find the jobber's marked price if he sells to gain 25% after deducting 25% and 20%.

8. A manufacturer sells bathtubs at \$35, less 40%. At what price should the retailer mark them to sell at 25% discount and still make 50%?

9. A merchant pays \$10.80, less 25% and 20%, a yard for lace. Find his marking price if he sells to gain 25% after allowing a discount of 10%.

10. A haberdasher bought ties as follows: $4\frac{1}{4}$ doz. @ \$4.50; $2\frac{1}{2}$ doz. @ \$15; 3 doz. @ \$10.50; $1\frac{1}{2}$ doz. @ \$18; $1\frac{1}{2}$ doz. @ \$15.75; less 20% and $16\frac{2}{3}\%$ on the whole lot. Find the retail selling price of one tie of each kind if sold to gain $33\frac{1}{3}\%$ of cost.

11. A jobber buys an article for \$54 less 25% and $16\frac{2}{3}\%$. He sells it for \$54 less $16\frac{2}{3}\%$ and 10%. The cost of the article increases so that the jobber buys for \$54 less 20% and $16\frac{2}{3}\%$. What discount from the list price of \$54 can the jobber allow so that he may have the same rate of profit as before?

12. Find the correct list price of an article that cost \$160 less $33\frac{1}{3}\%$, 25%, and 20%, so that a gross selling profit of $37\frac{1}{2}\%$ of cost may be realized after allowing a discount of 20% and $16\frac{2}{3}\%$.

13. The manufacturer's list price of a bicycle is \$37.50. He sells it to a wholesaler subject to a discount of 20%, $12\frac{1}{2}\%$, and 5%. The wholesaler sells it to make a profit of $33\frac{1}{3}\%$ of cost. The wholesaler's list price is \$40. What discount in addition to $12\frac{1}{2}\%$ does he allow his customers?

14. A retail clothier buys men's suits at \$35 less 20% and 10%. He sells them at \$40. Toward the end of the season he advertises a sale of men's suits at $33\frac{1}{3}\%$ off. Does he gain or lose, and what per cent of cost, by selling the regular \$40 suits at the sale?

PROFIT AND LOSS

In recent years there has grown up among many business firms a custom of reckoning profits on sales, instead of on cost.

If cost is used as a basis, cost represents unity or 100% of itself.

If sales is used as a basis, sales represents unity or 100% of itself.

Cost, that is, **gross cost**, in a trading business includes prime, or first, cost of the goods sold plus buying expenses.

Cost in a manufacturing business includes cost of raw material, direct labor, indirect labor, and all other factory expenses. It is called **factory cost**.

Sales, as a basis for reckoning profits, is the amount of gross sales less "returns and allowances," that is, the net sales.

Gross trading profit (sometimes called **gross selling profit**) is the excess of net sales over gross cost.

Gross profit in a manufacturing business is the excess of net sales over factory cost.

Selling expenses and other expenses of conducting the business are sometimes grouped under the name of "Cost of doing business."

PROFIT AND LOSS DRILL CHART

	COST	% GAIN	% LOSS	GAIN	LOSS	SELLING PRICE	SELLING PRICE
	1	2	3	4	5	6	7
1.	\$ 16.00	.12 $\frac{1}{2}$	.10	\$ 4.00	\$ 1.00	\$ 18	\$ 12
2.	18.00	.16 $\frac{2}{3}$	.11 $\frac{1}{9}$	6.00	2.00	24	15
3.	22.50	.33 $\frac{1}{3}$	.16 $\frac{2}{3}$	3.75	11.25	30	15
4.	28.00	.25	.12 $\frac{1}{2}$	4.00	7.00	30	20
5.	36.00	.11 $\frac{1}{8}$	.25	12.00	3.00	42	30
6.	48.00	.08 $\frac{1}{3}$	.16 $\frac{2}{3}$	4.00	2.00	64	36
7.	62.50	.20	.10	25.00	12.50	75	50
8.	96.00	.12 $\frac{1}{2}$	.37 $\frac{1}{2}$	6.00	24.00	128	80
9.	120.00	.37 $\frac{1}{2}$	.06 $\frac{2}{3}$	15.00	12.00	160	108
10.	150.00	.33 $\frac{1}{3}$	.04	15.00	7.50	210	141
11.	240.00	.15	.06 $\frac{1}{4}$	15.00	24.00	280	228
12.	360.00	.05	.03 $\frac{1}{3}$	54.00	18.00	450	324

1. Combine columns 1 and 2, and find selling price.
2. Combine columns 1 and 3, and find selling price.
3. Combine columns 1 and 4, and find gain per cent.
4. Combine columns 1 and 5, and find loss per cent.
5. Combine columns 1 and 6, and find gain per cent.
6. Combine columns 1 and 7, and find loss per cent.
7. Combine columns 2 and 4, and find cost and selling price, if gain is based on sales.
8. Combine columns 3 and 5, and find cost and selling price, if loss is based on cost.
9. Combine columns 4 and 6, and find gain per cent.
10. Combine columns 5 and 7, and find loss per cent.

EXAMPLES

1. Last year a merchant's prime cost of goods sold was \$475,000. His buying expenses were \$16,625 and his selling and general expenses were \$81,937.50. His sales were \$655,500. Find the per cent (a) of buying expense; (b) of selling and general expense; (c) of net profit based on cost; (d) of net profit based on sales.

$$(a) \$16,625 \div \$475,000 = .035; 3\frac{1}{2}\%, \text{ buying expense}$$

$$(b) \$81,937.50 \div \$655,500 = .125; 12\frac{1}{2}\%, \text{ selling and general expense}$$

$$(c) \$475,000 + \$16,625 = \$491,625, \text{ gross cost}$$

$$\$655,500 - \$491,625 = \$163,875, \text{ gross trading profit}$$

$$\$163,875 - \$81,937.50 = \$81,937.50, \text{ net profit}$$

$$\$81,937.50 \div \$491,625 = .16\frac{2}{3}; 16\frac{2}{3}\%, \text{ profit on cost}$$

$$(d) \$81,937.50 \div \$655,500 = .125; 12\frac{1}{2}\%, \text{ profit on sales}$$

2. Using the data in the preceding example, find the correct list price of an article whose prime cost is \$20, if a discount of $16\frac{2}{3}\%$ and 10% is to be allowed to customers.

$$3\frac{1}{2}\% \text{ of } \$20 = \$.70, \text{ buying expense}$$

$$\$20 + \$.70 = \$20.70, \text{ gross cost}$$

$$12\frac{1}{2}\% + 12\frac{1}{2}\% = 25\%, \text{ rate of selling expense and of profit combined}$$

$$100\% - 25\% = 75\%, \text{ ratio of cost to net selling price}$$

$$\$20.70 \div .75 = \$27.60, \text{ net selling price}$$

The net cost rate factor corresponding to a discount of $16\frac{2}{3}\%$ and 10% is 75%

$$\text{Hence, } 75\% \text{ of L. P.} = \$27.60$$

$$\$27.60 \div .75 = \$36.80, \text{ list price}$$

Problems

1. A merchant's purchases for one year were \$72,900 and his buying expenses were \$1638.90. Find (a) the gross cost of an article whose prime cost was \$32, and (b) the selling price if a gross profit of 25% of the cost was made.

✓ 2. During a sale, a merchant marked overcoats down $33\frac{1}{3}\%$, which made the selling price \$16.50. At that price he made a gross profit of 10% of the cost. What per cent of cost did he make before marking them down?

✓ 3. A quantity of hats was bought at \$3.60 less $33\frac{1}{3}\%$ and $16\frac{2}{3}\%$. The buying expense was 5%. At what price should these hats be sold to net a profit of 12% of sales, when selling and other expenses amount to 18% of sales?

4. A wholesale furniture dealer bought an upholstered rocker for \$48 less 20% and $16\frac{2}{3}\%$. His buying expenses amounted to $3\frac{1}{8}\%$ of prime cost of purchases. At what price should he list the chair so that he can make a net profit of 10% of his sales and allow 15% of sales for general and selling expenses, $6\frac{1}{4}\%$ for bad debts, and 20% discount to customers?

5. A given kind of cloth costs \$.95 a meter (39.37 inches). What per cent of cost is gained by selling it at \$1.25 a yard?

6. The prime cost of a pound of green coffee is \$.11. Buying expenses amount to 20% of prime cost. Coffee shrinks about 16% of its weight when roasted. At what price per pound should it be sold to clear 14% of sales, when selling and other expenses amount to $14\frac{4}{7}\%$ of sales?

7. Unsponged woolen cloth costs \$2.50 a yard, and buying expenses are 4%. Allowing 1 inch a yard for shrinkage in sponging and $14\frac{2}{7}\%$ of sales for expenses, find the selling price to clear $28\frac{4}{7}\%$ of sales.

8. Some goods cost \$33.75 a dozen articles, less 20% and $16\frac{2}{3}\%$, and are sold at \$3.00 each, less $16\frac{2}{3}\%$. What per cent of sales is the net profit if the expenses are 14% of sales?

9. In a given year a merchant's sales were \$750,000. His selling and other expenses were \$225,000. His net profit was 12% of sales, and his buying expenses averaged $8\frac{3}{4}\%$ of prime cost. Find the prime cost of the goods sold.

10. From the following abstract of a trading and profit and loss statement set up a statement showing the percentage of each item based (a) on cost, (b) on sales. Keep results to nearest tenth per cent.

Sales	\$475,000
Return sales	<u>14,250</u>
Net sales	460,750
Cost of sales	<u>275,000</u>
Gross selling profit	185,750
Selling expense	<u>59,375</u>
Net profit on sales	126,375
General expense	<u>39,425</u>
Net profit	86,950
Other income	<u>15,675</u>
Total income	102,625
Allowances, etc.	<u>21,375</u>
Profit and loss surplus	81,250

11. If labor and other manufacturing costs amount to 25% more than the cost of raw material, find the cost of raw material when the cost of the manufactured article is \$5.40.

12. A merchant sold an article for \$225, which was at a discount of 25% from his usual selling price. This gave him a gross profit of $12\frac{1}{2}\%$ of the cost. What per cent of the usual selling price was the gross profit before marking it down?

13. A quantity of imported cloth costs \$2.75 a meter. Duty and buying expenses together amount to 20% of prime cost. If it costs $14\frac{1}{2}\%$ of the selling price to sell the cloth, at what price per yard should it be sold to make a net profit of 10% of sales?

14. To manufacture a given article costs for raw material \$7.64, and for labor and other expenses, \$17.32. The manufacturer lists the article in his catalogue so that he can allow a discount of 25% and 20% and make a profit of 20% of his sales. Find the list price.

15. A jobber bought men's clothing at \$22.50 a suit, less 20% and 10%. He sold the clothing through an agent who charged 5% of sales for selling. The jobber marked the suits so as to make a profit of 10% of sales and allow 6% of sales for general expenses, 4% of sales for bad debts, and a discount of $16\frac{2}{3}\%$ and 10% from list price to customers. Determine his marking price.

16. If you buy at \$.485 a pound, less 25% and $12\frac{1}{2}\%$, and sell at the same list price less $16\frac{2}{3}\%$ and 10%, what rate of discount can you allow so that you can have the same rate of profit if you have to buy at \$.485 a pound, less 20% and 10%?

Miscellaneous Problems

Unless otherwise stated, cost is to be used as the basis for reckoning profits in the following problems.

1. The following table shows the number of organized wage earners (males) in New York State employed one or more days during the first quarter of a certain year. Find, to the nearest tenth per cent, the percentage for each limit of time. Prove the work.

PERIOD	NUMBER EMPLOYED
1-29 da.	15,497
30-59 da.	63,695
60-79 da.	189,756
80 da. or more	60,029

2. A grocer buys 6 bags of Rio coffee, 125 lb. in a bag, at 12¢ a pound, and 9 bags of Java coffee, 75 lb. in a bag, at 20¢ a pound. If he makes a mixture of the two kinds, and sells it at a profit of 90% of the cost, how much does he receive per pound?

3. A speculator bought a lot and sold it so as to gain 20%. He immediately bought a second lot at the selling price of the first, which he afterwards sold at a profit of $16\frac{2}{3}\%$. If his entire gain was \$1400, how much did he pay for each lot?

4. In selling a quantity of oranges and pineapples a huckster gained \$12.50, gaining 25% on the oranges but losing 10% on the pineapples. The loss on the pineapples was 20% of the net gain. Find the cost of each.

5. If coal can be bought at \$4.75 a long ton (2240 lb.) and the selling expense is \$.80 a short ton (2000 lb.), what rate of profit is realized if the coal is sold at \$6.50 a short ton?

6. At what price shall I mark an article that cost me \$18, so that I can instruct my agent to allow 25% discount on the marked price,

pay him 15% commission for selling, allow 5% of sales for bad debts, and still make a profit of $33\frac{1}{3}\%$ of cost?

7. Find, from the following data, the per cent of profit or loss based on cost of goods sold: inventory at beginning, \$12,000; purchases, \$35,000; sales, \$34,000; return purchases, \$1500; return sales, \$1250; freight on purchases, \$125; inventory at closing, \$17,500.

8. A produce dealer bought a carload of potatoes containing 650 bushels at 58¢ a bushel; freight was 3¢ a bushel and other expenses were 5% of the prime cost of the potatoes. He sold the potatoes at a gross profit of 20% of total cost. One merchant who bought 300 bushels of the potatoes on account failed and paid his creditors only 45¢ on the dollar. Find what per cent of the total cost the produce dealer gained or lost.

9. A merchant marks a certain lot of goods at 25% above cost. One quarter of the stock is ruined by fire and water. What per cent must the merchant increase the marked price of the remainder of the stock so that he may neither gain nor lose on the transaction?

10. A haberdasher bought ties at \$5.00 a dozen, less $12\frac{1}{2}\%$ and 5%. At what price per tie should he sell them to make a profit of 30% of cost if he makes an allowance of 10% of sales for bad debts, left-overs, etc.?

11. A real estate dealer bought 2 houses for \$4200. 30% of the cost of the first house equaled 40% of the cost of the second. He sold the first at a profit of 15% of its cost and the second at a loss of $12\frac{1}{2}\%$ of its cost. Did he gain or lose, and how much, on both?

12. A coffee dealer imported from Brazil a quantity of green coffee weighing 280,000 lb. at 12.3¢ a pound. Transportation charges were \$9.50 a ton. Cost of unloading was $62\frac{1}{2}\%$ an hour for 160 hours. Cartage cost \$1.50 a ton. Roasting cost 1¢ a pound, and caused a shrinkage in weight of about 16%. If selling expenses amounted to 15% of sales, and the coffee was sold at 25¢ a pound less 10%, what per cent of sales was the net profit?

13. A merchant's sales for one year were \$985,750. He owned his store, valued at \$40,000. Fixtures, etc. were valued at \$5000. Salesmen's salaries were \$48,750; delivery expense was \$17,860; light

and heat cost \$4175; insurance cost \$2400; general expenses were \$43,500; depreciation on real estate was estimated at 4%, on fixtures at 6%, and on delivery equipment, valued at \$8500, at 15%. Find to the nearest tenth per cent the cost of doing business.

The gross cost of goods sold was \$725,490. Determine to the nearest tenth per cent the rate of net profit based on sales.

14. A grocer's sales for one year amounted to \$91,250. At the close of the year his financial statement showed the following charges and expenses: rent, \$3200; fuel, heat, and light, \$512.50; salaries, \$8984.30; delivery expense, \$1788.50; insurance, \$250; general expense, \$572.50; depreciation, \$1000; bad debts, \$472.55. Find, to the nearest tenth per cent, and tabulate, the percentage for each item of expense, and show the total per cent cost of doing business.

15. In problem 14, assume the net profit to have been 15% of sales. Using the year's experience as a guide, at what average per cent above cost should the grocer sell his goods the following year to make the same rate of profit?

16. For the first quarter of a given year the following facts regarding organized male wage workers were reported in the state of New York. Arrange them in a table with columns showing: (1) the per cent of workers in each industry employed; (2) the average number of days worked by each man employed; (3) the average wage per day; and (4) the average earnings for the quarter for each person employed.

In the building industry there were 116,755 men reported, of whom 100,304 were employed an aggregate of 5,376,294 da., earning \$20,161,104. In transportation, 60,367 were reported, of whom 53,549 were employed an aggregate of 4,396,373 da., earning \$12,155,623. In clothing and textiles, 46,481 were reported, of whom 45,353 were employed an aggregate of 2,820,957 da., earning \$7,392,539. In metals and machinery, 30,428 were reported, of whom 29,850 were employed an aggregate of 2,092,485 da., earning \$6,537,150. In printing, binding, etc., 23,995 were reported, of whom 23,216 were employed an aggregate of 1,694,768 da., earning \$5,943,946.

IX. INTEREST AND DISCOUNT

For the discussion of interest and the methods of calculation see pages 27-40.

Bank discount is interest paid in advance. If one borrows \$100 at a bank for 60 days, and the interest rate is 6%, the bank retains \$1, the interest on \$100 for 60 days at 6%, and the borrower receives \$99. That is, the interest is deducted from the amount of the loan.

The procedure is the same if one wishes to discount a note at a bank. The bank deducts from the maturity value of the note the interest on the maturity value from the date of discount to the date of maturity.

An additional charge, called **exchange**, is made by the bank when discounting out-of-town paper. The exchange rates vary from a minimum charge of ten cents, on notes of small face value, to a certain percentage of face value on notes of greater value.

The **term of discount** is the time from the date of discount to the date of maturity of the paper. It is generally reckoned in exact days. In some places the date of discount as well as the date of maturity is included, one day being thereby added to the term of discount. The more common custom, however, is to exclude the date of discount and include the date of maturity. Thus, if a note matures August 16 and is discounted June 24, the term of discount is made up of the 6 remaining days in June, 31 days in July, and 16 days in August—a total of 53 days. The student should familiarize himself with the custom in his own locality and use that plan. This text follows the more common custom.

The difference between the maturity value of the paper and the bank's total charge for discount and exchange is called the **proceeds**.

The Negotiable Instruments law provides in most states that notes and drafts falling due on Saturday, Sunday, or a legal holiday become due and payable the next succeeding business day. In some states notes and drafts falling due on Saturday are payable the same

day, and those falling due on a Sunday or a legal holiday are payable on the preceding business day. The student should follow the custom in his own locality.

True discount differs from bank discount in that it is reckoned on the present value of a note or debt instead of on the maturity value.

The **present value** (also called **present worth**) of a debt is such a sum which, if put at interest for the given time and rate, will amount to the debt at maturity. **True discount** is the interest on the present worth of the debt for the given time and rate. **Bank discount** is the interest on the maturity value for the given time and rate. True discount is less than bank discount because the present worth of a debt is always less than its future, or maturity, value.

EXAMPLES

1. A 3-months' note of \$2575, dated Aug. 18, was discounted Sept. 24, at 6%. Find the proceeds.

Aug. 18 + 3 mo. = Nov. 18, due date

From Sept. 24 to Nov. 18 = 55 da., term of discount

Interest on \$2575 for 55 da. = \$23.60, bank discount

$\$2575 - \$23.60 = \$2551.40$, proceeds

2. A 60-day 6% interest-bearing note of \$4450, dated Mar. 23, payable at a Chicago bank, was discounted at a New York bank Apr. 14, at 6%. Exchange charge being \$.50, find the proceeds

Mar. 23 + 60 da. = May 22, due date

From Apr. 14 to May 22 = 38 da., term of discount

Interest on \$4450 for 60 da. at 6% = \$44.50

$\$4450 + \$44.50 = \$4494.50$, maturity value of note

38 da. interest on \$4494.50 = \$28.47, bank discount

$\$28.47 + \$.50 = \$28.97$, bank's total charges

$\$4494.50 - \$28.97 = \$4465.53$, proceeds

3. A draft of \$3000, dated Aug. 15, 1923, payable 90 days after sight, was accepted Aug. 21. It was discounted Sept. 13, at 6%. Find the proceeds.

Aug. 21 + 90 da. = Nov. 19, due date

From Sept. 13 to Nov. 19 = 67 da., term of discount

Interest on \$3000 for 67 da. = \$33.50, bank discount

\$3000 - \$33.50 = \$2966.50, proceeds

4. A debt of \$1392.50 is due Dec. 13, 1923. Allowing interest at 6%, find the present value of the debt Sept. 29, 1923.

From Sept. 29 to Dec. 13 is 75 days

(The problem is to find what sum of money placed at interest at 6% on Sept. 29 will amount to \$1392.50 on Dec. 13, that is, in 75 days.)

The interest on \$1 for 75 days at 6% is \$.0125

The amount of \$1 in 75 days at 6% is \$1.0125

Dividing \$1392.50 by 1.0125 gives \$1375.31. Hence the present value of the debt Sept. 29 is \$1375.31.

5. A 90-day note dated Sept. 19, 1923 was discounted Oct. 1, at 6%. The proceeds were \$1658.16. Find the face value of the note.

Sept. 19 + 90 da. = Dec. 18, due date

From Oct. 1 to Dec. 18 = 78 da., term of discount

Interest on \$1 for 78 da. at 6% = \$.013, discount on \$1

\$100 - \$.013 = \$.987, proceeds of \$1

\$1658.16 ÷ .987 = \$1680, face value of note

Problems

Find the proceeds of the following notes and drafts:

	FACE	TIME	DATE OF PAPER	DATE OF ACCEPTANCE	DATE OF DISCOUNT	RATE OF DISCOUNT
1.	\$1800.00	4 months	Aug. 1		Aug. 11	6%
2.	1640.00	90 days	May 13		June 4	6%
3.	1990.00	30 days' sight	Apr. 27	May 3	May 6	5%
4.	3675.00	2 months	July 17		Aug. 1	7%
5.	8478.00	30 days' sight	May 28	June 4	June 11	4½%
6.	5900.00	1 month	Jan. 31		Feb. 4	5½%
7.	4750.00	4 months	Oct. 6		Nov. 6	7½%
8.	6000.00	90 days after date	Sept. 5	Sept. 25	Oct. 11	6%
9.	4500.00	30 days' sight	July 5	July 11	July 11	7½%
10.	412.50	3 months	Apr. 12		May 1	9%

Find the proceeds of the following notes and drafts:

	FACE	TIME	DATE OF PAPER	DATE OF DISCOUNT	RATE OF DISCOUNT	RATE OF COLLECTION
11.	\$5000	4 mo.	Jan. 10	Mar. 8	6%	$\frac{1}{20}\%$
12.	2500	90 da.	June 8	July 1	6%	$\frac{1}{8}\%$
13.	1748	60 da.	Aug. 12	Sept. 1	6%	
14.	4382.75	90 da.	Nov. 17	Nov. 29	6%	$\frac{1}{10}\%$
15.	2600	3 mo.	Mar. 8	Mar. 13	6%	
16.	943.75	4 mo.	Oct. 1	Oct. 2	6%	
17.	416.50	60 da.	Mar. 31	Apr. 13	$4\frac{1}{2}\%$	$\frac{1}{4}\%$

	FACE	TIME	DATE OF PAPER	INTEREST	DATE OF DISCOUNT	RATE OF DISCOUNT	RATE OF COLLECTION
18.	\$1400	2 mo.	Aug. 1	6%	Aug. 10	6%	$\frac{1}{8}\%$
19.	2160	90 da.	Feb. 15	8%	Mar. 17	7%	$\frac{1}{8}\%$
20.	940	5 mo.	July 1	6%	Sept. 10	5%	$\frac{1}{4}\%$
21.	3125	75 da.	Apr. 20	$7\frac{1}{2}\%$	May 5	$4\frac{1}{2}\%$	$\frac{1}{8}\%$
22.	5100	1 mo.	Feb. 28	6%	Mar. 1	6%	$\frac{1}{20}\%$

23.

No. 1645 Binghamton, N. Y., July 10, 1923

Ninety days after date I promise to pay to the
order of George Haswell \$ 6380 $\frac{00}{100}$

Sixty-three hundred eighty $\frac{00}{100}$ Dollars

Value received, with interest at 6%.

Leo Boutell

Discounted Aug. 1 at 6%; collection $\frac{1}{8}\%$.

24. Find the present worth of a debt of \$100 due in 45 days, interest at 6%.

25. How large a 6% interest-bearing note should be given April 1 to cancel a debt of \$1000 due June 20?

26. Find the difference between the true and the bank discount on a note of \$1890 due in 93 days, the discount rate being $4\frac{1}{2}\%$.

27. You buy a bill of goods invoiced at \$2750.95 on 4 months' credit, with the understanding that you give a 4-months' 5% interest-bearing note which at maturity will amount to the face value of the bill of goods. Find the face of the note.

28. Allowing interest at 4%, what amount of ready cash will settle on May 15 a bill of \$15,472.50, due July 7?

29. April 10, I bought a bill of merchandise valued at \$5600 on 5 months' credit. June 20, I paid \$3000 on account. Aug. 1, I paid the present worth of the balance of the debt. Reckoning interest at 6%, what was the amount of my final payment? (Exact time.)

30. On May 15 the First National Bank discounted for its customers the following notes and acceptances:

FACE	DATED	TIME	ACCEPTANCE	RATE OF DISCOUNT
(a) \$11,475	May 8	90 da.	May 8	6%
(b) \$15,932.50	Apr. 28	90 da. sight		6%
(c) \$10,000	Apr. 23	4 mo.		6%
(d) \$18,922.75	May 3	60 days after date	May 5	6%

Owing to the need for a larger cash balance on May 27, the First National Bank rediscounted at The Federal Reserve Bank the foregoing notes and drafts at the following rates:

- (a) at 5%
- (b) at $4\frac{3}{4}\%$
- (c) at 5%
- (d) at $4\frac{3}{4}\%$

Find the First National Bank's gain in discount.

31. A. B. Gordon discounted July 17 a 3-months' note dated June 28, at 6%. He received as proceeds \$1337.60. Find the face of the note.

32. Simpson and Company on Aug. 19 had need for \$25,000. Their bank account showed a balance of \$9475.64. They had in their cash drawer a 90-day note of \$8500 dated July 16. They discounted the \$8500 note at 6%. To provide the remainder of the required funds and also a balance of \$5000 in the bank after drawing their check for \$25,000, they made and discounted their own 45-day note. How large a note did they write?

Problems in Interest

1. A banker pays 3% interest on a deposit of \$45,000. He loans it 3 mo. at 6%, 4 mo. at $5\frac{1}{2}\%$, and 48 da. at 7%. If it lies idle the remainder of the year, does he gain or lose, and how much?

2. Aug. 1, 1922, a wholesaler sold a bill of goods amounting to \$1875.28, terms net 30 da. If the bill was not paid until Jan. 8, 1923, what amount was due, interest being 6%? (Exact time.)

3. On July 8, 1922, a man borrowed money at 6% interest and bought 3000 bu. of wheat at \$1.08 a bushel. He sold the wheat Aug. 17, 1922, at \$1.21 a bushel and returned with interest the money he had borrowed. What was his gain?

4. A young man had \$1600. He purchased a house and lot for \$4000, borrowing \$2400 at a savings bank, giving as security a mortgage on the house and lot. The house rented for \$37.50 a month. He paid $5\frac{1}{2}\%$ interest on the mortgage, $\frac{1}{8}\%$ premium on an insurance policy of \$3000, 1.8% tax on a valuation of \$3500, \$14 water rent, and \$50 for repairs. What rate of interest did he receive on his investment?

5. A real estate broker offers me a house and lot for \$7500, guaranteeing an 8% investment. The assessed valuation of the property is \$5000 and the tax rate is $1\frac{3}{4}\%$. Insurance and other expenses amount to \$212.50 a year. At what price per month must the property rent to make good the broker's guaranty?

6. An apartment house has 12 four-room apartments, 10 five-room apartments, and 4 stores. The four-room apartments rent, on the average, for \$36 a month, the five-room apartments for \$44 a month, and the stores for \$75 a month. The house and lot cost \$140,000, of which \$100,000 was borrowed on bond and mortgage at $4\frac{1}{2}\%$. Expenses are: taxes 2% on full valuation, insurance $\frac{1}{4}\%$ on a policy of \$80,000, coal, lights, repairs, janitor service, etc., \$3164 a year. Assuming that all the apartments and stores are rented all the time, what rate of interest does the owner receive on his investment?

7. A merchant bought a bill of goods amounting to \$13,875—terms 3 mo., 5% cash. He accepted the cash terms. How much did he save if money was worth 6% ?

8. An invoice of merchandise is bought on the following terms: 4 mo. or 5% cash. The cash discount is equivalent to what rate of interest per annum on the gross amount? on the net amount?

9. On an invoice of \$4378.60, a merchant is offered 3 mo. credit or a discount of 5% for cash. Not having the ready money, he accepts the credit terms. What rate per cent of interest does he pay on the net amount of the bill? How much would he have saved if he had borrowed the money at 6% and paid cash?

10. A clerk is instructed to accept the best terms on all bills, assuming that money is worth 15% to his employer. Under these conditions, which terms should the clerk accept in the following invoices?

\$ 428.75, terms 30 da., 2% cash
\$ 769.50, terms 90 da., 4/30, 5% cash
\$1287.50, terms 4 mo., 5/30, 6% cash
\$1760.80, terms 30 da., 1% cash

11. An invoice of goods dated May 11, 1923 is valued at \$32,647.50, terms 5/10, n/90. To make payment in 10 days the buyer discounts a note of \$15,750, due August 9, at 6% , and deposits the proceeds in his bank. He then draws his check to pay for the bill of goods. If the bank allows 3% on daily balances, find the buyer's profit by early settlement.

12. A merchant buys a bill of goods valued at \$22,500, dated May 12, 1923, terms 3/10, n/60. On May 22 he finds he has in his bank but \$15,000 available cash. To provide the remainder of the funds needed to make settlement on that date, the merchant writes a 50-day note for such a sum that, when discounted at 6%, the proceeds will, with his \$15,000, enable him to make early payment. Find (a) the face value of the note he writes, and (b) his gain by early settlement if his bank pays $2\frac{1}{2}\%$ on daily balances.

13. Simpson and Company received a bill of goods valued at \$28,572.40, terms 4/10, n/60. The date of the invoice was Aug. 25. To enable them to make early payment they discounted at 6% a 4 mo. note of \$16,925, dated July 31. Their bank allowed them $2\frac{1}{2}\%$ interest on daily balances. Find their gain by settling on Sept. 4.

14. On April 3, Davis and Son received an invoice of merchandise valued at \$36,932.60, terms 5/10, n/90. The invoice was dated April 1. On April 11, Davis and Son, not having sufficient funds to make payment in full, asked if they might have the discount privilege on a payment of \$20,000. Such permission was granted by the seller. It was further agreed that Davis and Son might, at any time before the expiration of the term of credit, pay the remainder of the bill on a true discount basis, allowing interest at 6%. Accordingly, Davis and Son made on April 11 a payment of \$20,000 and on May 17 they paid the present worth of the balance due. If Davis and Son got 2% on daily balances at their bank, how much did they gain by paying the bill before the expiration of the term of credit?

15. If a merchant gets $2\frac{1}{2}\%$ interest on daily balances, how much per cent on the net amount of the bill is it better to accept the early terms, on invoices having terms as follows :

2/10, n/30? 3/10, n/60? 5/10, n/90?

16. On June 15, A. M. Watson wishes to borrow \$10,000 for 6 mo. He has a choice between writing an interest-bearing demand note for \$10,000, which he will pay off in 6 months, or a 6-months' non-interest-bearing note for such a sum that the proceeds when discounted will be \$10,000. Allowing interest and discount at 5%, which is the better choice and how much better?

17. On July 26, 1924 a large commercial bank borrowed \$50,000 from a savings bank. The loan was to run for 6 months at $5\frac{1}{2}\%$. On the due date of the note it was renewed by writing a new note for a further period of 6 months. Interest on the old note was due at maturity, but upon renewing the note the savings bank requested that the new note be discounted, that is, have the interest paid in advance. Compute the interest on the old note and the discount on the new note. What was the total amount of interest and discount received by the savings bank?

18. The proceeds of a 90-day 6% interest-bearing note dated Sept. 3 and discounted Oct. 7 at 6% were \$928.46. Find the face value of the note.

19. Assume that your bank balance on the morning of May 17 was \$3425.60. During the day you deposited checks as follows: \$56.75, \$3.56, \$1456.35, \$3476.50. On the last check there was an exchange charge of $\frac{1}{20}\%$. You also discounted at 6% a note of \$2345.50, which was due July 7. You drew checks as follows: \$35.67, \$234.45, \$47.79, one to pay for an invoice of merchandise dated May 7, valued at \$5342.50 — terms 3/10, n/60 — and another to prepay a 90-day 5% interest-bearing note of \$728.50 dated April 11, less discount at 6%. Determine your correct bank balance at the close of the day.

20. A manufacturer purchases a piece of machinery for \$45,000, on which there is a discount of 5% for cash. The purchaser has the option of paying cash or giving 36 monthly installment notes of \$1250 each with interest at 6%. To pay cash the purchaser must borrow funds at his bank. Assume that he could borrow at 6% discount on a 3-months' note dated April 15, 1922 with the privilege of renewal to cover a period of 18 months (approximately the average time of the installment notes). Which method of payment will be more advantageous, and by how much?

NOTE. Theoretically it would seem that a uniform amount of proceeds, namely \$42,750, should be received from the several notes. Practically, the notes would be renewed at a uniform face value, which would make the discount and the proceeds of some of the notes vary slightly from that of some of the others.

21. If, in the preceding problem, the purchaser had been able to borrow the funds needed for paying cash on an 18-months' note, would he have been better or worse off, and how much, than he would have been by discounting 6 notes of 3 months each? (Use the same dates as in problem 20.) Explain.

EQUATION OF ACCOUNTS

Equation of accounts is the process of finding the date when (1) several items due at different dates may equitably be paid in one amount, or (2) the balance of an account may be equitably settled, or (3) the proceeds of an account sales is due.

The theory on which equation of accounts is based is that the interest on overdue items in an account is canceled by the discount on other items in the account paid before they are due.

To find the equated date of a one-sided account

The following example will illustrate several terms and their definitions:

EXAMPLE

Find the equated date of the following bills: July 8, \$600; July 20, \$1200; Aug. 17, \$1800.

July 8,	\$ 600	40 da.	\$4.00, interest
July 20,	\$1200	28 da.	\$5.60, interest
* Aug. 17,	\$1800	0 da.	\$0.00, interest
	\$3600		\$9.60, interest

Interest on \$3600 for 1 da. = \$3.600 ÷ 6 = \$.60

\$9.60 ÷ \$.60 = 16; 16 da., average term of credit

Aug. 17 - 16 da. = Aug. 1, equated date

Arrange the several items and their respective due dates as shown in the solution. For the purpose of the solution, the latest date, Aug. 17, is assumed as the date of settlement. The assumed date of settlement is called the **focal date**. (Any date may be used as a focal date. In practice either the earliest or latest date is generally used, owing to the fact that they are the most convenient dates. In this book, the latest date is used, and is indicated by an asterisk (*).) If the \$600 due on July 8 is not paid till Aug. 17, it is overdue

40 da., and the accrued interest amounts to \$4. So, also, if the \$1200 due July 20 is not paid till Aug. 17, it is overdue 28 da., and the interest is \$5.60. If the \$1800 is paid on its due date, Aug. 17, there is no interest charged or discount allowed. Now, if the total amount of the bill, \$3600, is paid on Aug. 17, the seller loses \$9.60 interest on the overdue items. To avoid this loss the account must be settled earlier.

The interest on \$3600 for 1 da. is \$.60; hence if the payment were made one day earlier, the loss to the seller would be \$.60 less. Therefore, the account should be settled as many days before Aug. 17 as \$.60, the interest on the debt for 1 da., is contained times in \$9.60, the amount of interest due, which is 16 times, or 16 da. Counting back 16 da. from Aug. 17 gives Aug. 1, the equated date.*

To prove the correctness of the work and to show that there is neither loss nor gain to either party, find the interest on all items due before Aug. 1 from the date they are due to Aug. 1. Then find the discount (simple interest) on the item due after Aug. 1 from Aug. 1 to its due date. Thus:

The interest on \$600 from July 8 to Aug. 1, 24 da., is	\$2.40
and the interest on \$1200 from July 20 to Aug. 1, 12 da., is	<u>\$2.40</u>
Total interest due is	\$4.80

The discount on \$1800 from Aug. 1 to Aug. 17, 16 da., is \$4.80; that is, the interest on the overdue items is equal to the discount on the item that was prepaid. Hence the work is correct, and Aug. 1 is the average due date.

Problems

Find the average due date of the following bills:

1923	1923	1923
1. May 1, \$1500	2. June 18, \$ 560	3. Aug. 10, \$360
May 20, \$ 600	June 30, \$ 720	Aug. 20, \$480
June 8, \$ 300	July 15, \$ 900	Aug. 28, \$660
June 20, \$ 900	July 25, \$1000	Sept. 15, \$840

EXAMPLE

On what date may the following items be paid in one amount without loss of interest to either party?

Jan. 3, 1923, Mdse., 30 da., \$600
 Jan. 14, 1923, Mdse., 1 mo., \$900
 Jan. 20, 1923, Mdse., 60 da., \$800
 Feb. 10, 1923, Mdse., 1 mo., \$300

* The "equated date," the "average date of payment," the "average due date," and similar expressions are synonymous, and are the names given the date of settlement as determined by equation.

1923	1923			
Jan. 3 + 30 da. =	Feb. 2,	\$ 600	47 da.	\$ 4.70
Jan. 14 + 1 mo. =	Feb. 14,	\$ 900	35 da.	5.25
Jan. 20 + 60 da. =	* Mar. 21,	\$ 800	0 da.	.00
Feb. 10 + 1 mo. =	Mar. 10,	\$ 300	11 da.	.55
		<u>\$2600</u>		<u>\$10.50</u>

The interest on \$2600 for 1 da. = $\$2.600 \div 6 = \$4.333$

$\$10.50 \div \$4.333 = 24.2$; 24 da., ** average term of credit

Mar. 21 - 24 da. = Feb. 25, equated date

When various terms of credit are given on the items to be equated, it is necessary first to find the due date of the separate items. Having found the several due dates, we perform the solution as in the preceding exercises.

Find when the following bills may be equitably paid in one amount. Prove the work.

4.

1923

Apr. 1, Mdse., 15 da., \$ 500
 Apr. 10, Mdse., 1 mo., \$ 800
 May 1, Mdse., 30 da., \$1000
 May 10, Mdse., Net, \$ 300

5.

1923

July 1, Mdse., 1 mo., \$ 856.60
 July 18, Mdse., 2 mo., \$1260.00
 July 30, Mdse., 30 da., \$1000.00
 Aug. 10, Mdse., 10 da., \$1268.40

6. Daniel Reardon bought of Curran and Mann merchandise on 60 days' credit as follows: Aug. 12, 1923, a bill for \$1388.75; Sept. 10, a bill for \$1564.25; and Sept. 20, a bill for \$975.50. Reardon wishes to give a 2-months' interest-bearing note † in settlement of the entire amount. When should the note be dated? Write the note.

To find the equated date of a two-sided account

EXAMPLE

What is the equated date of payment of the balance of the following account?

1923

May 1, Mdse., \$1840
 June 1, Mdse., \$1200

1923

May 10, Cash, \$600
 May 20, Cash, \$900

** A fraction of a day less than $\frac{1}{2}$ is rejected; a fraction greater than $\frac{1}{2}$ is called 1 da.

† An account becomes interest-bearing at the equated date.

DUE	AMT.	DAYS	INT.	DUE	AMT.	DAYS	INT.
1923				1923			
May 1	\$1840	31	\$9.506	May 10	\$ 600	22	\$2.20
*June 1	\$1200	0	0.000	May 20	\$ 900	12	\$1.80
	<u>\$3040</u>		<u>\$9.506</u>		<u>\$1500</u>		<u>\$4.00</u>

\$3040 - \$1500 = \$1540, balance due on the account

\$9.506 - \$4.00 = \$5.506, balance of interest due

Interest on \$1540 for 1 da. = $\$1.540 \div 6 = \$0.2566$

$\$5.506 \div \$0.2566 = 21.4$; 21 da.

21 da. back from June 1 = May 11, the equated date

All the debit items and all the credit items are arranged as shown in the solution. The latest date on either side is taken as a focal date. If the bill of \$1840 were not paid until June 1, it would have been 31 da. overdue, and there would have been \$9.506 interest due on it. There would be no interest due on the bill of \$1200 if paid on June 1. The total interest due, then, by settling the account on June 1, would be \$9.506. But two payments have been made. The \$600 paid on May 10 was paid 22 da. before the assumed date of settlement, hence the total amount of interest due as shown on the debit side is reduced by the interest on \$600 for 22 da., or \$2.20. In like manner the \$900 paid on May 20, or 12 da. before June 1, reduces the debit interest by \$1.80 more. The total credit interest is \$4, leaving a balance of \$5.506 interest due. The balance of the account is \$1540. The interest on \$1540 for 1 da. is \$.2566, hence if the assumed date of settlement had been 1 da. earlier, the balance of interest would have been \$.2566 less. To find the number of days earlier than June 1 the account should have been settled, divide \$5.506 by \$.2566, which gives 21 da. Therefore the equated date is 21 da. before June 1, or May 11.

PROOF

DUE	AMT.	DAYS	INT.	DIS.	DUE	AMT.	DAYS	INT.	DIS.
1923					1923				
May 1	\$1840	10	\$3.066		May 10	\$ 600	1	\$1.10	
June 1	\$1200	21 (dis.)		\$4.20	May 20	\$ 900	9 (dis.)		\$1.35
					May 11	\$1540	0	.00	
	<u>\$3040</u>		<u>\$3.066</u>	<u>\$4.20</u>		<u>\$3040</u>		<u>\$1.10</u>	<u>\$1.35</u>

\$4.20 - \$3.066 = \$1.134, debit discount

\$1.35 - \$1.10 = \$1.25, credit discount

\$1.25 - \$1.134 = \$.116, less than half a day's interest on the balance, \$1540

Write the balance, \$1540, with equated date on the credit side of the account, as shown. Equate the account with May 11 as the focal date and balance the items of interest and discount. This makes the credit side of the account \$.116 more than the debit side. Since the interest on \$1540 for 1 da. is \$.2566, the difference in discount between the two sides is less than half the interest for 1 da. Inasmuch as the law does not recognize a fraction of a day, the account as it now stands is considered as being in balance, and the solution proves.

Counting back or forward from the focal date

1. *When to count back*

If the balance of the account and the balance of interest are on the same side of the account, that is, both on the debit side or both on the credit side, the person who owes the balance of the account owes the balance of interest also; hence he should have settled the account *earlier*.

2. *When to count forward*

If the balance of the account and the balance of interest are on opposite sides of the account, that is, one balance on the credit side and the other on the debit side, the person who owes the balance of the account has the balance of interest due him; hence he has the right to keep the balance of the account *after* the focal date long enough to earn the balance of interest.

Problems

Equate the following accounts :

1. L. D. WALTHAM AND COMPANY

1923				1923			
Nov.	1	Mdse., 60 da.	840	Dec.	7	Cash	1500
	15	" 30 da.	560	1924			
Dec.	1	" 3 mo.	1860	Jan.	4	"	1000
	15	" 1 mo.	1000		10	Note, 10 da.	500

2.

S. L. MATTHEWS

1923				1923			
Feb.	1	Mdse., net	125	Feb.	8	Cash	100
	11	" 30 da.	450		15	Note, 30 da.	500
	20	" 60 da.	750	Mar.	1	" 60 da.	
	28	" 30 da.	1000			with int.	1200

3.

BRAINERD AND COMPANY

1923				1923			
Jan.	1	Balance	1875	Jan.	10	Cash	1000
	15	Mdse., 2 mo.	1983 70	Mar.	1	"	1000
Feb.	14	" 90 da.	3684 60	Apr.	10	Note, 2 mo.	2000

4. A bank lent Edmunds and Barry money as follows: \$1000 on a 3-months' note, dated Oct. 3, 1923; \$1500 on a 2-months' note, dated Nov. 10, 1923; \$2500 on a 60-day note, dated Dec. 14, 1923; and \$5000 on a 90-day note, dated Dec. 18, 1923. On Jan. 3, 1924, Edmunds and Barry settled their account at the bank as follows: They gave the bank a check for \$5000, and a 4-months' note for the balance. All notes were interesting-bearing. Determine by equation the correct date of the new note given on Jan. 3, and state the amount required to cancel it on its due date.

EQUATION OF ACCOUNT SALES

An **account sales** is an itemized statement made by a commission agent, showing the sales of merchandise, the charges on the consignment, the advances, if any, to the consignor, and the net proceeds of the sales.

Cash is considered due on sales at the due date of the several sales. Charges and advances are due when paid. Commission, unless otherwise specified, may be considered due on the date of the last sale. (Some merchants use the average date of sales as the date when the commission is due, and some use the average *due* date of sales.)

In equating an account sales, the sales form the credit side of the account and the charges and advances, the debit side.

EXAMPLE

Find the equated date of paying the proceeds of the following account sales:

Philadelphia, Pa., June 20, 1923

Account Sales 500 bbl. Flour

Sold for Account of Union Milling Company, Rochester, New York

June	8	200 bbl. Flour	11.50				2300
	14	150 bbl. Flour	12.25	60 da.			1837 50
	19	150 bbl. Flour	12.00	30 da.			1800
							5937 50
		Charges					
	5	Freight and Cartage			275		
	16	Cash advanced			1500		
	19	Storage 5¢ per bbl.			25		
	19	Commission 5%			296 88		
July	25	Net Proceeds due by equation			3840 62		
					5937 50		5937 50

1923			DA.		1923		DA.
June	5	\$ 275	69	\$ 3.16	June	8	\$2300
	16	1500	58	14.50	* Aug.	13	1837.50
	19	321.88	55	2.95	July	19	1800
		\$2096.88		\$20.61			\$5937.50
							\$32.80

$\$32.80 - \$20.61 = \$12.19$, credit balance of int.

$\$5937.50 - \$2096.88 = \$3840.62$, credit balance of acct.

$\$3840.62 \div 6 = \$640.1$, int. on balance of acct. for 1 day

$\$12.19 \div \$640.1 = 19$, number of days to count back

Aug. 13 - 19 da. = July 25, equated date

By making the sales the credit side of the account, and the charges and advances the debit side of the account, the equated due date of paying the proceeds is found the same as in equation of accounts.

If the average date of sales, or the average due date of sales, is used as the date of charging the commission and other post-sale charges, the average date, or due date, of the sales must be found before the account can be equated.

Find the equated date for the payment of the proceeds of the following account sales :

Problems

1. Baltimore, Md., Sept. 3, 1923

Account Sales Cotton

Sold for the account of George Bartow, Atlanta, Ga.

Aug.	15	250 bales* 125,000 # @ 13.35¢			
	18	140 bales 70,000 # @ 13.60¢			
	25	110 bales 55,000 # @ 13.45¢			
		Charges			
	12	Freight and cartage	525		
	20	Cash advanced	10,000		
	25	Storage 10¢ a bale			
		Commission \$5 per 100 bales			

2. Pittsburgh, Pa., Oct. 17, 1923

Account Sales Potatoes

Sold for account of A. M. Warner, Nelson, Pa.

Sept.	30	125 bbl. @ \$3.25	30 da.			
Oct.	5	75 bbl. @ \$3.15	10 da.			
Oct.	13	100 bbl. @ \$3.20	30 da.			
Oct.	16	200 bbl. @ \$3.30	45 da.			
		Charges				
Sept.	22	Freight 50¢ a barrel				
	23	Cartage 10¢ a barrel				
	25	Cooperage		15		
	25	Advertising		10		
Oct.	16	Storage 5¢ a barrel				
		Commission 10%				

In this problem assume the commission to be due at the average *due* date of the sales.

*Cotton averages 500 # (500 lb.) to the bale.

3.

Boston, Mass., March 2, 1923

Account Sales Broadcloth

Sold for account of Myers and Company, Worcester, Mass.

Feb.	15	457 ¹ yd. @ \$4.50	30 da.				
	21	328 ³ yd. @ \$4.40	10 da.				
	28	672 ² yd. @ \$4.45	15 da.				
		Charges					
	11	Express			35	40	
	13	Advertising			15		
	20	Cash advanced			2500		
		Commission 3½%					

In this problem assume the commission to be due on the average date of sales.

ACCOUNTS CURRENT

An **account current** is an itemized statement of the debits and credits between two business houses for a given period of time, showing the balance due at the current date. The balance due is called the **cash balance**. Accounts current are often spoken of as "cash balance."

The cash balance of an account may or may not include interest charges and allowances. Problems in this text will involve interest.

EXAMPLE

Find the balance due on the following account July 1, 1923, interest at 6%:

CHARLES BROWN AND COMPANY

1923						1923					
Jan.	1	Balance	J	5	2462 50	Feb.	1	Cash	C	8	1500
Feb.	15	Mdse. 30 da.	S	16	1800	Mar.	10	Note 1 mo.	J	7	1000
Apr.	19	Mdse. 60 da.	S	19	2145 60	May	31	Cash	C	12	2500
June	10	Mdse. 10 da.	S	24	1150						

BURNS AND SON IN ACCOUNT WITH CHARLES BROWN AND COMPANY

1923			DA.	INT.	AMT.	1923			DA.	INT.	AMT.
Jan.	1	Balance	181	74.29	2462 50	Feb.	1	Cash	150	37.50	1500
Feb.	15	Mdse. as Mar. 17	106	31.80	1800	Mar.	10	Note as Apr. 10	82	13.67	1000
Apr.	19	" " June 18	13	4.65	2145 60	May	31	Cash	31	12.92	2500
June	10	" " " 20	11	2.11	1150	July	1	Bal. of int.		48.76	
July	1	Bal. of interest			48 76		1	Bal. of acct.			2606 86
					112.85					112.85	7606 86
1923											
July	1	Balance			2606 86						

The due date of each item is used as the date from which to reckon interest. The days are the exact number of days from each due date respectively to July 1.

If any due date is later than the current settlement date, the interest on such an item should be transferred to the opposite side of the account, or entered in red on its own side and deducted from the total of the other interest items on its side of the account.

Using a form like the foregoing, prepare a statement showing the amount due Jan. 1, 1924, for each of the following accounts. Assume that the accounts are on your own books, and that the interest rate is 6%.

Problems

Find the balances due on the following accounts :

1.

ROBERTSON BROTHERS

1923						1923					
July	1	Balance	J	14	1375 80	Aug.	1	Cash	C	32	1000
"	24	Mdse. 1 mo.	S	23	1619 75	Sept.	5	Cash	C	38	3000
Aug.	11	" 60 da.	S	27	2750 65	Oct.	20	Note 60 da.	J	15	2500
Oct.	19	" 30 da.	S	36	1893 44	Dec.	1	Cash	C	48	500
Dec.	13	" net	S	44	634 50						

2.

L. B. JAMESON

1923						1923					
Aug.	9	Mdse. 60 da.	S	28	3264 75	Oct.	8	Cash	C	34	2500
Sept.	28	" 30 da.	S	33	2831 45	Nov.	1	Note 3 mo.	J	16	2500
Oct.	26	" 30 da.	S	37	4532 60	Dec.	15	Cash	C	46	2000
Dec.	4	" 60 da.	S	42	3845 50		28	Note 60 da.	J	17	3000

3. For the following account render an account current for July 1, 1923, and Jan. 1, 1924. Any balance of interest unpaid July 1, 1923 will automatically become a part of the account from that date. Allow interest at 6%.

KELLY BROTHERS

1923					1923				
Jan.	1	Balance	J	11	752	60	Feb.	10	Note 3 mo. J 11 2500
Feb.	9	Mdse. 30 da.	S	17	1945	80	Apr.	15	Cash C 18 2250
Mar.	16	" 60 da.	S	19	3148	90	June	16	Cash C 24 1500
Apr.	4	" 10 da.	S	22	1231	50	July	5	Cash C 28 1000
May	11	" 30 da.	S	26	1745	75	Aug.	17	Note 60 da. J 18 2500
June	21	" 60 da.	S	31	1860		Sept.	13	Cash C 34 1200
Aug.	17	" 30 da.	S	36	3284	75	Nov.	24	Note 3 mo. J 22 3500
Oct.	19	" net	S	41	631	25			
Dec.	20	" 30 da.	S	44	2263	40			

Bankers, in making foreign exchange settlements with European banks, often reckon the cash balance of such accounts by the English, French, or German method, as the case may be.

ENGLISH METHOD

By the English method, interest is reckoned on daily balances. Credit interest as well as credit items of the account are called **positive** (+) and debit items of interest and of the account are called **negative** (-). Interest is reckoned on the positive or negative balance of the account for the number of days the balance is undisturbed. The interest on a positive balance is positive, and on a negative balance is negative. The following solution illustrates the method.

EXAMPLE

Find the cash balance of the following account Jan. 31, interest at 6%:

MERCHANTS' BANK, LTD.

1923					1923				
Jan.	15			3785	20	Jan.	5		4920 06
	20			2918	75		22		3250
	30			3284	60		29		4100

Let it be assumed that the account in the example on page 116 was in account with a French Bank instead of Merchants' Bank, Ltd. By the French method the solution would be as follows:

DAYS PRODUCTS				DAYS PRODUCTS			
Jan. 15	\$3785.20	10	380	Jan. 5	\$4920.06	0	
20	2918.75	15	435	22	3250.00	17	561
30	3284.60	25	825	29	4100.00	24	984
31	2281.51	26	598	31	Bal. of Products		693
			<u>2238</u>				<u>2238</u>

$693 \div 60 = 11.55$. Hence interest to be added to the balance of the account is \$11.55.

$\$2281.51 + \$11.55 = \$2293.06$, cash balance due French bank.

The variation of 3 cents in the result from that obtained by the English method is due to approximating the products to the nearest 100.

Owing to the use of the earliest date as a focal date, interest obtained from a debit balance of products is to be *added* to a credit balance of the account. If the balance of the account and the balance of products are both debit or both credit, their difference is to be taken.

GERMAN METHOD

The German method is very similar to the French method, except that the date of settlement, instead of the earliest date, is used as a focal date.

Treated as an account with a German bank, the solution of the foregoing account would be as follows:

DAYS PRODUCTS				DAYS PRODUCTS			
Jan. 15	\$3785.20	16	608	Jan. 5	\$4920.06	26	1274
20	2918.75	11	319	22	3250	9	297
30	3284.60	1	33	29	4100	2	82
31	Bal. No.		<u>693</u>				
			<u>1653</u>				<u>1653</u>

$693 \div 60 = 11.55$. Hence \$11.55 is to be added to the balance of the account.

$\$2281.51 + \$11.55 = \$2293.06$, balance due German Bank.

Note that the balance of products is a credit balance in this solution, the same as the balance of the account. Since the latest date was used as a focal date, credit interest is added to a credit balance, etc.

Problems

1. Find the cash balance of the following account, Nov. 30, interest at 6%, by each of the three methods just explained:

ANY FOREIGN BANK

Nov.	1			2475	60	Nov.	11			1548	40
	8			3852	42		14			5632	58
	15			6345	50		20			3867	70
	26			2045	65		22			15364	80

2. In the following account find by the English method the monthly cash balance, allowing interest at 6%. Check the first month's result by the method explained on page 115, the second month by the French method, and the third month by the German method:

THE EUROPEAN BANK

Apr.	3			5643	25	Apr.	7			7923	50
	11			8438	50		14			4938	60
	18			6372	96		20			9537	90
	28			5976	80		25			3721	75
May	4			8724	37		29			6372	95
	15			7486	50	May	10			1972	00
	20			9538	38		17			8500	00
	24			7219	11		26			5807	06
June	1			6400			28			5775	00
	12			3750		June	5			8327	65
	15			7500			11			9584	21
	21			7231	13		18			2132	60
							27			3218	75

X. FOREIGN EXCHANGE

Foreign exchange treats, broadly speaking, of the settlement of international indebtedness by way of bills of exchange (checks, etc.), by telegraphic transfers, or by the actual shipment of gold (or silver). It treats also of instruments of credit in the form of letters of credit, travelers' cheques, etc.

A **bill of exchange** is a draft for a stipulated amount drawn in one country on another country, payable, according to its tenor, either on presentation (as checks, demand, and sight bills) or after a stated period of days or months.

It may be a documentary bill of exchange, that is, it may have documents attached (bills of lading, insurance, inspection, weighing, certificates, invoice, etc.) in evidence of the underlying transaction; or it may be a clean bill of exchange, that is, it may have no documents attached. In the case of a documentary draft, the documents, according to a memorandum attached, may be delivered to the drawee either on actual payment of the draft or on his mere promise to pay the draft at maturity (called **acceptance**), which takes the form of a written notice across the face of the draft, "Accepted, payable on (date)," with the signature of the acceptor. Before the consignee of the underlying goods can get possession of his bills of lading and other documents, he must, therefore, either pay or accept — as the case may be — the bill of exchange drawn on him.

Bills of exchange are generally drawn in sets of two, called either "First and Second of Exchange," or, in the case of checks and demand drafts, "Original and Duplicate." These are sent by different mails, but the payment of one of them cancels the other.

A bill of exchange payable on demand and drawn by one banker on another is generally called a **check**. A check is drawn against funds on deposit with the drawee.

Sight bills and **demand bills** are generally drawn by individuals or mercantile firms. They may be drawn on other individuals or firms, or on a bank.

Time bills may be drawn either by banks or by individuals. If the time is 30 days or less, they are called short bills; if the time is more than 30 days, they are called long bills.

Sight or demand bills and **time bills** do not imply any deposit of funds in the hands of the drawee to the credit of the drawer.

A SET OF EXCHANGE

1	Exchange for _____ <i>Boston, U. S. A.</i> _____
	_____ <i>days after sight of this First</i>
	<i>of Exchange (Second unpaid) pay to the Order of</i>
	Brown, Lee, and Company
	Value received and charge the same to account of
	To _____ } _____
	No. _____ } _____

2	Exchange for _____ <i>Boston, U. S. A.</i> _____
	_____ <i>days after sight of this Second</i>
	<i>of Exchange (First unpaid) pay to the Order of</i>
	Brown, Lee, and Company
	Value received and charge the same to account of
	To _____ } _____
	No. _____ } _____

NOTE. Checks on Great Britain having "and Company" written cross-wise on their face are payable only through the mediation of a bank. This necessitates identification of the payee and prevents misuse.

ORIGINAL	No.	Brown, Lee, and Company	
	£	Stg	Boston, 19
	On Demand (Duplicate being unpaid) pay		
	to the order of		
	Sterling Value received		
	To	Brown, Lee, and Company, 180 Lombard Street, London, E.C. 5.	

DUPLICATE	No.	Brown, Lee, and Company	
	£	Stg	Boston, 19
	On demand (Original being unpaid) pay		
	to the order of		
	Sterling Value received		
	To	Brown, Lee, and Company, 180 Lombard Street, London, E.C. 5.	

SOLA	No.	Brown, Lee, and Company	
	£	Stg	Boston, 19
	On demand pay to the order of		
	Sterling Value received		
		To	Brown, Lee, and Company, 180 Lombard Street, London, E.C. 5.

Foreign bills of exchange are generally negotiable instruments covering a multitude of international transactions, the most important of which are :

1. The import or export of merchandise from one country to another.

2. The purchase or sale of securities — public or private — of one country, from or to the other.

3. The investment or loaning of funds in countries (markets) giving a higher interest return, and the reflux of such funds upon a change in monetary conditions.

4. The providing of funds or settling of expenses of domestic travelers in foreign countries, and the remittances of aliens to their mother countries.

A revenue stamp (frequently called a “bill stamp”) is required on foreign drafts as follows :

1. *Great Britain.* All drafts above 3 days' sight are subject to $\frac{1}{20}\%$ bill stamp.

2. *France.* All drafts, not checks, are subject to $\frac{1}{20}\%$ bill stamp. In order to exempt checks from bill stamp, it is necessary to spell the month and day with letters ; for instance, “December eighth, 1923.”

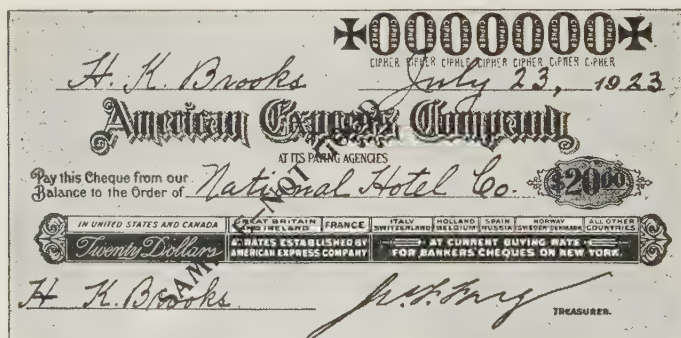
3. *Germany.* All drafts, not checks, that are not drawn on a German bank or banker are subject to $\frac{1}{20}\%$ bill stamp ; if drawn above 90 days' sight they are subject to $\frac{1}{10}\%$ bill stamp. In order to make checks exempt from bill stamp the following clause is necessary : “Drawn against balance.”

A **cable transfer** is a cabled order to pay a specified sum of money to a certain firm or person in a foreign country. In cable transfers between private parties, the cost is the cable rate of exchange plus $\frac{1}{4}\%$ commission plus the telegraph and cable charges. In cable transfers in trades between bankers, for a minimum amount of about \$50,000, there is generally no commission. Cable charge on transactions between bankers is \$2.50.

A **travelers' letter of credit** is a circular letter issued by a bank or banker, introducing the beneficiary and authorizing him to draw

money on demand from a large number of institutions all over the world up to the inscribed face value of the letter of credit. Foreign letters of credit, prior to 1914, were generally issued in pounds sterling. Since the World War, however, many letters of credit originating in the United States have been issued in dollars because of the superior value of dollar exchange throughout the world. When wishing to draw funds, the traveler presents his letters of credit to the most convenient bank or banker, states the amount he wishes to draw, and is required to sign a demand draft in dollars on the American bank or banker issuing the letter of credit. After careful comparison of his signature with the one appearing on his letter of credit, the foreign banker pays over to him the equivalent of the amount of the draft, and, after writing off the amount drawn on the letter of credit, returns it to the traveler.

A commission of 1% is charged by the issuing banker in addition to the face value of the letter of credit issued in foreign currency and $\frac{1}{2}\%$ on letter of credit issued in dollars.



A travelers' cheque is a circular cheque made payable for a stipulated amount of money in the currency indicated by the denomination of the cheque or an equivalent amount of other currency as stipulated on the face of the cheque.

A United States dollar travelers' cheque is made payable in United States dollars, or in an equivalent amount of other currency determined by the "current buying rate for bankers' cheques on New York." Such cheques are issued in denominations of \$10, \$20, \$50, \$100, and \$200.

Brown, Lee, and Company

CABLE ADDRESS
BROWNLEE, BOSTON,
U. S. A.

N. B.
Drafts to be drawn on
Clark and Company's
West End Office
480, Strand, London, W. C.

Travelers' Letter of Credit

Boston, Mass., _____ 19____

No. _____

Gentlemen:

This Letter will be presented to you by

whom we commend to your usual courtesies.

*We hereby authorize _____ to draw upon
Clark and Company, of London, eight drafts
to an aggregate amount of £ _____ Ster-
ling, say _____ Pounds Ster-
ling which you will please negotiate (less your
charges) according to our agreement with you.*

*Each draft must be plainly marked as drawn
under Brown, Lee, and Company's Travelers'
Letter of Credit No. _____ and we engage that
the same shall meet due honor by Clark and
Company if drawn before _____
provided the issue of such drafts is indorsed on
the back hereof and this credit has not been ex-
hausted by drafts and indorsements prior to
said date.*

*M _____ furnished with a
Letter of Indication which containing _____
signature will serve to identify _____ and we do
not therefore send you any advice of this Letter*

We are, Gentlemen,

Your obedient servants

*To the Correspondents
mentioned in our Letter
of Indication*

*The Bearer is requested on his return to Boston to send back
this Letter to Brown, Lee, and Company*

Gold is the universal standard of value in international financial transactions, because in each country a fixed value has been given to a given weight of gold of a given fineness. The mint par of exchange is the value of the pure metal contained in the monetary unit of one country expressed in terms of the monetary unit of another country of the same standard.

When the mint laws of two countries are known, the mint par of exchange is easily determined. For example :

The United States coins \$10 out of 258 grains of gold, $\frac{9}{10}$ fine.

England coins £1 out of 123.274 grains of gold, $\frac{11}{12}$ fine.

France coins 3100 francs out of 1 kilogram of gold, $\frac{9}{10}$ fine.

Germany coins 2790 marks out of 1 kilogram of fine gold.

NOTE. Standard gold is $\frac{9}{10}$ fine in the United States, France, Germany, and most other countries. It is $\frac{11}{12}$ fine in England and a few other countries.

By using the chain rule,* parity between England and the United States is determined as follows :

\$? = £1

1 = 123.274 gr. ($\frac{11}{12}$ fine)

12 = 11 fine

9 = 10 standard ($\frac{9}{10}$ fine)

258 = \$10

$$\frac{123.274 \times 11 \times 10 \times \$10}{12 \times 9 \times 258} = \$4.866544$$

Hence £1 = \$4.8665.

The "1" in the second equation is £1. The "12" in the third equation is 12 grains English standard gold. "9" in the fourth equation is 9 grains fine gold. "258" in the fifth equation is 258 grains U. S. standard gold.

The solution consists in dividing the product of all factors on the right side by the product of all factors on the left side. The quotient is the desired result.

*The "chain" consists of a series of equations. Each equation constitutes a "link" in the "chain." Since links in a chain must be fastened together, so each equation must be "linked" to the equation following it.

The following principles must be borne in mind :

1. The first equation states the question to be solved.
2. The first member of each equation is of the same denomination as the second member of the preceding equation. (This principle is what connects one "link" with another.)
3. The second member of the last equation is of the same denomination as the first member of the first equation.

Parity between the United States and Germany is found in a similar manner.

$$\begin{aligned}
 ?\text{¢} &= \text{M. } 1 \\
 2790 &= 1 \text{ Kg. fine gold} \\
 1 &= 15432 \text{ grains} \\
 9 &= 10 \text{ standard } (\frac{9}{10} \text{ fine}) \\
 25.8 &= 100\text{¢} \\
 \frac{15432 \times 10 \times 100\text{¢}}{2790 \times 9 \times 25.8} &= 23.82\text{¢}
 \end{aligned}$$

That is, 1 gold mark is worth \$.238.

To find parity between France and Germany, the "chain" would be:

$$\begin{aligned}
 \text{F. ?} &= \text{M. } 1 \\
 2790 &= 1 \text{ Kg. fine gold} \\
 9 &= 10 \text{ standard } (\frac{9}{10} \text{ fine}) \\
 1 &= \text{F. } 3100 \\
 \frac{10 \times \text{F. } 3100}{9 \times 2790} &= \text{F. } 1.2345
 \end{aligned}$$

Hence, 1 mark = 1.2345 francs.

Prior to the World War the most important exchanges were quoted as follows:

$$\begin{aligned}
 \text{£}1 &= \$4.8665 \\
 \$1 &= \text{F. } 5.18\frac{1}{8} \\
 \text{M. } 4 &= \$95\frac{1}{4} \\
 1 \text{ florin} &= \$40
 \end{aligned}$$

Sterling rates varied by \$.0005 or \$.00025.

French rates varied by $\frac{5}{8}$ of a centime, as $5.18\frac{1}{8}$; $5.18\frac{3}{4}$; $5.19\frac{3}{8}$, etc.

These quotations were modified by adding or subtracting $\frac{1}{64}\%$, $\frac{1}{32}\%$, $\frac{1}{16}\%$, etc. Thus, to express a rate about midway between $5.18\frac{3}{4}$ and $5.19\frac{3}{8}$, the form of the quotation would be $5.18\frac{3}{4} - \frac{1}{16}$ or $5.19\frac{3}{8} + \frac{1}{16}$.

So, also, the mark and florin quotations varied by sixteenths, eighths, and quarters of 1 cent, and these quotations were in turn modified by adding or subtracting small fractions of 1%.

One effect of the war has been, for the present, at least, to bring about in certain exchanges a so-called "direct" method of quotation in the foreign exchange field. The current quotation (March 6, 1923) of some of the more important exchanges is

£1 = \$4.69 $\frac{7}{8}$	written conventionally	4.69 $\frac{7}{8}$
F. 1 = \$.0610	written conventionally	6.10
M. 1 = \$.0044	written conventionally	.44
1 florin = \$.3947	written conventionally	39.47

These quotations vary by from \$.000025 to \$.00025 per unit of foreign coin.

The advantage of this method of quoting is that it is less confusing to the average man than the "indirect" method.

Normally there are five quotations per unit of foreign coin. They are for exchange by cable, on demand, at 30 days, at 60 days, and at 90 days. The basis rate from which other rates are determined is the demand rate (sometimes called sight, or check, rate).

When a banker sells demand exchange, he has the use of the sale price of the draft for at least 6 to 10 days — that is, until the draft can be sent to its destination and presented for payment. In selling cable exchange he has at most only about 3 days' use of the funds. Hence, cable rates are higher than demand rates.

30-, 60-, and 90-day rates are lower than demand rates for the reason that the banker has the use of the funds for a longer time than when selling demand exchange.

Under normal conditions bankers and others dealing in foreign exchange have tables showing interest deductions and bill stamp charges which they use in determining the 30-, 60-, and 90-day quotations. The procedure of finding a time rate is as follows:

Assuming a demand rate of 4.8625, let it be required to find a 90-day rate, allowing discount at 4 $\frac{1}{2}$ %, and bill stamp $\frac{1}{10}$ %.

Demand rate is	4.8625
The exact interest on \$4.85 for 93 days at 4 $\frac{1}{2}$ % is	.0556
Bill stamp charge is $\frac{1}{10}$ % of \$4.85 or	<u>.0024</u>
Making a total deduction of	<u>.0580</u>
Hence the 90-day price is	4.8045

Three days of grace are allowed in England and interest is reckoned on the 365-day basis. Under normal conditions the interest (discount) and the bill stamp are both reckoned on a standard value of \$4.85 per pound sterling. The discount and the bill stamp together make a total deduction per pound of \$.0580. Hence the banker's buying price of a 90-day sight bill would be 4.8045 under the conditions stated.

When rates are abnormally low, as they were following the World War, the solution would be as follows:

Assume all conditions as given above, save that demand exchange is at 3.75.

Demand rate is	3.75
The exact interest on \$3.75 for 93 days at $4\frac{1}{2}\%$ is	.043
Bill stamp charge is $\frac{1}{20}\%$ of 3.75 or	<u>.0019</u>
Making a total deduction of	.0449
Hence the 90-day price is	<u>3.7051</u>
This price would be quoted as $3.70\frac{1}{2}$.	

This solution differs from the preceding one only in that the interest (discount) and bill stamp charge are reckoned on the demand rate instead of on some fixed standard value. In normal times 4.85 is about midway between the high and low demand rates. The use of a standard rate on which to reckon interest and bill stamp charges permits of the construction and use of a table showing amount of interest and bill stamp charges.

To convert a demand rate on Paris into a time rate (using pre-war quotations) the solution is as follows:

If demand exchange on Paris is $5.17\frac{1}{2}$, find the equivalent 90-day quotation when interest is at 4% , and bill stamp $\frac{1}{20}\%$.

Demand exchange is	5.1750
Interest* on F. 5.20 for 90 days at 4% is	.0520
Bill stamp, $\frac{1}{20}\%$ of F. 5.20, is	<u>.0026</u>
Hence the value of the 90-day quotation is	5.2296

French quotations are not expressed as 5.2296, but as regular, standard values, plus or minus a fraction. The nearest regular quotation to 5.2296 is $5.23\frac{1}{8}$. To determine the fraction, proceed as on page 130.

* Interest is reckoned on a standard value of F. 5.20 to the dollar, on a 360-day basis, and no days of grace.

$$5.23125 - 5.2296 = .00165$$

$$.00165 \div 5.23125 = .00031$$

$$.00031 = \frac{31}{10000}\% = \text{approximately } \frac{1}{32}\%$$

$$\text{Therefore } 5.2296 = 5.23\frac{1}{8} + \frac{1}{32}$$

The possible denominators of the fractions usable in these cases are 16, 32, 64, or 128. Hence in reducing the decimal to its equivalent common fraction per cent form, only a close approximation can be expected. In the foregoing illustration the sign is *plus* because 5.2296 is a higher quotation than $5.23\frac{1}{8}$.

Find the time quotations as indicated corresponding to the following demand quotations:

	<i>Demand rate</i>	<i>Time</i>	<i>Interest</i>
1.	4.6975	60 da.	$4\frac{1}{2}\%$
2.	4.6850	90 da.	$3\frac{7}{8}\%$
3.	$5.18\frac{1}{8}$	90 da.	$4\frac{1}{4}\%$
4.	$5.19\frac{3}{8}$	30 da.	$4\frac{3}{8}\%$

INTEREST ON STERLING EXCHANGE

Computed on basis of \$4.85 per pound sterling (365 days) with 3 days' grace and bill stamp $\frac{1}{20}\%$ -0.24¢.

RATE	33 DA. AND STAMP	63 DA. AND STAMP	93 DA. AND STAMP	RATE	33 DA. AND STAMP	63 DA. AND STAMP	93 DA. AND STAMP
$\frac{1}{2}$	.46	.66	.86	$2\frac{3}{4}$	1.45	2.54	3.64
$\frac{5}{8}$	.52	.77	1.01	$2\frac{7}{8}$	1.50	2.65	3.80
$\frac{3}{4}$	.57	.87	1.17	3	1.56	2.75	3.95
$\frac{7}{8}$	.63	.98	1.32	$3\frac{1}{8}$	1.61	2.86	4.10
1	.68	1.08	1.48	$3\frac{1}{4}$	1.67	2.96	4.26
$1\frac{1}{8}$	.74	1.18	1.63	$3\frac{3}{8}$	1.72	3.07	4.41
$1\frac{1}{4}$	.79	1.29	1.79	$3\frac{1}{2}$	1.78	3.17	4.57
$1\frac{3}{8}$	.85	1.39	1.94	$3\frac{5}{8}$	1.83	3.28	4.72
$1\frac{1}{2}$	.90	1.50	2.10	$3\frac{3}{4}$	1.89	3.38	4.88
$1\frac{5}{8}$	.96	1.60	2.25	$3\frac{7}{8}$	1.94	3.49	5.03
$1\frac{3}{4}$	1.01	1.71	2.40	4	2.00	3.59	5.19
$1\frac{7}{8}$	1.06	1.81	2.56	$4\frac{1}{8}$	2.05	3.70	5.34
2	1.12	1.92	2.71	$4\frac{1}{4}$	2.11	3.80	5.49
$2\frac{1}{8}$	1.17	2.02	2.87	$4\frac{3}{8}$	2.16	3.90	5.65
$2\frac{1}{4}$	1.23	2.13	3.02	$4\frac{1}{2}$	2.22	4.01	5.80
$2\frac{3}{8}$	1.28	2.23	3.18	$4\frac{5}{8}$	2.27	4.11	5.96
$2\frac{1}{2}$	1.34	2.34	3.33	$4\frac{3}{4}$	2.33	4.22	6.11
$2\frac{5}{8}$	1.39	2.44	3.49	5	2.43	4.43	6.42

With reference to the solution on page 129 note that the interest \$.0556 and the bill stamp charge \$.0024 are together equal to \$.0580 (that is, 5.80¢), the amount given in the table for 93 days' interest and bill stamp.

Foreign exchange bankers and brokers use conversion tables for determining the amount of foreign currency equivalent to a given amount of United States money, or vice versa. The following is a sample of one such conversion table.

CONVERSION OF STERLING INTO DOLLARS AND CENTS, AND VICE
VERSA

RATE 4.4675

	0	£100	£200	£300	£400	£500
0		\$446.7500	\$893.5000	\$1340.2500	\$1787.0000	\$2233.7500
1	\$ 4.4675	451.2175	897.9675	1344.7175	1791.4675	2238.2175
2	8.9350	455.6850	902.4350	1349.1850	1795.9350	2242.6850
3	13.4025	460.1525	906.9025	1353.6525	1800.4025	2247.1525
4	17.8700	464.6200	911.3700	1358.1200	1804.8700	2251.6200
5	22.3375	469.0875	915.8375	1362.5875	1809.3375	2256.0875
6	26.8050	473.5550	920.3050	1367.0550	1813.8050	2260.5550
7	31.2725	478.0225	924.7725	1371.5225	1818.2725	2265.0225
8	35.7400	482.4900	929.2400	1375.9900	1822.7400	2269.4900
9	40.2075	486.9575	933.7075	1380.4575	1827.2075	2273.9575
10	44.6750	491.4250	938.1750	1384.9250	1831.6750	2278.4250

	0s.	1s.	2s.	3s.	4s.	5s.
0d.		\$.2234	\$.4467	\$.6701	\$.8935	\$1.1169
1d.	\$.0186	.2420	.4654	.6887	.9121	1.1355
2d.	.0372	.2606	.4840	.7074	.9307	1.1541
3d.	.0558	.2792	.5026	.7260	.9493	1.1727
4d.	.0745	.2979	.5212	.7446	.9680	1.1913
5d.	.0931	.3165	.5398	.7632	.9866	1.2099
6d.	.1117	.3351	.5584	.7818	1.0052	1.2286
7d.	.1303	.3537	.5770	.8004	1.0238	1.2472
8d.	.1489	.3723	.5957	.8190	1.0424	1.2658
9d.	.1675	.3909	.6143	.8377	1.0610	1.2844
10d.	.1861	.4095	.6329	.8563	1.0796	1.3030
11d.	.2048	.4282	.6515	.8749	1.0983	1.3216

VALUES OF FOREIGN COINS AS PROCLAIMED BY THE TREASURY
DEPARTMENT, WASHINGTON, D. C. (April 1, 1923)

COUNTRY	LEGAL STANDARD	MONETARY UNIT	VALUE IN TERMS OF U. S. MONEY
Argentine Republic	Gold	Peso	\$0.9648
Austria	Gold	Krone	.2026
Belgium	Gold and silver	Franc	.1930
Bolivia	Gold	Boliviano	.3893
Brazil	Gold	Milreis	.5462
British Colonies in Aus- tralia and Africa	Gold	Pound Sterling	4.8665
Bulgaria	Gold	Lev	.1930
Canada	Gold	Dollar	1.0000
Chile	Gold	Peso	.3650
China	Silver	{ Tael Dollar	.7174 to .7991 .5089 to .5204
Colombia	Gold	Peso	.9733
Cuba	Gold	Peso	1.0000
Denmark	Gold	Krone	.2680
Ecuador	Gold	Sucre	.4867
Egypt	Gold	Pound (100 piasters)	4.9431
Finland	Gold	Markka	.1930
France	Gold and silver	Franc	.1930
Germany	Gold	Mark	.2382
Great Britain	Gold	Pound sterling	4.8665
Greece	Gold and silver	Drachma	.1930
Guatemala	Silver	Peso	.4791
India (British)	{ Gold Silver	{ Mohur and sovereign Rupee	{ 4.8665 .2277
Indo-China	Silver	Piaster	.5174
Italy	Gold	Lira	.1930
Japan	Gold	Yen	.4985
Mexico	Gold	Peso	.4985
Netherlands	Gold	Guilder (Florin)	.4020
Newfoundland	Gold	Dollar	1.0000
Norway	Gold	Krone	.2680
Panama	Gold	Balboa	1.0000
Paraguay	Gold	Peso	.9648
Persia	Silver	Kran	.0882
Peru	Gold	Libra	4.8665
Philippine Islands	Gold	Peso	.5000
Portugal	Gold	Escudo	1.0805
Rumania	Gold	Leu	.1930
Russia	Gold	Ruble	.5146
Spain	Gold and silver	Peseta	.1930
Sweden	Gold	Krona	.2680
Switzerland	Gold	Franc	.1930
Turkey	Gold	Piaster	.0440
Uruguay	Gold	Peso	1.0342
Venezuela	Gold	Bolivar	.1930

EXAMPLES

1. Find the cost of a sterling sight draft of £1925 9s. 8d. at $4.45\frac{1}{8}$.

$$8d. = .66\frac{2}{3}s.$$

$$9.66\frac{2}{3}s. = £.48\frac{1}{3}$$

$$\text{Hence } £1925 \text{ 9s. 8d.} = £1925.48\frac{1}{3}$$

$$1925.48\frac{1}{3} \times \$4.45\frac{1}{8} = \$8570.81, \text{ cost}$$

2. How large a banker's check on London may be purchased for \$2750 when sight exchange is selling at $4.45\frac{1}{4}$?

$$\$2750 \div \$4.45\frac{1}{4} = 617.6305, \text{ number of pounds that can be bought}$$

$$.6305 \times 20s. = 12.61s.$$

$$.61 \times 12d. = 7.32d.$$

Hence sterling check is £617 12s. 7d.

3. Find the proceeds of a 60-day sight draft on London of £3475 10s. when demand exchange is at $4.46\frac{1}{2}$, interest $4\frac{1}{4}\%$, bill stamp $\frac{1}{20}\%$, and commission $\frac{1}{8}\%$.

Demand exchange is	\$4.465
Int. on \$4.465 at $4\frac{1}{4}\%$ for 63 da. is	.03275
Bill stamp, $\frac{1}{20}\%$ of \$4.465 is	.00223
Commission $\frac{1}{8}\%$ of \$4.465 is	<u>.00558</u>
Total deduction is	.04056
Proceeds of £1	<u>\$4.42444</u>

$$£3475 \text{ 10s.} = £3475.5$$

$$3475.5 \times \$4.42444 = \$15,377.14, \text{ proceeds}$$

4. Find the cost of a 90-day sight draft on London of £5482 4s. when demand exchange is at $4.47\frac{1}{4}$, interest $4\frac{3}{4}\%$, bill stamp $\frac{1}{20}\%$, and commission $\frac{1}{8}\%$.

Demand exchange is	\$4.4725
Int. on \$4.4725 for 93 da. at $4\frac{3}{4}\%$ is	.05413
Bill stamp $\frac{1}{20}\%$ of \$4.4725 is	.00223
Commission $\frac{1}{8}\%$ of \$4.4725 is	<u>.00559</u>
	<u>.00782</u>
	.04631
Cost of £1	<u>\$4.42619</u>

$$£5482 \text{ 4s.} = £5482.2$$

$$5482.2 \times \$4.42619 = \$24,265.26, \text{ cost}$$

Problems

Find the cost of the following sight bills :

- | | |
|---|---|
| 1. £937 5s. 6d. at $4.45\frac{1}{4}$ | 5. M. 38,750 at $1.16\frac{7}{8}\text{¢}$ |
| 2. £1156 6s. 7d. at $4.43\frac{7}{8}$ | 6. F. 29,040 at $8.75\frac{1}{2}\text{¢}$ |
| 3. £3145 9d. at $4.41\frac{3}{8}$ | 7. F. 48,150 at $8.87\frac{3}{4}\text{¢}$ |
| 4. M. 15,490 at $1.17\frac{1}{4}\text{¢}$ | 8. Y. 33,985 at 49.875¢ |

Find how large a banker's check could be bought for each of the following amounts :

- | | |
|---|---|
| 9. \$2500 at $4.47\frac{1}{4}$ | 13. \$1250 at $1.121\frac{3}{4}\text{¢}$ per mark |
| 10. \$15,925 at $4.47\frac{7}{8}$ | 14. \$4500 at $1.19\frac{3}{4}\text{¢}$ per mark |
| 11. \$750 at $8.67\frac{3}{4}\text{¢}$ per franc | 15. \$15,000 at 49.125¢ per yen |
| 12. \$1875 at $8.93\frac{3}{8}\text{¢}$ per franc | 16. \$3692.50 at 31.73¢ per florin |

Using the chain rule find :

17. How many gold marks are equal to \$1.
18. How many francs are equal to £1.
19. Find the cost of a draft at 60 days' sight on London of £992 5s. 9d. when demand exchange is at 4.4775, interest at $3\frac{3}{4}\%$, and bill stamp $\frac{1}{20}\%$.
20. How much can a banker afford to pay for a bill on London for £355 16s. 3d., 60 days after sight, demand rate 4.4815, interest 4%, bill stamp $\frac{1}{20}\%$, and commission $\frac{1}{4}\%$?
21. J. P. Morgan remits to London via Paris when the rates are as follows :—New York on London, 4.8675 ; New York on Paris, $5.19\frac{3}{8}$ (francs to the dollar) ; Paris on London, 25.04 (francs to the pound). How much is the gain on a remittance of £50,000?
22. Find the proceeds on May 15 of a sterling draft of £3475 7s. 4d., drawn March 27 at 90 days' sight, and accepted April 10, interest $4\frac{1}{4}\%$, bill stamp $\frac{1}{20}\%$, commission $\frac{1}{8}\%$, and demand exchange at $4.46\frac{3}{4}$.
23. On Oct. 28 a New York exporter shipped a cargo of wheat to a customer in London. The same day he drew a draft of £28,945 8s. at 60 days' sight on the London firm against the shipment, attached the bill of lading, insurance certificate, etc., and discounted it at the Chemical National Bank. The demand rate for the day was $4.44\frac{1}{2}$;

the London discount rate was $4\frac{3}{4}\%$, bill stamp $\frac{1}{20}\%$, and banker's commission $\frac{1}{32}\%$. What were the net proceeds of the draft?

GOLD EXPORTS AND IMPORTS

Imports of merchandise are paid for either by buying here and forwarding a bankers' bill of exchange, or by accepting and paying here a draft drawn by the exporter on the other side. By the former method, the banker issuing the bill maintains a balance on the other side to meet the draft. By the latter method, payment of the draft is made to an American banker, and he forwards the proceeds. In either case, funds tend to accumulate here.

Exports of merchandise are paid for on the other side in practically the same manner, and the tendency is for funds to accumulate over there.

In case of an excess of imports over exports, there will be a greater demand here for drafts payable to the foreign merchant than on the other side for drafts payable here. This excessive demand for cashing drafts payable in a foreign country gives rise to the necessity for gold exports. Likewise an excess of exports over imports would have a tendency to cause gold to flow in the opposite direction.

At the United States mint an ounce of fine gold is worth \$20.67183. No seigniorage charge is made for converting standard gold bullion into gold coins. The mint does, however, charge 2¢ per ounce for the copper required to standardize gold bullion. The United States Treasury charges 40¢ per \$1000 premium (called **bar charge**) when selling gold bars.

The Bank of England's minimum buying price for United States gold coin is 76*s.* 3½*d.* per ounce; for gold bars, $\frac{11}{12}$ fine, it is 77*s.* 9*d.* per ounce. Normally, the maximum price is, for gold coin 76*s.* 7*d.*, and for bars 78*s.* 1*d.*

The expenses of a shipment of gold are about as follows:

Freight, $\frac{3}{16}\%$ of the value of the gold.

Insurance, $\frac{1}{20}\%$ of the value of the gold.

Boxing, carting, loss of interest, etc., $\frac{1}{10}\%$.

Bar charge at United States Treasury when selling gold bars, 40¢ per \$1000.

In addition to the foregoing expenses, there is a slight loss of weight, variously estimated at from $\frac{1}{20}\%$ to $\frac{1}{10}\%$, due to abrasion when shipping gold coin.

The Bank of France pays 3437* francs per kilogram of fine gold. United States gold coin is reckoned by the Bank of France to be .8992 fine instead of .900 fine.

The Reichsbank (of Germany) pays 2784† marks per kilogram of fine gold. United States gold coin is accepted as being .900 fine.

EXAMPLES

1. When the Bank of England is paying 77*s.* 10*d.* per ounce for gold bars $\frac{1}{2}$ fine, at what price per pound sterling would it become profitable to export gold instead of buying exchange to settle a balance due in London? (Expenses of the shipment are given on page 135.)

£1 = 240*d.*

77*s.* 10*d.* = 934*d.* = 1 oz. standard gold ($\frac{1}{2}$ fine English standard)

934*d.* $\div \frac{1}{2}$ = 1018.909*d.* = 1 oz. fine gold

1 oz. fine gold = \$20.67183 (U. S. Treasury buying and selling price)

Therefore the parity of exchange is

$(\$20.67183 \div 1018.909) \times 240 = \$4.8692 = £1$

Cost of gold to produce £1	\$4.8692
----------------------------	----------

Add: Freight, about $\frac{3}{16}\%$	.0091
--------------------------------------	-------

Insurance, about $\frac{1}{20}\%$	.0024
-----------------------------------	-------

Boxing, carting, loss of interest, etc., about $\frac{1}{20}\%$	.0024
---	-------

Bar charge, 40¢ per \$1000	.0019
----------------------------	-------

Total cost of £1	<u>\$4.8850</u>
------------------	-----------------

Hence if the price of demand exchange rose above 4.8850, it would be cheaper to export gold.

The importation of gold is a somewhat different problem. Gold is imported when the price of exchange is low here. The exchange rate declines when there is offered for sale a quantity of bills of exchange in excess of the demand for such bills.

*The *coining value* of a kilogram of fine gold is 3444 $\frac{2}{3}$ francs (3100 francs $\div .9$). The Bank of France makes a seigniorage charge of 6.7 francs per kilogram when buying gold. Hence the *mint* price of a kilogram of fine gold is 3437 francs [(3100 francs - 6.7 francs) $\div .9$].

†Seigniorage charge is 6 marks per kilogram.

To buy the gold in London the American banker forwards a sufficient quantity of bills of exchange to the Bank of England to buy at their selling rate the quantity of gold he wishes.

2. An American banker wishes to import gold to the value of about \$2,500,000 in the form of English standard bars, at a price of 77*s.* 10*d.* per ounce. Allowing $\frac{3}{16}\%$ for freight, $\frac{1}{20}\%$ for insurance, $\frac{1}{32}\%$ for boxing and carting, and interest at $4\frac{1}{2}\%$ for 16 days, find the rate of exchange below which it would be profitable for the banker to import gold.

$$\begin{aligned} \$2,500,000 \div \$20.67183 &= 120,937.5 \text{ oz. of fine gold} \\ 120,937.5 \text{ oz.} \div \frac{11}{12} &= 131,921.8 \text{ oz. standard gold } (\frac{11}{12} \text{ fine}) \\ 77*s.* 10*d.* &= 934*d.* \end{aligned}$$

$$\text{Cost of gold in London} = \frac{131,931.8 \times 934*d.*}{240*d.*} = \text{£}513,434.6$$

Expenses in London

Freight $\frac{3}{16}\%$	962.69
Insurance $\frac{1}{20}\%$	256.72
Boxing and carting $\frac{1}{32}\%$	160.45
Total cost in sterling	<u>£514,814.46</u>

Gross proceeds of importation \$2,500,000

Deduct:

Loss of interest * 16 days at $4\frac{1}{2}\%$	<u>4,990</u>
Present worth of proceeds	\$2,495,010

$\$2,495,010 \div 514,814.46 = \$4.8464$, cost per pound of importing gold.

* The considerable item of interest arises from the fact that the American banker first buys a sufficient quantity of bills of exchange drawn on London to pay for the gold he wishes to import. He forwards them to London with an order for the gold. The time required for the bills to reach London and for the gold to reach New York (or other American city) is upwards of 2 weeks, depending on conditions. The 16 days of the problem is an arbitrary number in this case. The banker would use the exact number of days involved. The interest is determined by finding the true discount for 16 days on the gross proceeds.

Bankers' Finance Bills. When an American banker draws a long bill of exchange on a foreign bank with the intention of himself redeeming the bill at maturity, such a bill is called a "finance bill." Its prime purpose is to provide funds for the use of the American banker for the term of the bill. It is a method of borrowing. It is accomplished by having what is known as an "acceptance account" with the foreign bank. Arrangements are made in advance whereby the foreign bank will accept bills drawn on it up to a certain agreed limit. Having completed these arrangements, all the American has to do is to draw a 60-day or a 90-day bill, and either (1) sell it on this side at the prevailing rate of exchange for 60-day or 90-day bills, or (2) send it to the foreign bank to be discounted and placed to his credit there. In the first case the funds are immediately available here for whatever use is desired of them, and in the second case, the funds are available as a deposit against which the American banker may sell demand bills at once.

EXAMPLES

1. A New York banker sells a 90-day bill of £10,000 on a London bank when demand exchange is at 4.25, London discount rate is $3\frac{1}{2}\%$, bill stamp $\frac{1}{20}\%$, and commission $\frac{1}{16}\%$. Allowing that interest in New York is at 4% and that a demand draft can be bought 93 days later at $4.23\frac{1}{2}$, to cover the long bill when due, determine the New York banker's gain or loss by the transaction.

Demand exchange is at		4.25
Less 93 days' interest at $3\frac{1}{2}\%$	.0379	
$\frac{1}{20}\%$ bill stamp	<u>.002125</u>	.040025
Proceeds of £1 at 90 days' sight		<u>4.209975</u>

STERLING LOAN ACCOUNT

<i>Cost</i>			<i>Proceeds</i>
£10,000 at $4.23\frac{1}{2}$	\$42,350	£10,000 at 4.209975	\$42,099.75
Commission $\frac{1}{16}\%$	26.47	Interest 93 da. at 4%	429.07
Net profit	152.35		
	<u>\$42,528.82</u>		<u>\$42,528.82</u>

While the rates and prices in this illustration are chosen arbitrarily, they reflect a condition that is by no means unusual in the foreign exchange market. At certain periods of the year demand rates on London tend to fluctuate, depending on an increase or a decrease of exports and imports. Rates of interest and discount may vary. Any material change in the rates, etc. will, of course, affect the net profit.

If the New York banker, instead of selling the draft in New York, had sent it to London for discount, he would have established a credit in London as follows :

DISCOUNT STATEMENT

90-day sight draft		£10,000
Interest 93 da. at $3\frac{1}{2}\%$	£89 - 3 - 7	
Bill stamp $\frac{1}{20}\%$	5 - 0 - 0	94 - 3 - 7
Net proceeds of discounted bill		£9905 - 16 - 5
Sale of check at demand £9905 - 16 - 5 @ 4.25		\$42,099.76
Interest earned 93 days at 4%		429.07
		<u>\$42,528.83</u>
Cost of check to cover at maturity		
£10,000 at $4.23\frac{1}{2}\%$	\$42,350	
Commission $\frac{1}{8}\%$	26.47	\$42,376.47
Net profit		<u>\$152.36</u>

Problems

1. An importer can purchase certain goods in Amsterdam for 12,000 guilders at $38\frac{3}{4}$ (cents per guilder). He can purchase the same goods in Paris for F. 52,500, at 8.75 (cents per franc). Which is the cheaper, and how much, in United States money?

2. What are the proceeds of a documentary bill of exchange on London for £528 8s. 6d. at 60 days' sight, exchange at $4.47\frac{1}{2}$, interest 4%, stamp $\frac{1}{20}\%$, commission $\frac{1}{8}\%$?

3. Johnson and Company import an invoice of statuettes from Florence, Italy, amounting to 14,725.35 lira. They buy a 30-days' sight draft on a Florence banker. If demand exchange is at 4.25

(cents per lira), discount 3%, stamp $\frac{1}{20}\%$, commission $\frac{1}{8}\%$, find the cost of the draft.

4. A banking concern dealing in foreign exchange has the following transactions on its account with its London correspondent :

Debits

Sept. 1	Remittance, 30-day bill	£400	@ 4.46
10	Remittance, sight bill	£100 10s.	@ 4.47
15	Remittance, demand bill	£200 0s. 6d.	@ 4.467½

Credits

Sept. 2	Draft, sight	£300	@ 4.475
12	Draft, demand	£200 12s. 5d.	@ 4.47
20	Cable, demand	£100	@ 4.48

Ascertain the profit or loss on the account for the month of September; state the balance as of September 30 in foreign and in domestic currency, the current rate on that date being 4.49½.

5. Find the cost of exporting gold enough to settle a debt of £20,000 if gold bars ($\frac{1}{12}$ fine) are shipped to London at 77s. 9½d. per ounce, charges as given on page 135.

6. A New York bank, to cover a 90-day sight bill on London, must cable funds at once. The cable rate on London is 4.42½. In Paris a cable on London can be bought at 47.84 francs per pound sterling. What is the highest rate the New York bank can afford to pay for a cable on Paris to be used in purchasing the required London cable?

7. (a) A Boston bank receives for collection a 60-day sight bill drawn by a Buenos Aires exporter on a local leather merchant for \$4000. The instructions are that the proceeds shall be invested immediately in 90-day sterling exchange, and forwarded to a London bank for the credit of the Buenos Aires bank. With the Boston discount rate for 60-day bills at 3½%, the Boston bank's charges for collection $\frac{1}{4}\%$ of the proceeds of the bill (face of bill, minus discount), the rate on 90-day sight bills on London at 4.45, the London discount rate for 90-day bills at 4%, and English stamp charges $\frac{1}{20}\%$ of the face of the bill, what amount in pounds will be placed to the credit of the Buenos Aires bank in London?

(b) Would it have been more or less advantageous (and how much) to invest the proceeds in a demand bill at $4.49\frac{1}{2}$?

8. Prepare an exchange table for dollars and francs at the rate of $8.85\frac{3}{4}$ (cents per franc) for any sum up to 1,000,000 francs or dollars. The following is a franc table for the rate 8.935:

FRANCS	DOLLARS	DOLLARS	FRANCS
1	.08935	1	11.19194
2	.17870	2	22.38388
3	.26805	3	33.57582
4	.35740	4	44.76777
5	.44675	5	55.95971
6	.53610	6	67.15165
7	.62545	7	78.34359
8	.71480	8	89.53553
9	.80415	9	100.72747

9. Using the table made in problem 8, find the cost of a sight draft of F. 25,675 at $8.85\frac{3}{4}$. Find how large a draft on Paris can be bought with \$1496.50.

10. (a) A banker exports gold bars to London to cover an amount of £750,000. The price of English standard bars is $77s. 10\frac{3}{4}d.$ per ounce, freight is $\frac{3}{16}\%$, insurance $\frac{1}{20}\%$, boxing, carting, loss of interest, etc. $\frac{1}{60}\%$, and bar charge $40¢$ per \$1000. Find the cost of the shipment.

(b) Would it have cost more or less, and how much, to have exported gold coin, at $76s. 5d.$, if an allowance of $\frac{1}{20}\%$ is made for abrasion, other charges, except bar charge, being the same as given for exporting bars?

11. A New York banker arranges for an importation of 72,000 cz. of standard gold bars from London by sending on April 13 a sufficient quantity of demand drafts on London to purchase the gold at $77s. 9\frac{1}{2}d.$ an ounce, and to pay the expenses in London as follows: freight, $\frac{3}{16}\%$; insurance, $\frac{1}{20}\%$; boxing, carting, etc., $\frac{1}{32}\%$. The gold is delivered in New York on May 3. Allowing interest at 4%, find the present worth of the proceeds at time demand exchange was bought, and the dollar value of each pound sterling remitted at time of remittance.

12. Assume, in problem 11, that the New York banker wished to have the importation of bars converted into American coin. The government makes no charge for coining, but does charge 2 cents an ounce for the copper required to convert the bars to American standard gold. How many dollars in gold coin will the banker receive if the government deducts the copper charge from the coins when minted? (Reckon in nearest \$5.)

13. An American banker sells a 60-day sight bill of £25,000 on a London bank when demand exchange is at 4.40, the London discount rate is at 4%, bill stamp $\frac{1}{20}\%$, and commission $\frac{1}{8}\%$. If a demand draft can be bought at the expiration of 63 days at $4.40\frac{1}{2}$ to cover the long bill, the American banker's profit (or loss) equals (a) what sum in dollars? (b) what rate of (exact) interest per annum on the proceeds of the long bill?

14. Assume that the banker in problem 13 had drawn his 60-day sight bill against an acceptance account and had sent it to London for discount and credit, and then had sold demand drafts at 4.40 against the credit. Using all other conditions as stated in problem 13, find (a) the amount of the credit (in pounds) allowed at the London bank; (b) the gain or loss in dollars on the entire transaction.

XI. PROGRESSIONS

ARITHMETIC PROGRESSION

An **arithmetic progression** is a series of values increasing or decreasing by a common difference.

Each of the following series of values forms an arithmetic progression:

$$\begin{array}{cccccc} 1, & 4, & 7, & 10, & 13, & 16 \\ x, & x + y, & x + 2y, & x + 3y, & x + 4y, & x + 5y \\ 12, & 9\frac{1}{2}, & 7, & 4\frac{1}{2}, & 2, & -\frac{1}{2} \end{array}$$

The first two of the foregoing series are **ascending series** because each succeeding value is greater than the preceding one. In the first series the common difference is 3; in the second it is y .

The third series is a **descending series** because the values decrease as the progression continues. The common difference is $-2\frac{1}{2}$.

In arithmetic progression the following nomenclature is used:

The *first term* is represented by the letter a .

The *common difference* is represented by the letter d .

The *last term* is represented by the letter l .

The *number of terms* is represented by the letter n .

The *sum of the series* is represented by the letter s .

In the series

$$x, x + y, x + 2y, x + 3y, x + 4y, x + 5y$$

$$a = x$$

$$d = y$$

$$l = x + 5y$$

$$n = 6$$

$$s = 6x + 15y$$

How to find the last term of an arithmetic progression

Referring to the preceding series, note that the coefficient of y in any term is 1 less than the number of the term.

The 4th term is $x + (4 - 1) y$.

The 5th term is $x + (5 - 1) y$.

The 6th term is $x + (6 - 1) y$.

The n th term would be $x + (n - 1) y$.

Hence we have for the last term of an arithmetic progression the formula,

$$l = a + (n - 1) d \quad (1)$$

How to find the sum of the series in an arithmetic progression

The sum of the preceding series is indicated thus :

$$s = x + (x + y) + (x + 2y) + (x + 3y) + (x + 4y) + (x + 5y)$$

or in reverse order

$$s = (x + 5y) + (x + 4y) + (x + 3y) + (x + 2y) + (x + y) + x$$

Adding we have

$$2s = (2x + 5y) + (2x + 5y) + (2x + 5y) + (2x + 5y) + (2x + 5y) + (2x + 5y)$$

Observe (1) that the result is 2 times the *sum* of the series.

(2) that the terms in the result are all alike.

(3) that each term in the result is equal to the sum of the first term + the last term of the original series.

(4) that the number of terms in the result equals the number of terms in the original series.

Hence, using the letters given on page 143, the result found may be written

$$2s = n(a + l)$$

Therefore

$$s = \frac{n}{2}(a + l) \quad (2)$$

Substituting for l its equivalent as shown in (1) gives

$$s = \frac{n}{2}[2a + (n - 1)d] \quad (3)$$

EXAMPLES

1. Find (a) the 20th term and (b) the sum of 20 terms of the series 10, 13, 16, 19, 22, etc.

$$(a) \quad l = a + (n - 1)d \quad \left. \begin{array}{l} a = 10 \\ n = 20 \\ d = 3 \end{array} \right\}$$

Therefore $l = 10 + (19 \times 3) = 67$

$$(b) \quad s = \frac{n}{2}(a + l)$$

Therefore $s = 10 \times (10 + 67) = 770$

2. The first term of an arithmetic progression is 5; the last term is 77; and the number of terms is 19. Find (a) the common difference and (b) the sum of the series.

$$(a) \quad \text{Since} \quad \begin{aligned} l &= a + (n - 1)d \\ d &= \frac{l - a}{n - 1} \end{aligned}$$

Therefore $d = \frac{77 - 5}{19 - 1} = 4$

$$(b) \quad s = \frac{n}{2}(a + l)$$

Therefore $s = \frac{19}{2}(5 + 77) = 779$

3. In the series 6, $4\frac{1}{2}$, 3, $1\frac{1}{2}$, 0, $-1\frac{1}{2}$, -3 , etc., find the 15th term and the sum of 15 terms.

$$\begin{aligned} a &= 6 \\ d &= -1\frac{1}{2} \\ n &= 15 \end{aligned}$$

$$\begin{aligned} l &= [6 + (15 - 1)(-1\frac{1}{2})] = 6 + (14) \times (-1\frac{1}{2}) = 6 - 21 = -15 \\ s &= \frac{15}{2}(6 - 15) = \frac{15}{2} \times -9 = -67\frac{1}{2} \end{aligned}$$

4. A man bought an automobile for \$2500, paying \$500 down and giving ten 6% interest-bearing notes of \$200 each due in 1, 2, 3, etc. months respectively. Find the total amount of interest paid on all the notes.

$$s = \frac{n}{2}(a + l)$$

$$s = \frac{10 \times (\$200 + \$2000)}{2 \times 200} = \$55$$

On the first note 1 month's interest on \$200 will be due; on the second note, 2 months' interest on \$200 (equivalent to 1 month's interest on \$400); on the third note, 3 months' interest on \$200 (equivalent to 1 month's interest on \$600) etc.; till the last note, on which there will be due 10 months' interest on \$200 (equivalent to 1 month's interest on \$2000). The interest items form an arithmetic progression, the first term of which is the interest on \$200, and the last term, the interest on \$2000. The interest on any sum for 1 month is found by pointing off two places and dividing the result by 2. That is, it is equivalent to dividing the principal sum by 200. By the formula, $s = \frac{n}{2}(a + l)$. Multiplying the sum of the interest on the first and last notes by half the number of notes gives the total interest.

Problems

1. In the series 3, 7, 11, 15, etc., find the 25th term and the sum of 25 terms.
2. Find the sum of 30 terms of the series $2\frac{1}{2}$, 5, $7\frac{1}{2}$, 10, etc.
3. The first term of an arithmetic progression is 84, the last term is 14, and the number of terms is 21. Find the common difference and the sum of the series.
4. In a descending arithmetic progression the 22d term is -56 and the common difference is $-3\frac{1}{3}$. Find the 10th term and the sum of all the terms.
5. A boy during his first week as a newsboy sold 222 papers. For the next 25 weeks he increased his sales 16 papers per week. If the papers cost him $1\frac{1}{4}\text{¢}$ each, and he sold them at 2¢ each, find (a) his sales for the last week and (b) his total profit.
6. A piece of machinery costing \$17,500 was purchased on the following terms: cash \$2500; balance payable in monthly installments of \$500 with interest at 6%. Find the total amount of interest paid.

7. A young man purchased a piano for \$525, paying \$25 down and agreeing to pay the balance in monthly installments of \$10 plus interest at 6% on unpaid balances. Set up the first 6 terms of the series, showing the monthly interest payments, and find the total amount of interest paid in making all payments on the piano.

GEOMETRIC PROGRESSION

A **geometric progression** is a series of values increasing or decreasing according to some fixed ratio.

Each of the following series of values constitutes a geometric progression.

2, 6, 18, 54, 162, etc.

12, 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, etc.

1, 1.1, 1.21, 1.331, 1.4641, etc.

a , ar , ar^2 , ar^3 , ar^4 , ar^5 , etc.

In the first of these series the ratio is 3; in the second, $\frac{1}{2}$; in the third, 1.1; in the fourth, r .

The **ratio** is the common factor by which any given term is multiplied to produce the next succeeding term.

The nomenclature of geometric progression is as follows:

The *first term* is represented by the letter a .

The *last term* is represented by the letter l .

The *ratio* is represented by the letter r .

The *number of terms* is represented by the letter n .

The *sum of the series* is represented by the letter s .

How to find the last term of a geometric progression

In the series a , ar , ar^2 , ar^3 , ar^4 , ar^5 , etc., note that in the 2d term the exponent of r is 1; in the 3d term the exponent of r is 2; in the 4th term the exponent of r is 3, and so on. That is, the exponent of r in any term is 1 less than the number of the term. Hence, if the number of terms is 11, the last term is ar^{10} ; if the number of terms is n , the last term is ar^{n-1} . That is,

$$l = ar^{n-1} \quad (1)$$

How to find the sum of the series in a geometric progression

The sum of the preceding series may be indicated thus :

$$s = a + ar + ar^2 + ar^3 + \dots + ar^{n-1} \quad (1)$$

$$r \times (1): \quad rs = \frac{ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n}{(2)}$$

$$(2) - (1): \quad rs - s = ar^n - a$$

$$s(r - 1) = ar^n - a$$

$$s = \frac{ar^n - a}{r - 1} \quad (2)$$

$$\text{Since } l = ar^{n-1}$$

$$rl = ar^n$$

Hence, equation (2) may be written,

$$s = \frac{rl - a}{r - 1} \quad (3)$$

EXAMPLES

1. Find the eighth term of the series 4, 12, 36, 108, etc.

$$a = 4 \quad l = ar^{n-1}$$

$$r = 3 \quad l = 4 \times 3^7 = 8748$$

$$n = 8$$

2. The first term is 16, the ratio is $\frac{1}{4}$, and the number of terms is 7.
Find the last term.

$$l = ar^{n-1}$$

$$l = 16 \times \left(\frac{1}{4}\right)^6 = \frac{1}{256}$$

3. Find the sum of 9 terms of the series $\frac{1}{2}$, $1\frac{1}{2}$, $4\frac{1}{2}$, $13\frac{1}{2}$, etc.

$$\begin{aligned} a &= \frac{1}{2} & s &= \frac{ar^n - a}{r - 1} \\ r &= 3 & s &= \frac{(\frac{1}{2} \times 3^9) - \frac{1}{2}}{3 - 1} = \frac{(\frac{1}{2} \times 19,683) - \frac{1}{2}}{2} \\ n &= 9 & &= 4920\frac{1}{2} \end{aligned}$$

4. A man agreed to work for 2 weeks, receiving \$.05 the first day, \$.10 the second day, \$.20 the third day, and so on, each day's wages

being double those of the preceding day. How much were his wages for the 2 weeks (12 days)?

$$\begin{array}{ll}
 a = \$.05 & s = \frac{ar^n - a}{r - 1} \\
 r = 2 & s = \frac{(\$.05 \times 2^{12}) - \$.05}{2 - 1} = \frac{(\$.05 \times 4096) - \$.05}{1} \\
 n = 12 & = \$204.75
 \end{array}$$

Problems

1. The first term of a series is 5, the ratio is 5, and the number of terms is 5. Find (a) the last term and (b) the sum of the series.

2. The series 27, 9, 3, 1, etc. has 9 terms. Find the last term and the sum of the series.

3. A agreed to work for B for 10 days for wages as follows: \$.01 the first day, \$.03 the second day, \$.09 the third day, and so on, his wages being tripled daily. Find his wages for the last day and his total wages for the 10 days.

4. A farmer in the Middle West obtained a peck of choice seed oats from Canada. He sowed the seed and harvested therefrom 5 bushels of oats. The next year he sowed the 5 bushels and again harvested a crop proportionally as good as the preceding year. Assuming that the entire crop of oats was sowed each year for a total of 5 years with the same proportional increase annually, find the number of bushels of oats in the fifth year's crop.

5. A sum of money was invested for 8 years without any of its earnings being withdrawn. The ratio of increase per annum was $1\frac{1}{12}$. Show the ratio of the final amount to the original investment.

XII. LOGARITHMS

A **logarithm** is an exponent. Hence the principles and rules governing the use of exponents apply to logarithms.

For the purposes of this work only the *common* or *Briggs system* of logarithms will be considered. In this system of logarithms 10 is used as the base or root. That is, if we know that the logarithm of some number is 2, we know that the number is 100, because

$$10^2 = 100$$

If we know that the logarithm of a number is 3, we know that the number is 1000, because

$$10^3 = 1000$$

Or conversely, since

$10^3 = 1000,$	$\therefore \log 1000$	$=$	3
$10^2 = 100,$	$\therefore \log 100$	$=$	2
$10^1 = 10,$	$\therefore \log 10$	$=$	1
$10^0 = 1,$	$\therefore \log 1$	$=$	0
$10^{-1} = .1,$	$\therefore \log .1$	$=$	-1
$10^{-2} = .01,$	$\therefore \log .01$	$=$	-2
$10^{-3} = .001,$	$\therefore \log .001$	$=$	-3

The expression " $\log 1000 = 3$ " is read

"The logarithm * of 1000 is 3."

Were it not for the uniform base of 10, it would be necessary to indicate the base of the logarithm in each expression. It would be written thus:

$$\log_{10} 1000 = 3$$

which reads in full "the log of 1000 to the base 10 is 3."

The base 10 is always to be understood in using the common logarithms.

* For brevity's sake read "log."

The logarithm of unity

Referring to the foregoing schedule of logarithms, note that $10^0 = 1$; hence $\log 1 = 0$. Any quantity having a 0 exponent is equal to 1, because $\frac{10^1}{10^1} = 10^0$, but $\frac{10^1}{10^1} = 1$. Therefore $10^0 = 1$. Also $\frac{x^1}{x^1} = x^0$, but $\frac{x^1}{x^1} = 1$. Therefore $x^0 = 1$. Hence the logarithm of unity = 0.

Meaning of a negative exponent

$$x^n \times x^n = x^{n+n}$$

$$x^{-n} \times x^n = x^{-n+n} = x^0 = 1$$

$$\text{Since } x^{-n+n} = 1$$

$$x^{-n} = \frac{1}{x^n}, \text{ and } x^n = \frac{1}{x^{-n}}$$

Therefore any quantity may be transferred from the numerator to the denominator of an expression (or vice versa) by changing the sign of its exponent.

Referring again to the above-mentioned schedule of logarithms, note that

$$10^{-1} = .1, \text{ that is, } \frac{1}{10}$$

$$10^{-2} = .01, \text{ that is, } \frac{1}{10^2} = \frac{1}{100}$$

$$10^{-3} = .001, \text{ that is, } \frac{1}{10^3} = \frac{1}{1000}$$

Note that the logarithms of the integers bear a given relation to the number of digits required to write the integer.

An integer of 4 digits has a logarithm of 3.

An integer of 3 digits has a logarithm of 2.

An integer of 2 digits has a logarithm of 1.

An integer of 1 digit has a logarithm of 0, etc.

Observe these expressions:

$$\log .1 = -1$$

$$\log .01 = -2$$

$$\log .001 = -3$$

$$\log .0001 = -4$$

Note that the numerical value of the logarithm of each of these values is 1 greater than the number of zeros between the decimal point and the first significant figure of the decimal.

Thus far only logarithms having zero or integral values have been considered. Thus, $\log 1 = 0$; $\log 10 = 1$; $\log 100 = 2$, etc.

Since $\log 1 = 0$, and $\log 10 = 1$, the logarithms of all values between 1 and 10 must be somewhere between 0 and 1. Thus :

$$\log 2 = .30103$$

$$\log 3 = .47712$$

$$\log 5 = .69897$$

$$\log 7 = .84510$$

$$\log 9 = .95424$$

As the numbers approach 10 in value, the logarithms approach 1.

Since $\log 10 = 1$, and $\log 100 = 2$, the logarithms of all values between 10 and 100 must be between 1 and 2. Thus :

$$\log 20 = 1.30103$$

$$\log 30 = 1.47712$$

$$\log 50 = 1.69897$$

$$\log 70 = 1.84510$$

$$\log 90 = 1.95424$$

A logarithm, then, may be an *integer*, a *decimal*, or an *integer and a decimal*.

The integral part of a logarithm is called the **characteristic**, and the decimal part is called the **mantissa**.

The characteristic may be either positive or negative, but the *mantissa is always positive*.

Compare the logarithms of 2, 3, 5, 7, 9, and of 20, 30, 50, 70, 90, given above and note that the mantissas are alike. The difference consists only in the addition of the characteristic in the logarithms of 20, 30, 50, etc. Hence tables of logarithms show only the mantissas. The characteristics are to be determined by inspection.

EXAMPLES

1. Find the logarithm of 324.

Turn to page 270 of the table of logarithms and find in the margin the number 324. In the column of mantissas headed 0 and

opposite 324, find 51055. The characteristic is 2 because the number 324 has three digits. Hence

$$\log 324 = 2.51055$$

2. Find the logarithm of 5476.

Turn to page 274 of the table of logarithms, and find the number 547 in the margin. In the column of mantissas headed 6, opposite 547, find 73846. The characteristic is 3 because the number 5476 has four digits. Hence

$$\log 5476 = 3.73846$$

3. Find the logarithm of 54.76.

The mantissa of $\log 54.76$ is the same as the mantissa of $\log 5476$. The difference lies only in the characteristic. The characteristic is 1 because the integer 54 has two digits. Hence

$$\log 54.76 = 1.73846$$

4. Find the number of which 1.82197 is the logarithm.

In the table of logarithms look for the mantissa 82197. It is on page 277 in column headed 7. In the left margin of the table, opposite 82197, is the number 663. To 663 annex the 7 at the top of the column in which 82197 was found. Hence the number of which 1.82197 is the logarithm is 66.37. That is,

$$\log 66.37 = 1.82197$$

The decimal point is placed between the 6 and the 3 because the integral part of the number must contain 1 digit more than the numeric value of the characteristic, 1.

Since a logarithm is an exponent, the principles governing the use of exponents govern the use of logarithms as well. In multiplication the logarithms are added, in division, they are subtracted, etc.

5. Multiply 79 by 86.

$$\log 79 = 1.89763$$

$$\log 86 = \underline{1.93450}$$

$$\log \text{ of product} = 3.83213$$

In the table on page 277, column 4, find the mantissa 83213. It is opposite 679 in column headed 4. Annexing the 4 gives the number 6794. That is,

$$\log 6794 = 3.83213$$

Hence

$$79 \times 86 = 6794$$

6. Find the logarithm of 54,927.

The table gives the logarithms of numbers of four digits only. Hence to find $\log 54927$ requires an approximation. This is done by a process called **interpolation**. Interpolation is based on the assumption that the logarithm varies in the same proportion as the number. This assumption is not exactly true, but is nearly true for small differences.

It has been shown that the mantissas are alike for a given order of digits in any values. Hence the mantissas in

$$\begin{aligned}\log 5492.7 \text{ and} \\ \log 54927 \text{ are alike.}\end{aligned}$$

The number 5492.7 lies .7 of the way between 5492 and 5493

The mantissa in $\log 5493$ is 73981

The mantissa in $\log 5492$ is 73973

Their difference is 8

$$.7 \times 8 = 5.6 \text{ (Approximately 6)}$$

Hence the mantissa of $\log 5492.7$ is 73979 ($73973 + 6$)

Therefore $\log 54927 = 4.73979$

The **antilogarithm** is the number corresponding to a given logarithm.

7. Find the antilogarithm of 2.471637.

Turn to page 269 of the table and find the two numbers nearest equal to 47163. They are in columns 2 and 3.

The number for the mantissa 471730 = 2963

The number for the mantissa 471637 is desired

The number for the mantissa 471590 = 2962

Difference between 2d and 3d mantissa is 47

Difference between 1st and 3d mantissa is 140

Difference between the numbers is 1

Hence the number corresponding to the mantissa 471637 is $\frac{47}{140}$ of 1 greater than the number corresponding to the mantissa 471590.

$\frac{47}{140}$ of 1 = .33 (ratio of differences)

$2962 + .33 = 2962.33$, the number corresponding to the mantissa 471637. Three places must be pointed off because the characteristic of the logarithm is 2.

Therefore $\text{antilog } 2.471637 = 296.233$

8. Find the logarithm of .09728.

$$\log .09728 = \bar{2}.98802 \text{ (page 283 of tables)}$$

The characteristic is -2 because there is 1 zero between the decimal point and the first significant figure of the decimal.

There are two ways of writing a logarithm having a negative characteristic. One method is shown above. The minus sign is written above the characteristic 2 to indicate that it alone is negative, the mantissa being always positive.

The other, and more common, way of writing the logarithm of a decimal is as follows :

$$\log .09728 = 8.98802 - 10$$

This form is the result of adding 10 to the negative characteristic 2 and indicating the subtraction of 10 from the result.

9. Multiply 42.76 by .007654.

$$\log 42.76 = 1.63104$$

$$\log .007654 = \overline{7.88389 - 10}$$

$$\log \text{ of the product} = \overline{9.51493 - 10}$$

The number for the mantissa 51495 is 3273

The number for the mantissa 51493 is desired

The number for the mantissa 51481 is 3272

$$\text{The ratio of differences} = \frac{12}{14} = .8571$$

Hence the number for the mantissa 51493 is 32728571

Since the characteristic is -1 , the decimal point is placed next to the left-hand digit of the number.

$$\text{That is,} \quad \log .32728571 = 9.51493 - 10$$

$$\text{Therefore} \quad 42.76 \times .007654 = .32728571 \text{ (very nearly)}$$

10. Find the value of $(1.05)^{40}$.

Since $(1.05)^{40}$ indicates the use of 1.05 forty times as a factor, $\log (1.05)^{40} = 40 \text{ times } \log 1.05$.

$$\log 1.05 = .02119$$

$$40 \times .02119 = .84760$$

$$\text{Hence} \quad \log (1.05)^{40} = .84760$$

The number for the mantissa 84763 is 7041

The number for the mantissa 84760 is desired

The number for the mantissa 84757 is 7040

$$\text{The ratio of differences} = \frac{3}{6} = .5$$

Hence the number for the mantissa 84760 is 70405

That is, $\log 7.0405 = .84760$

Therefore $(1.05)^{40} = 7.0405$

11. Divide 37.92 by .032.

$$\log 37.92 = 1.57887$$

$$\log .032 = 8.50515 - 10$$

Since $8.50515 - 10$ cannot be subtracted from 1.57887, 10 is both added to 1.57887 and subtracted from it, thus :

$$\log 37.92 = 11.57887 - 10$$

$$\log .032 = \frac{8.50515 - 10}{}$$

$$\log \text{ of quotient} = 3.07372$$

The number for the mantissa 07372 is 1185

That is, $\log 1185 = 3.07372$

Therefore $37.92 \div .032 = 1185$

12. Divide .1875 by 750.

$$\log .1875 = 9.27300 - 10$$

$$\log 750 = \frac{2.87506}{}$$

$$\log \text{ of quotient} = 6.39794 - 10$$

The number for the mantissa 39794 is 25

That is, $\log .00025 = 6.39794 - 10$

Therefore $.1875 \div 750 = .00025$

13. Find the square root of 5625.

$$\log 5625 = 3.75012$$

$$\frac{1}{2} \text{ of } 3.75012 = 1.87506$$

Hence $\log \sqrt{5625} = 1.87506$

The number for the mantissa 87506 is 75

That is, $\log 75 = 1.87506$

Therefore $\sqrt{5625} = 75$

14. Find the 13th root of 5.472.

$$\log 5.472 = .73815$$

$$\frac{1}{13} \text{ of } .73815 = .05678$$

Hence $\log \sqrt[13]{5.472} = .05678$

The number for the mantissa 05690 is 1140

The number for the mantissa 05678 is desired

The number for the mantissa 05652 is 1139

The ratio of differences = $\frac{26}{38} = .68$

Hence the number for the mantissa 05678 is 113968

That is, $\log 1.13968 = .05678$

Therefore $\sqrt[13]{5.472} = 1.13968$

15. Find the 7th root of .00476.

$$\log .00476 = 7.67761 - 10$$

Before dividing by 7, add 60 - 60 to the logarithm.

Thus :

$$\begin{array}{r} 7.67761 - 10 \\ 60. \quad - 60 \\ \hline 67.67761 - 70 \end{array}$$

Now divide by 7.

$$\begin{array}{r} 7 \overline{)67.67761 - 70} \\ \underline{9.66823 - 10} \end{array}$$

The result is in the familiar form of logarithm of a decimal.

Hence $\log \sqrt[7]{.00476} = 9.66823 - 10$

The number for the mantissa 66829 is 4659

The number for the mantissa 66823 is desired

The number for the mantissa 66820 is 4658

The ratio of differences is $\frac{3}{9} = .333$

Hence the number for the mantissa 66823 is 4658333

That is, $\log .4658333 = 9.66823 - 10$

Therefore $\sqrt[7]{.00476} = .4658333$

The **cologarithm** of a number is the logarithm of the reciprocal of the number.

16. Find the cologarithm of 3.046.

The reciprocal of 3.046 is $\frac{1}{3.046}$

$$\begin{array}{r} \log 1 = 0 = 10 \quad - 10 \\ \log 3.046 = .48373 \\ \hline \text{colog } 3.046 = 9.51627 - 10 \end{array}$$

The cologarithm of a number may be written directly from the table. Note in the foregoing exercise that, in subtracting .48373, log 3.046, from $10 - 10$, the result is easily obtained by beginning at the left and subtracting each digit of .48373 from 9, excepting the last or right-hand digit, which is subtracted from 10. $\text{Log } 472 = 2.67394$. $\text{Colog } 472 = 7.32606 - 10$, written from left to right by taking the difference between 9 and the successive digits till the 4 is reached, when the difference between 10 and 4 is taken. A little practice renders the exercise easy.

17. Find the value of $\frac{372 \times .0456 \times 13.5}{8.56 \times 11500 \times .00593}$.

$$\begin{array}{rcl}
 \log 372 & = & 2.57054 \\
 \log .0456 & = & 8.65896 - 10 \\
 \log 13.5 & = & 1.13033 \\
 \text{colog } 8.56 & = & 9.06753 - 10 \\
 \text{colog } 11500 & = & 5.93930 - 10 \\
 \text{colog } .00593 & = & 2.22695 \\
 & & \hline
 & & 29.59361 - 30 \\
 \text{subtract } 20. & & - 20 \\
 & & \hline
 & & 9.59361 - 10
 \end{array}$$

Hence $\log \text{ of quotient} = 9.59361 - 10$

The number for the mantissa 59362 is 3923

The number for the mantissa 59361 is desired

The number for the mantissa 59351 is 3922

The ratio of differences is $\frac{10}{11} = .9$

Hence the number for the mantissa 59361 is 39229

That is. $\log .39229 = 9.59361 - 10$

Therefore $\frac{372 \times .0456 \times 13.5}{8.56 \times 11500 \times .00593} = .39229$

NOTE. The right-hand digit in the mantissas in a table of logarithms is only approximately correct. Logarithms are not in the same ratio as the numbers of which they are the logarithms. Hence in dealing with large numbers, or with numbers having several decimal places, and therefore requiring interpolations, accurate results cannot be obtained. The more places there are in the table of logarithms, the higher the degree of accuracy.

Exercises

1. Find the logarithm of 87; of 594; of 63,482; of .004567; of 1.0365.

2. Find the antilogarithm of 2.59184; of 9.93832 - 10; of 4.862055; of 6.74410 - 10.

3. Find the cologarithm of 932; of .0037; of .01476; of $(1.0275)^{50}$.

Using logarithms, perform the following indicated operations:

4. 13.45×765.9

13.
$$\frac{(1.06)^{30} - 1}{.06}$$

5. $.7235 \times .002638$

14.
$$\frac{1 - \frac{1}{(1.055)^{40}}}{.055}$$

6. $\left(\frac{1}{1.045}\right)^{25}$

15.
$$\frac{.05}{(1.05)^{30} - 1}$$

7. $392.5 \div 95.25$

16.
$$\frac{.04 \times (1.04)^{50}}{(1.04)^{50} - 1}$$

8. $.04264 \div 5.565$

9. $\sqrt[11]{008776}$

17.
$$\frac{15,750 \times .035 \times (1.035)^{15}}{(1.035)^{15} - 1}$$

10.
$$\frac{689 \times .156 \times 25,500}{9764 \times .005963 \times .3175}$$

11. $\sqrt[24]{823.5}$

12. $\sqrt[13]{\frac{15}{16}}$

18.
$$125,000 \times \frac{1 - \frac{1}{(1.048)^{60}}}{.048}$$

XIII. COMPOUND INTEREST

Compound interest derives its name from the fact that at regular periodic intervals accumulated simple interest is added to the principal sum at interest, and hence becomes a part of the principal.

As the principal becomes greater with each interest accretion, so likewise each periodic interest accumulation becomes greater. The rapidity of increase in interest accumulation depends upon three factors, namely:

1. Rate of interest.
2. Frequency of conversion period.
3. Time in years or periods over which the indebtedness or investment extends.

The rate of interest is generally expressed on an annual basis, as $4\frac{1}{2}\%$, 5%, or 6% per annum, respectively.

By *frequency of conversion* is meant the frequency of converting interest into, or combining interest with, principal. Interest may be added to the principal annually, semiannually, quarterly, or at other regular periods of time as desired. The more frequent the conversion period, the more rapid the accumulation of interest.

The time is generally expressed in years, or years, months, etc., instead of in periods. A *period*, as used in this connection, means the time from one interest addition to the next. Since the rate is expressed on an annual basis and the time is expressed in years, etc., instead of in periods, the rate must be adjusted to the period whenever the period is other than 1 year. That is, if the rate is 6% and the conversion period is 6 months, or $\frac{1}{2}$ of a year, the rate is 3% per period, and the number of periods is 2 per annum. Again, if interest is converted quarterly, that is, every 3 months, the rate per period is $\frac{1}{4}$ of 6%, or $1\frac{1}{2}\%$, and the number of periods is 4 per annum.

The following illustration shows the amount of \$1 for 3 years, at 6%, converted annually, semiannually, and quarterly, respectively.

To find the amount of any sum for a given time and rate

AMOUNT OF \$1 FOR 3 YEARS AT 6% CONVERTED ANNUALLY,
SEMIANNUALLY, AND QUARTERLY

TIME	6% ANNUALLY	6% ANNUALLY 3% SEMIANNUALLY	6% ANNUALLY 1½% QUARTERLY
0	1.	1.	1.
3 mo.			1.015
6 mo.		1.03	1.030225
9 mo.			1.0456784
1 yr.	1.06	1.0609	1.0613636
3 mo.			1.0772840
6 mo.		1.092727	1.0934433
9 mo.			1.1098449
2yr.	1.1236	1.1255088	1.1264926
3 mo.			1.1433900
6 mo.		1.1592741	1.1605408
9 mo.			1.1779489
3 yr.	1.191016	1.1940523	1.1956182

Compare the annual amounts in each column. Note the increase in value as the frequency of conversion increases.

In any column of the foregoing table (or any compound interest table) each succeeding value bears a uniform ratio to the value next preceding it. Thus, in the column showing semiannual values, each value is 1.03 times the value preceding it.

The first amount is 1.03×1 , or 1.03.

The second amount is 1.03×1.03 , or 1.0609.

The third amount is 1.03×1.0609 , or 1.092727.

The fourth amount is 1.03×1.092727 , or 1.1255088; and so on.

The amount of 1 at 6% for 5 years, converted quarterly, is expressed thus, $(1.015)^{20}$. Since interest is converted quarterly, the rate per period is $\frac{1}{4}$ of 6%, or 1.5%, and the number of periods is 4 times the number of years, or 20. So also, the amount of 1 for any number of years at any rate may be similarly expressed.

For actuarial purposes it is customary to represent the rate by the small letter i , the number of years by the small letter n , and the amount of 1 for any time and rate by the small letter s .

Hence we have the expression

$$s = (1 + i)^n \quad (1)$$

If $i = .04$ and $n = 6$, by substitution we have

$$s = (1.04)^6 = 1.04 \times 1.04 \times 1.04 \times 1.04 \times 1.04 \times 1.04 = 1.2653190.$$

That is, the amount of 1 for 6 periods at 4% per period is 1.265319.

The compound interest is $1.265319 - 1 = .265319$.

That is, if $I =$ interest,

$$I = (1 + i)^n - 1 \quad (2)$$

Since the amount of 1 for any rate and any number of periods may be expressed by the formula $s = (1 + i)^n$, the compound interest table may be represented by the following geometric progression:

$$(1 + i), (1 + i)^2, (1 + i)^3, (1 + i)^4, (1 + i)^5, (1 + i)^6, \dots (1 + i)^n.$$

In the progression just given, the ratio is $(1 + i)$. The second value is $(1 + i)$ times the first value, that is, $(1 + i)(1 + i) = (1 + i)^2$; the third value is $(1 + i)$ times the second value; and so on.

Each value in the progression is called a **term**.

By the law of exponents,

$$(1 + i)^6 = (1 + i)^2 (1 + i)^4 = (1 + i)^3 (1 + i)^3.$$

$$\text{Also } (1 + i)^{40} = (1 + i)^{10} (1 + i)^{30} = (1 + i)^{28} (1 + i)^{12}, \text{ etc.}$$

Tables are provided (see page 284) showing the amounts of 1 for various rates and for periods up to 100. Should it be desired to find the amount of 1 for more than 100 periods, it may be found as above explained. Thus the amount of 1 for 60 years at 4%, converted semi-annually, is expressed by the formula $s = (1.02)^{120}$. Since the table does not give values for more than 100 periods, the value for 120 periods may be found by multiplying the value for 100 periods by the value for 20 periods (or the values for any other two numbers of periods whose sum is 120).

ILLUSTRATION

$$(1.02)^{120} = (1.02)^{100}(1.02)^{20}$$

$$(1.02)^{100} = 7.2446461$$

$$(1.02)^{20} = 1.4859474$$

$$(1.02)^{120} = 7.2446461 \times 1.4859474 = 10.7651630$$

OR

$$(1.02)^{120} = (1.02)^{75}(1.02)^{45}$$

$$(1.02)^{75} = 4.4158355$$

$$(1.02)^{45} = 2.4378542$$

$$(1.02)^{120} = 4.4158355 \times 2.4378542 = 10.7651630$$

To find the compound amount of any principal sum, multiply the amount of \$1 per annum by the principal sum. Thus, if S = amount, and P = the principal,

$$S = P(1 + i)^n \quad (3)$$

This formula is important because by its use there can be found either the time or the interest rate required for a given principal to increase to a given amount. From the formula we have, by division,

$$(1 + i)^n = \frac{S}{P}$$

$$\text{Also } n \log (1 + i)^* = \log \left(\frac{S}{P} \right)$$

$$\text{Then } n = \frac{\log \left(\frac{S}{P} \right)}{\log (1 + i)} \quad (4)$$

$$\text{or } \log (1 + i) = \frac{\log \left(\frac{S}{P} \right)}{n} \quad (5)$$

(See examples, page 166.)

Nominal and effective rates. By referring to the table (page 161) showing amount of \$1 converted annually, semiannually, and quarterly, it is to be observed that the interest accumulation at the end of one year differs in each case, though the annual rate is expressed as 6% in each case. 6% is called the "nominal" rate. In the semi-annual column the amount of \$1 for 1 year is \$1.0609. That is, the real interest accumulation is \$.0609. In the quarterly column the

* See example 10, page 155, on how to find value of $(1.05)^{40}$.

real interest accumulation is \$.0613636. Hence if the nominal rate of 6% is converted semiannually, the effective rate is 6.09%, and if converted quarterly, the effective rate is 6.13636%.

The **nominal rate** is the rate realized if interest is converted annually.

The **effective rate** is represented by the total return on \$1 of principal for 1 year. The nominal rate and the effective rate are identical in those cases in which the conversion period is 1 year.

Because the conversion period is so frequently less than 1 year, it is desirable to distinguish the notation representing the two rates. Therefore for notation purposes,

i represents the effective rate per annum

and j represents the nominal rate per annum.

If interest is convertible p times a year, the effective rate would be indicated thus :

$$i = \left(1 + \frac{j}{p}\right)^p - 1 \quad (6)$$

which, in terms of figures, when $j = 6\%$ and $p = 2$, would be

$$i = [(1.03)^2 - 1] = 1.0609 - 1 = .0609; \text{ that is, } 6.09\%$$

That is, for finding the amount of 1 for any number of years at nominal rate j , convertible p times a year,

$$s = \left(1 + \frac{j}{p}\right)^{np} \quad (6a)$$

EXAMPLES

1. Find the compound interest on \$1900 for 15 years at 5%.

The table, page 287, gives the amount of 1 for 15 years at 5% as \$2.0789282.

The compound interest on \$1 = \$2.0789282 - \$1 = \$1.0789282

$$1900 \times \$1.0789282 = \$2049.96, \text{ compound interest}$$

2. Find the compound amount of \$3500 for 30 years at 5%, if interest is converted semiannually.

The interest rate per period is $2\frac{1}{2}\%$.

The number of periods is 60.

The table, page 285, gives the amount of \$1 for 60 periods at $2\frac{1}{2}\%$ as \$4.3997898.

$$3500 \times \$4.3997898 = \$15,399.26, \text{ compound amount}$$

The following examples illustrate the application of logarithms to compound interest calculations. If the student is unfamiliar with the use of logarithms, he should carefully review pages 150-159.

3. Find the compound amount of 1 for 25 years at 4.4 %.

$$s = (1 + i)^n = (1.044)^{25}$$

$$\log (1.044)^{25} = .4675 = \log 2.93426$$

Therefore $(1.044)^{25} = 2.93426$

4. If the nominal rate is 6%, find the effective rate, interest being converted every 2 months.

$$i = \left[\left(1 + \frac{j}{p} \right)^p - 1 \right] = (1.01)^6 - 1$$

$$\log (1.01)^6 = .02592 = \log 1.0615$$

Therefore $(1.01)^6 = 1.0615$

$$1.0615 - 1 = .0615; 6.15\%, \text{ effective rate}$$

Since interest is converted every 2 months, the number of conversion periods is 6, and the rate per period is $\frac{1}{6}$ of 6%, or 1%.

5. Find the compound amount of \$12,750 for 40 years at $4\frac{3}{4}\%$.

$$\log 12750 = 4.10551$$

$$\log (1.0475)^{40} = \frac{.8062}{4.91171}$$

The number for the mantissa 91171 = 81604

That is, $\log 81604 = 4.91171$

Therefore the compound amount is \$81,604

6. Find the compound amount of \$1250 at 5% from Apr. 1, 1896 to Feb. 15, 1923, interest being converted semiannually.

$$\begin{array}{r} 1923 - 2 - 15 \\ 1896 - 4 - 1 \\ \hline 26 \text{ yr. } 10 \text{ mo. } 14 \text{ da.} \end{array}$$

26 yr. 10 mo. 14 da. = 53 periods and 4 mo. 14 da. over.

$$\log 1250 = 3.09691$$

$$\log (1.025)^{53} = \frac{.56816}{3.66507}$$

The number for the mantissa 66507 is 462455

That is, $\log 4624.55 = 3.66507$

Therefore amount of \$1250 for 53 periods at $2\frac{1}{2}\%$ is \$4624.55.

The simple interest on \$4624.55 for 4 mo. 14 da. at 5% per annum is \$86.07.

\$4624.55 + \$86.07 = \$4710.62, compound amount of \$1250 for 26 yr. 10 mo. 14 da.

7. A deposit of \$1500 was made in a savings bank paying 4% compounded semiannually. In how many years will the deposit amount to \$3500, on the assumption that no interest is drawn and that no more deposits are made?

$$np = \frac{\log \left(\frac{S}{P} \right)}{\log \left(1 + \frac{j}{p} \right)} \quad \text{Formula (4)}$$

$$np = \frac{\log \left(\frac{3500}{1500} \right)}{\log 1.02} = \frac{\log \left(\frac{7}{3} \right)}{\log 1.02} = \frac{.36798}{.00860} = 42.79$$

That is, the number of periods, or half years, is, in round numbers, 43.

Hence the time is 21 yr. 6 mo.

8. At what rate per cent will \$2000 amount to \$4000 in $17\frac{1}{2}$ years, if interest is convertible semiannually?

$$\log (1 + i) = \frac{\log \left(\frac{S}{P} \right)}{n}, \text{ that is, } \log \left(1 + \frac{j}{p} \right) = \frac{\log \left(\frac{S}{P} \right)}{np} \quad \text{Formula (5)}$$

$$\log \left(1 + \frac{j}{2} \right) = \frac{\log \left(\frac{4000}{2000} \right)}{35} = \frac{\log 2}{35} = \frac{.30103}{35} = .0086$$

The number for the mantissa .0086 = 1.02

That is, $1 + \frac{j}{2} = 1.02$, and $\frac{j}{2} = .02$

Therefore $j = .04 = 4\%$

NOTE. The preceding two solutions should be studied until every step in the process is entirely clear. The principles involved will be required in future solutions.

Problems

1. Compare the compound interest on \$2500 for 20 years at 5% converted annually with the compound interest on the same amount for the same time and rate with interest converted semiannually.

2. Find the compound amount of \$2500 at 6% for 15 yr.; for 30 yr.; for 60 yr. Explain the rapid increase in amount.

3. A sum of \$40,000 was left in trust to a boy on May 27, 1922, to be paid to him with compound interest at 4%, accumulated semiannually, when he reached the age of 21 years. The boy was born July 29, 1913. Find the amount of the fund when the boy was 21 years of age.

4. A bequest of \$15,000 was left in trust for a boy 13 years of age, with instructions to pay principal and accumulated interest to him when the fund amounted to \$40,000. The fund was invested at 5% convertible semiannually. At what age should the young man receive the \$40,000? (Give answer in nearest half year.)

5. At the birth of a son, his father set aside the sum of \$5000, which, with interest compounded semiannually, was to be paid to the son at the age of 21 years. The amount paid to the son on becoming of age was \$14,105. At what nominal rate was the money invested?

To find the present value of a future sum*The present value of 1*

Another phase of compound interest is to find the present value of a future sum. Theoretically, as in simple interest (see pages 98-99), the present value of a future sum is found by dividing the future sum by the amount of 1 for the given time and rate.

The present worth of a future value, as stated on page 98, is a sum which, with interest accumulation for the given time and rate, will amount to the future value. Discount, that is, true discount, is equal to the interest for the given time and rate on the present value.

Plainly, then, if v stands for present value, and the future value is 1, we have

$$v(1+i) = 1$$

$$\text{or } v = \frac{1}{1+i}$$

$$\text{also } v^2 = \frac{1}{(1+i)^2}$$

$$\text{and } v^{15} = \frac{1}{(1+i)^{15}}$$

$$v^n = \frac{1}{(1+i)^n} \quad (7)$$

By the law of negative exponents (see page 151) formula 7 may be written

$$v^n = (1+i)^{-n} \quad (8)$$

Present values, therefore, are indicated by the series

$$(1+i)^{-1}, (1+i)^{-2}, (1+i)^{-3} \quad . \quad . \quad . \quad (1+i)^{-n}$$

Present-value tables are useful and convenient for rates and periods within their scope. For periods beyond the range of the table, the same principle applies as in finding the amount of 1 for periods outside the table. The present value of 1 for 125 periods at 2% per period is indicated thus:

$$v^{125} = (1.02)^{-125}$$

The table gives

$$v^{100} = (1.02)^{-100} = .1380330$$

and

$$v^{25} = (1.02)^{-25} = .6095309$$

but

$$(1.02)^{-100} \times (1.02)^{-25} = (1.02)^{-125}$$

Hence

$$.1380330 \times .6095309 = .0841354$$

That is,

$$(1.02)^{-125} = .0841354$$

Therefore the present value of 1 for 125 periods at 2% per period is .0841354.

Since S = amount at maturity (formula 3), the present value of any sum may be represented by $S(1+i)^{-n}$.

Hence if V represents present value, we have

$$V = S(1+i)^{-n} \quad (9)$$

If one has not access to a table of present values, such values may be found by actual calculation or by the application of logarithms.

The following indicated operations will help to make clear the present value series.

Assume $i = .02$

$$(1.02)^{-1} = 1 \div 1.02 = .9803922$$

$$(1.02)^{-2} = .9803922 \div 1.02 = .9611688$$

$$(1.02)^{-3} = .9611688 \div 1.02 = .9423223$$

$$(1.02)^{-4} = .9423223 \div 1.02 = .9238454, \text{ and so on}$$

Or, since $(1.02)^{-1} = .9803922$

$$(1.02)^{-2} = .9803922 \times .9803922 = .9611688$$

and $(1.02)^{-3} = .9611688 \times .9803922 = .9423223, \text{ etc.}$

The method above outlined becomes too tedious for problems other than those for a very few periods.

The application of logarithms to the computation of present values will be illustrated under "Examples," page 170.

Compound discount

The difference between the present value and the future value of a debt is the **compound discount**. Or, to put it differently, the compound discount is the compound interest on the present value for the given time and rate.

If D represents compound discount,

$$D = 1 - v^n = 1 - \frac{1}{(1 + i)^n} \quad (10)$$

or $D = \frac{1}{(1 + i)^n} \times [(1 + i)^n - 1] = \frac{(1 + i)^n - 1}{(1 + i)^n}$

but $(1 + i)^n - 1 = I$ (Formula 2)

Hence $D = \frac{I}{(1 + i)^n} \quad (11)$

That is, the compound discount is equal to the quotient of the compound interest divided by the compound amount.

As an illustration of the preceding formulas, assume that it is desired to find the compound discount on 1 for 5 years at 4% per annum.

By formula 10, $1 - .8219271 = .1780729$, compound discount.

By formula 11, the amount of 1 for 5 yr. at 4% is 1.2166529 and the interest is .2166529.

Therefore $.2166529 \div 1.2166529 = .1780729$, compound discount.

EXAMPLES

1. Find the present value and compound discount of \$1875 due in 8 years at 5% per annum.

By the table (page 291) the present value of 1 for 8 yr. at 5% is .6768394.

$$1875 \times \$.6768394 = \$1269.07, \text{ present value}$$

$$\$1875 - \$1269.07 = \$605.93, \text{ compound discount}$$

2. Find the present value of 1 for 25 periods at $5\frac{1}{4}\%$ per period.

$$v^n = (1 + i)^{-n} = (1.0525)^{-25} = \frac{1}{(1.0525)^{25}}$$

$$\log 1 = 10 \qquad - 10$$

$$\log (1.0525)^{25} = \underline{.555625}$$

$$\log \text{ quotient} = \underline{9.444375 - 10}$$

The number for the mantissa 444375 is 27821

That is, $\log .27821 = 9.444375 - 10$

Therefore $.27821 = \text{present value}$

3. A debt of \$10,000, due in 15 years, is to be provided for by a present deposit with a trust company. If the deposit accumulates at 4.8% per annum, convertible semiannually, find the amount of the deposit.

$$v^n = 10,000 \left(1 + \frac{j}{p}\right)^{-np} = 10,000 \frac{1}{\left(1 + \frac{j}{p}\right)^{np}} = \frac{10,000}{(1.024)^{30}}$$

$$\log 10,000 = 4$$

$$\log (1.024)^{30} = \underline{.30900}$$

$$\log \text{ quotient} = \underline{3.69100}$$

The number for the mantissa 69100 = 490911

That is, $\log 4909.11 = 3.69100$

Therefore $\$4909.11 = \text{present value}$

Problems

1. Given $(1 + i)^n = 2.4117140$, find the compound discount for the same time and rate.

2. Given $(1 + i)^n - 1 = 2.3863549$, find the present value of 1 for the same time and rate.

3. The compound interest on \$1500 for a given time and rate is \$1209.17. Find the present worth of \$1500 for the same time and rate.

4. At 4.6% per annum, what sum of money will amount to \$9285.31 in 45 years?

5. A debt of \$2150 is due 7 years hence. The debtor wishes to pay the debt at once. Allowing interest at 5% converted semiannually, to how much discount should he be entitled?

6. Find the present value of 1 for 46 periods at 3.7% per period.

7. What sum invested so as to accumulate at $5\frac{1}{2}\%$ per annum nominally will amount to \$25,000 in 40 years if interest is converted semiannually?

XIV. ANNUITIES

An **annuity** is a series of equal payments made at regular intervals of time.

Payments made on account of interest on bonds, insurance premium, rents, etc., are illustrations of annuities.

The sum of the payments made in one year is called the *annual rent*.

The interval of time between two successive payments may be 1 year, $\frac{1}{2}$ year, 1 month, or any other fixed period of time.

Unless otherwise specified it is to be understood that the first payment of an annuity is to be made at the end of the first interval or period of time.

Annuities are variously designated as follows :

Annuity certain — an annuity having a fixed number of periods to run.

Annuity due — an annuity whose first payment is made at the beginning of the period.

Contingent annuity — an annuity whose first or last payment depends upon the occurrence of some event the date of which cannot be foretold.

Deferred annuity — an annuity whose first payment is to be made after a specified number of periods have elapsed. If an annuity dated April 1, 1922, is deferred 4 years, the annuity is said to be “entered on” at the expiration of the 4 years, that is, April 1, 1926. The first payment would be due April 1, 1927. Hence in an annuity deferred m years, the first payment would be made at the end of $(m + 1)$ years. A deferred annuity becomes a regular annuity from the date it is entered on.

Perpetuity — an annuity whose payments continue forever.

Forborne annuity — an annuity whose payments are allowed to accumulate at compound interest during its status.* The accumulated sum at the expiration of the status is called the *amount* of the annuity.

The customary notation used in dealing with annuities is as follows :

- R = the annual rent ; that is, the annual payment.
 $s_{\overline{n}|}$ = the amount of 1 per annum for n years.
 $S_{\overline{n}|}$ = the amount of an annuity for n years.
 $a_{\overline{n}|}$ = the present value of 1 per annum for n years.
 $A_{\overline{n}|}$ = the present value of an annuity for n years.
 $s_{\overline{n}|}^p$ = the amount of 1 per annum for n years payable in p installments a year.
 $a_{\overline{n}|}^p$ = the present value of 1 per annum for n years payable in p installments a year.
 $t|s_{\overline{n}|}$ = the amount of an annuity of 1 per annum for n years, the first payment being deferred $(t + 1)$ years.
 $t|a_{\overline{n}|}$ = the present value of 1 per annum for n years, the first payment being deferred $(t + 1)$ years.
 $t|a_{\overline{n}|}^p$ = the present value of 1 per annum payable in p installments a year, the first installment being deferred $\left(t + \frac{1}{p}\right)$ years.
 a_{∞}^{**} = the present value of a perpetuity.

When the symbols $s_{\overline{n}|}$, $a_{\overline{n}|}$, $s_{\overline{n}|}^p$, etc., are written in black or bold-face type as they appear in this sentence, they refer to an **annuity due**.

To find the amount of an annuity

The problem of finding the amount of an annuity presupposes it to be a forborne annuity — that is, all interest increments as they become due are added to the principal.

By definition, an annuity is a series of equal payments made at regular intervals of time.

* The "status" of an annuity is the entire time over which payments are made.

** The symbol ∞ means infinity. Hence the expression a_{∞} means the present value of an annuity which runs forever.

Find the amount of 1 per annum for 5 years at the effective rate i .

Each payment draws compound interest from the time it is made to the termination of the term of the annuity. The first payment will draw interest for 4 years; the second for 3 years; etc. Hence

The amount of the 1st payment is $(1 + i)^4$.

The amount of the 2d payment is $(1 + i)^3$.

The amount of the 3d payment is $(1 + i)^2$.

The amount of the 4th payment is $(1 + i)$.

The amount of the 5th payment is 1.

These values in reverse order form a geometric progression.

Thus : $1, (1 + i), (1 + i)^2, (1 + i)^3, (1 + i)^4$

The first term is 1.

The ratio is $(1 + i)$.

Since $s = \frac{rl - a}{r - 1}$, by substitution we have, for the sum of the series:

$$s = \frac{(1 + i)(1 + i)^4 - 1}{(1 + i) - 1} = \frac{(1 + i)^5 - 1}{i}$$

In general terms the progression for n terms is

$$1, (1 + i), (1 + i)^2, (1 + i)^3, (1 + i)^4 \dots (1 + i)^{n-1}$$

Substituting in the formula $s = \frac{rl - a}{r - 1}$ gives

$$s = \frac{(1 + i)(1 + i)^{n-1} - 1}{(1 + i) - 1} = \frac{(1 + i)^n - 1}{i}$$

$$\text{Hence} \quad s_{\overline{n}|} = \frac{(1 + i)^n - 1}{i} \quad (12)$$

To find the amount of an annuity due, the series is written thus:

$$(1 + i), (1 + i)^2, (1 + i)^3, (1 + i)^4 \dots (1 + i)^n$$

Again, substituting in the formula $s = \frac{rl - a}{r - 1}$ gives

$$\begin{aligned} s &= \frac{(1 + i)(1 + i)^n - (1 + i)}{(1 + i) - 1} = \frac{(1 + i)^{n+1} - 1 - i}{i} \\ &= \frac{(1 + i)^{n+1} - 1}{i} - 1 \end{aligned}$$

$$\text{Hence} \quad s_{\overline{n}|} = \frac{(1 + i)^{n+1} - 1}{i} - 1 \quad (13)$$

To find the amount of an annuity payable in p installments annually at nominal rate j , the series is

$$\frac{1}{p}, \frac{1}{p}\left(1 + \frac{j}{p}\right), \frac{1}{p}\left(1 + \frac{j}{p}\right)^2, \frac{1}{p}\left(1 + \frac{j}{p}\right)^3 \dots \frac{1}{p}\left(1 + \frac{j}{p}\right)^{np-1}$$

The sum of the series, then, is expressed thus:

$$s = \frac{1}{p} + \frac{1}{p}\left(1 + \frac{j}{p}\right) + \frac{1}{p}\left(1 + \frac{j}{p}\right)^2 + \frac{1}{p}\left(1 + \frac{j}{p}\right)^3 + \dots \frac{1}{p}\left(1 + \frac{j}{p}\right)^{np-1}$$

Since $\frac{1}{p}$ is a common factor, the expression may be written:

$$s = \frac{1}{p} \left[1 + \left(1 + \frac{j}{p}\right) + \left(1 + \frac{j}{p}\right)^2 + \left(1 + \frac{j}{p}\right)^3 + \dots \left(1 + \frac{j}{p}\right)^{np-1} \right]$$

Substituting in the formula $s = \frac{rl - a}{r - 1}$ gives

$$s = \frac{1}{p} \left[\frac{\left(1 + \frac{j}{p}\right)\left(1 + \frac{j}{p}\right)^{np-1} - 1}{\left(1 + \frac{j}{p}\right) - 1} \right] = \frac{1}{p} \left[\frac{\left(1 + \frac{j}{p}\right)^{np} - 1}{\frac{j}{p}} \right]$$

$$\text{Hence} \quad s_{\overline{n}|}^p = \frac{1}{p} \left[\frac{\left(1 + \frac{j}{p}\right)^{np} - 1}{\frac{j}{p}} \right] \quad (14)$$

If an annuity of \$1200 is assumed to run for 5 years, payable in semiannual installments, with nominal rate of interest at 6%, formula 14 reduces to this simple form:

$$S_{\overline{n}|} = \frac{1200}{2} \left[\frac{\left(1 + \frac{.06}{2}\right)^{10} - 1}{\frac{.06}{2}} \right] = 600 \left(\frac{1.03^{10} - 1}{.03} \right)$$

EXAMPLES

1. Find the amount of an annuity of \$1200 for 30 years at 5%.

In the table, page 295, the amount of 1 per annum for 30 years at 5% is given as 66.4388475.

$$1200 \times \$66.4388475 = \$79,726.62, \text{ amount}$$

2. An annuity of \$2400, payable in quarterly installments, is invested at 6% per annum. The term of the annuity is 20 years. Find its amount at the expiration of its status.

Since the annuity is payable in quarterly installments, each payment is \$600, the interest rate per period is $1\frac{1}{2}\%$, and the number of periods is 80.

In the table, page 292, find 152.7108525 as the amount of 1 per period for 80 periods at $1\frac{1}{2}\%$.

$$600 \times \$152.7108525 = \$91,626.51, \text{ amount}$$

3. An insurance premium of \$450 is payable annually for 20 years. Allowing interest at $4\frac{1}{2}\%$, find the amount of the payments at the expiration of the 20 years.

Insurance premiums are payable in advance, hence this is an annuity due.

By reference to the formula

$$s_{\overline{n}|} = \frac{(1+i)^{n+1} - 1}{i} - 1 = \frac{(1.045)^{21} - 1}{.045} - 1$$

it is to be noted that by taking the value for 21 years in the table, page 294, and deducting 1 therefrom, we have the amount of an annuity due of 1, for 20 years.

For 21 years the amount is 33.7831368

Deducting

leaves

1.

32.7831368, the amount of 1 for
20 years

$$450 \times \$32.7831368 = \$14,752.41, \text{ amount}$$

4. Find the amount of an annuity of \$5000 payable in semi-annual installments for a period of 35 years, interest being $5\frac{1}{2}\%$ per annum.

$$s_{\overline{n}|}^2 = \frac{\left(1 + \frac{j}{2}\right)^{2n} - 1}{\left(1 + \frac{j}{2}\right) - 1} = \frac{(1.0275)^{70} - 1}{.0275}$$

$$\log (1.0275)^{70} = .82460$$

$$\text{antilog } .82460 = 6.677285$$

$$\text{Hence } \frac{(1.0275)^{70} - 1}{.0275} = \frac{5.677285}{.0275} = 206.44673, \text{ amount of 1}$$

$$2500 \times \$206.44673 = \$516,116.83, \text{ amount}$$

5. In how many years will an annual saving of \$500 amount to \$10,000 at 4% effective? (Reckon to nearest whole year.)

$$\text{Since } S_n = \frac{R [(1 + i)^n - 1]}{i}$$

$$10,000 = \frac{500 [(1.04)^n - 1]}{.04}$$

Clearing of fractions and multiplying gives

$$400 = 500(1.04)^n - 500$$

Transposition gives

$$900 = 500(1.04)^n$$

Dividing by 500 gives

$$1.8 = (1.04)^n$$

Hence

$$\log 1.8 = n \times \log 1.04$$

That is,

$$n = \frac{\log 1.8}{\log 1.04} = \frac{.25572}{.01703} = 15$$

Therefore the time is 15 years.

Problems

1. By the terms of a will a boy 7 years of age was left an annuity of \$750 which was to be invested at 5% and left to accumulate until he became of age. What amount did he receive on becoming 21 years of age?

2. A piece of property rents for \$18,000 a year, payable quarterly in advance. If the quarterly installments are invested regularly at 5% interest, to what sum will the rentals amount in 15 years?

3. To redeem at maturity a bond issue due in 40 years, a corporation sets aside every 6 months \$40,260 as a sinking fund. If the fund accumulates nominally at 5%, find to the nearest \$1000 the amount of the bond issue.

4. A young man aged 21 decides to begin a savings fund and deposits therein \$125 each 6 months for the next 5 years, at which time

his earnings are sufficient to enable him to save \$250 each half year. He continues to save at that rate for 10 years. He then begins to save at the rate of \$500 semiannually. Assuming that his savings are invested at 5%, nominal, find the amount that he will have to his credit at the age of 60.

5. Compare the amount of an annuity of \$3500 for 30 years at $4\frac{3}{4}\%$ per annum, payable annually, with the amount if paid in semi-annual installments, at the same nominal rate of interest.

6. Using the data given in problem 5 and regarding the annuity as an annuity due, compare the amount on an annual basis with the amount on a semiannual basis. How do you explain the variation in answers?

7. At the age of 21 a young man determines to save \$250 a year until his savings amount to \$25,000. Assuming that he invests his savings semiannually at 5% nominal, at what age will he have \$25,000 to his credit? (Reckon to nearest birthday.)

To find the present value of an annuity

Finding the present value of an annuity is a very important phase of investment calculations.

The present value of an annuity of 1 for n years is the sum of the several present values of 1 for from 1 to n years respectively.

Let n equal 5 and the rate 4%; find the present value of 1 per annum.

The present value of 1:

$$\text{Due 1 year hence is } \frac{1}{(1+i)} = \frac{1}{1.04} = .9615385$$

$$\text{Due 2 years hence is } \frac{1}{(1+i)^2} = \frac{1}{(1.04)^2} = .9245562$$

$$\text{Due 3 years hence is } \frac{1}{(1+i)^3} = \frac{1}{(1.04)^3} = .8889964$$

$$\text{Due 4 years hence is } \frac{1}{(1+i)^4} = \frac{1}{(1.04)^4} = .8548042$$

$$\text{Due 5 years hence is } \frac{1}{(1+i)^5} = \frac{1}{(1.04)^5} = .8219271$$

$$\text{The total present value is } 4.4518224$$

The values

$$\frac{1}{1+i}, \frac{1}{(1+i)^2}, \frac{1}{(1+i)^3}, \frac{1}{(1+i)^4}, \frac{1}{(1+i)^5} \dots \frac{1}{(1+i)^n}$$

form a descending progression. Hence the present value of an annuity is the sum of a descending geometric progression.

Since $v = \frac{1}{(1+i)}$ and $v^2 = \frac{1}{(1+i)^2}$, etc., it is customary, for convenience, to write the foregoing progression thus :

$$v, v^2, v^3, v^4, v^5, \dots v^n$$

The first term is v .

The ratio is v .

Since $s = \frac{rl - a}{r - 1}$, by substitution the value is written:

$$s = \frac{vv^n - v}{v - 1}$$

That is,

$$a_{\overline{n}|} = \frac{vv^n - v}{v - 1} = \frac{v^{n+1} - v}{v - 1}$$

Dividing both numerator and denominator by -1 gives

$$a_{\overline{n}|} = \frac{v - v^{n+1}}{1 - v}$$

Dividing both numerator and denominator by v , or by its equivalent $\frac{1}{1+i}$, gives

$$a_{\overline{n}|} = \frac{\frac{v - v^{n+1}}{v}}{\frac{1 - v}{\frac{1}{1+i}}} = \frac{1 - v^n}{\frac{1+i-1}{1+i} \times \frac{1+i}{1}} = \frac{1 - v^n}{i}$$

Hence

$$a_{\overline{n}|} = \frac{1 - v^n}{i} \quad (15)$$

If the annuity consists of the payment of \$1000 per annum instead of \$1, its present value is $1000 \times \frac{1 - v^n}{i}$.

Since $A_{\overline{n}|}$ represents present value and R the annual rent, we have

$$A_{\overline{n}|} = R \frac{1 - v^n}{i} \quad (16)$$

An annuity due is an annuity whose first payment is made at the beginning instead of at the end of the first period. After the first payment is made, the remaining payments constitute a regular annuity for $(n - 1)$ periods.

That is,
$$a_{\overline{n-1}|} = \frac{1 - v^{n-1}}{i} \quad (16 a)$$

Since the first payment is cash, and its present value is equal to itself, the present value of an annuity due is 1 greater than an annuity for $(n - 1)$ periods.

Hence
$$a_{\overline{n}|} = 1 + \frac{1 - v^{n-1}}{i} \quad (16 b)$$

To find the present value of an annuity payable in p installments a year at nominal rate j , the series is :

$$\frac{1}{p} \left(\frac{1}{1 + \frac{j}{p}} \right), \frac{1}{p} \left(\frac{1}{\left(1 + \frac{j}{p}\right)^2} \right), \frac{1}{p} \left(\frac{1}{\left(1 + \frac{j}{p}\right)^3} \right), \dots, \frac{1}{p} \left(\frac{1}{\left(1 + \frac{j}{p}\right)^{np}} \right)$$

Taking out the common factor $\frac{1}{p}$ and substituting in the formula $s = \frac{rl - a}{r - 1}$ gives:

$$s = \frac{1}{p} \left[\frac{\left(\frac{1}{1 + \frac{j}{p}} \right) \left(\frac{1}{1 + \frac{j}{p}} \right)^{np} - \frac{1}{\left(1 + \frac{j}{p}\right)}}{\frac{1}{\left(1 + \frac{j}{p}\right)} - 1} \right] = \frac{1}{p} \left[\frac{\frac{1}{\left(1 + \frac{j}{p}\right)^{np+1}} - \frac{1}{\left(1 + \frac{j}{p}\right)}}{\frac{1}{\left(1 + \frac{j}{p}\right)} - 1} \right]$$

Changing the signs of all terms in both numerator and denominator gives:

$$s = \frac{1}{p} \left[\frac{\frac{1}{\left(1 + \frac{j}{p}\right)} - \frac{1}{\left(1 + \frac{j}{p}\right)^{np+1}}}{1 - \frac{1}{\left(1 + \frac{j}{p}\right)}} \right]$$

Dividing both numerator and denominator by the common factor $\frac{1}{1 + \frac{j}{p}}$, and writing $a_{\overline{n}|}^p$ for s gives:

$$a_{\overline{n}|}^p = \frac{1}{p} \left[\frac{1 - \frac{1}{\left(1 + \frac{j}{p}\right)^{np}}}{\frac{j}{p}} \right] \quad (17)$$

Assuming an annuity of \$1500, to run for 10 years, payable in semi-annual installments, with nominal interest rate of 6%, show the application of formula 17 to its solution.

$$A_{\overline{n}|}^2 = \frac{R}{p} \left[\frac{1 - \frac{1}{\left(1 + \frac{j}{p}\right)^{np}}}{\frac{j}{p}} \right] = 750 \left[\frac{1 - \frac{1}{(1.03)^{20}}}{.03} \right]$$

A **perpetuity** has been defined as an annuity whose payments last forever or indefinitely. It is possible to find the present value of a perpetuity, though its future sum may not be reckoned.

Since an investment of 1 will produce an annuity of i forever, it is evident that $I = ia_{\infty}$. Assume $I = 1$. Then $ia_{\infty} = 1$. Therefore the present value of a perpetuity of 1 is expressed thus:

$$a_{\infty} = \frac{1}{i} \quad (18)$$

A **deferred annuity** is one whose first payment has been deferred a given number of years or periods. The problem of finding the present value of a deferred annuity is theoretically a double problem.

A deferred annuity becomes a regular annuity from the date it is entered on. Theoretically, it is necessary to find two present values:

1. The present value of a regular annuity.
2. The present value of the result found in 1.

Thus, if an annuity payable for n years is deferred t years, the first result would be

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

The second result would be v^t times the first result. That is,

$$t|a_{\overline{n}|} = v^t \left(\frac{1 - v^n}{i} \right) \text{ or } v^t \cdot a_{\overline{n}|} \quad (19)$$

But an annuity deferred t years and payable for n years is equal to an annuity for $(n + t)$ years less an annuity for t years.

$$\text{Hence} \quad t|a_{\overline{n}|} = a_{\overline{n+t}|} - a_{\overline{t}|} \quad (20)$$

Similarly, the formula for finding the present value of an annuity deferred t years and payable for n years, in p installments a year, is

$$t|a_{\overline{n}|}^p = a_{\overline{n+t}|}^p - a_{\overline{t}|}^p \quad (21)$$

EXAMPLES

1. What is the present value of an annuity of \$1500 for 20 years, interest 5% per annum?

The present value of 1 per annum for 20 years at 5% is 12.4622103.

$$1500 \times \$12.4622103 = \$18693.32, \text{ present value}$$

2. An annuity of \$900 is payable in quarterly installments for a period of 15 years. Allowing interest at 6%, nominal, find the present value of the annuity.

Since the annuity is paid in quarterly installments, there are 60 installments. At 6% per annum the rate per quarter is $1\frac{1}{2}\%$. In the table, page 296, find the present value of 1 per period for 60 periods at $1\frac{1}{2}\%$. It is 39.3802689.

$$225 \times \$39.3802689 = \$8860.56, \text{ present value}$$

3. Find the present value of an annuity due of \$800 to run 25 years, at 5%.

$$a_{\overline{n}|} = 1 + \frac{1 - v^{n-1}}{i} = 1 + \frac{1 - \frac{1}{(1.05)^{24}}}{.05}$$

In the table, page 299, the present value of 1 for 24 years at 5% is given as 13.7986418. Adding 1 gives 14.7986418.

$$800 \times \$14.7986418 = \$11,838.91, \text{ present value}$$

4. Find the present value of a perpetuity of \$750 at 5% effective.

$$a_{\infty} = \frac{1}{i}$$

$$A_{\infty} = \frac{\$750}{.05} = \$15,000, \text{ present value}$$

5. An annuity of \$1250 payable for 15 years is entered on 5 years from the present time. Allowing interest at $4\frac{1}{2}\%$ effective, find its present value.

By formula 20, page 182, the present value of an annuity payable for 15 years after being deferred 5 years is equal to the present value of an annuity for 20 years less the present value of an annuity for 5 years.

The present value of an annuity of 1 at $4\frac{1}{2}\%$

for 20 years is 13.0079364 (table, page 298)

for 5 years is 4.3899767 (table, page 298)

The difference is 8.6179597

$$1250 \times \$8.6179597 = \$10,772.45, \text{ present value}$$

6. An annuity of \$2000, deferred 3 years, is payable in semiannual installments, for a period of 12 years. At a nominal rate of 5.6%, find its present value.

$$t|a_n^p = a_{n+\overline{t}|}^p - a_{\overline{t}|}^p = \frac{1 - \frac{1}{(1.028)^{30}}}{.028} - \frac{1 - \frac{1}{(1.028)^6}}{.028}$$

$$\log 1 = 10 \quad - 10$$

$$\log (1.028)^{30} = .35970$$

$$\log .43682 = 9.64030 - 10$$

$$1 - .43682 = .56318$$

$$.56318 \div .028 = 20.11357$$

$$\text{That is, } \frac{1 - \frac{1}{(1.028)^{30}}}{.028} = 20.11357$$

$$\log 1 = 10 \quad - 10$$

$$\log (1.028)^6 = .07194$$

$$\log .84734 = 9.92806 - 10$$

$$1 - .84734 = .15266$$

$$.15266 \div .028 = 5.45214$$

$$\text{That is, } \frac{1 - \frac{1}{(1.028)^6}}{.028} = 5.45214$$

$$20.11357 - 5.45214 = 14.66143, \text{ present value of 1}$$

$$1000 \times \$14.66143 = \$14,661.43, \text{ present value of annuity}$$

Problems

1. Find the present value of an annuity of \$1800 payable for 30 years, interest at 5%.

2. An annuity of \$1500 having 25 years to run, with first payment due at once, is for sale on a $4\frac{1}{2}\%$ basis. Find its present cash value.

3. Allowing interest at 5% nominal, what is the present cash value of a rental of \$5000 per annum, payable quarterly in advance for a period of 12 years?

4. To endow a chair in a university costs \$5000 per annum. Find the amount of the endowment fund, assuming that it is invested at 4% effective.

5. A store is leased for a term of 20 years, at \$15,000 a year payable annually in advance, the term of the lease to begin 2 years hence. Find the present cash value of the lease, allowing interest at $4\frac{1}{2}\%$.

6. A property is leased for a term of 30 years at an annual rental of \$25,000, payable annually at the end of the year. Allowing interest at 5.4% effective, find the present value of the lease.

7. J. D. Simmons leased a property for 35 years from April 1, 1918, at an annual rental of \$7500. Mr. Simmons also agreed to pay the taxes and insurance and to keep the property in repair. On April 1, 1922, Mr. Simmons sublet the property for the remainder of the term of his lease at \$12,000 per annum, and agreed to continue paying taxes, insurance, and repair bills. Mr. Simmons died in December, 1922. His executors desired to arrive at the value of his estate as of April 1, 1923. Taxes averaged \$1000, insurance \$250, and repairs \$500 a year. If money was worth 6% effective, find the value of the leasehold April 1, 1923.

8. C. K. Mentor has a 10-year lease of a factory, which expires December 31, 1925. The annual rental is \$6000, payable in semiannual installments at the end of each half year. In December, 1922, Mr. Mentor seeks to renew the lease for a further period of 10 years at the same rental. It is agreed that the property has increased in value, that \$7000 per annum is a fair and just price, and that Mr. Mentor shall make a cash payment December 31, 1922, which will offset the increase in rent for the renewal period of 10 years. Allowing money to be worth 5%, find the amount of the cash payment.

9. For the purpose of securing for his wife and children a definite income of \$25,000 per annum, payable in quarterly installments at the end of each quarter for a period of 30 years, a capitalist buys from an insurance company such an annuity. Assuming that the annuity is capitalized at 3.6% nominal, find its cost.

10. On September 15, 1922, L. A. Norton paid a trust company a sum sufficient, with interest accumulation, to pay his daughter's college expenses, estimated to be \$2500 a year, payable in advance each year for 4 years. If the daughter was to enter college in September, 1927, and the fund was to earn $5\frac{1}{4}\%$, exclusive of expense of administering it, what sum was paid to the trust company, September 15, 1922?

An interesting problem under "present value of an annuity" is to find the present value of a perpetuity payable at intervals of n years. To illustrate, let it be assumed that a given public improvement, costing \$15,000, has to be renewed every 25 years. Considering money worth 5%, what sum should be set aside when the improvement is first made, to provide for an indefinite number of renewals?

If the improvement were to be renewed but once, the present value of its cost would be expressed symbolically thus:

$$v^n = \frac{1}{(1+i)^n} = \frac{1}{(1.05)^{25}}$$

To provide for a second renewal would require additional funds whose present value would be expressed by

$$v^{2n} = \frac{1}{(1+i)^{2n}} = \frac{1}{(1.05)^{50}}$$

Also the present value of the funds for a third renewal would be indicated by

$$v^{3n} = \frac{1}{(1+i)^{3n}} = \frac{1}{(1.05)^{75}}, \text{ etc.}$$

That is, the series is

$$v^n, v^{2n}, v^{3n}, v^{4n}, \dots v^{\infty, n}$$

The present value sought is the sum of the series. Hence substituting in the formula $s = \frac{rl - a}{r - 1}$ gives

$$s = \frac{v^n \cdot v^{\infty, n} - v^n}{v^n - 1}$$

Changing signs in both numerator and denominator gives

$$s = \frac{v^n - v^n \cdot v^{\infty, n}}{1 - v^n}$$

Dividing both numerator and denominator by the common factor v^n , or by its equivalent $\frac{1}{(1+i)^n}$, gives

$$s = \frac{1 - v^{\infty, n}}{(1+i)^n - 1}$$

Since $v = \frac{1}{(1+i)}$, $v^{\infty, n} = \frac{1}{(1+i)^{\infty, n}}$. But the symbol ∞ means infinity, hence the denominator $(1+i)^{\infty, n}$ becomes infinitely large. Therefore the value of the expression $\frac{1}{(1+i)^{\infty, n}}$ is regarded as 0. Writing the commonly used symbol $a_{\infty, n}$ for s and eliminating $v^{\infty, n}$ (which = 0) gives

$$a_{\infty, n} = \frac{1}{(1+i)^n - 1} \quad (22)$$

as the symbolic expression of the present value of a perpetuity payable at intervals of n years.

Numerically expressed, the formula is $\frac{1}{(1.05)^{25} - 1}$.

$$(1.05)^{25} = 3.3863549$$

$$3.3863549 - 1 = 2.3863549$$

$$\$15,000 \div 2.3863549 = \$6285.74, \text{ present value}$$

If the original cost of \$15,000 is added to the present value of future renewals, the result is called the **capitalized cost** of the improvement.

$$\$15,000 + \$6285.74 = \$21,285.74, \text{ capitalized cost}$$

Since
$$\frac{(1+i)^n - 1}{i} = s_{\overline{n}|}$$

$$(1+i)^n - 1 = i s_{\overline{n}|}$$

Therefore
$$a_{\overline{\infty}, \overline{n}|} = \frac{1}{i s_{\overline{n}|}}$$

If C = original cost, the capitalized cost $= C + \frac{C}{i s_{\overline{n}|}}$

Factoring $C + \frac{C}{i s_{\overline{n}|}}$ gives $\frac{C}{i} \left(i + \frac{1}{s_{\overline{n}|}} \right)$

This in turn equals $\left(\frac{C}{i} \right) \left(\frac{1}{a_{\overline{n}|}} \right)$

Therefore capitalized cost may be expressed as $\left(\frac{C}{i} \right) \left(\frac{1}{a_{\overline{n}|}} \right)$

Working from this last formula, it is possible to find what additional sum may profitably be expended on an article at the time of the original outlay to extend its life a given number of years. Such problems as the chemical treatment of fence posts, railroad ties, the use of slate or copper roofing materials in the place of wooden shingles, etc., are but suggestions for the use of such a formula.

EXAMPLE

A wooden sidewalk costing \$150 will last 8 years. How much more can the owner afford to pay for a cement walk if it will last 20 years, money being worth 5%?

Let w = the additional cost
and x = the increase in the number of years of service

The capitalized cost of the wooden sidewalk is $\left(\frac{C}{i} \right) \left(\frac{1}{a_{\overline{n}|}} \right)$ and of the cement walk is $\left(\frac{C+w}{i} \right) \left(\frac{1}{a_{\overline{n+x}|}} \right)$.

The assumption is that, in the long run, the cost of the two sidewalks will be the same. Therefore

$$\left(\frac{C}{i} \right) \left(\frac{1}{a_{\overline{n}|}} \right) = \left(\frac{C+w}{i} \right) \left(\frac{1}{a_{\overline{n+x}|}} \right)$$

Clearing of fractions gives

$$(C + w)(ia_{\overline{n}|}) = C ia_{\overline{n+x}|}$$

Dividing and transposing to find value of w gives

$$w = C \frac{ia_{\overline{n+x}|}}{ia_{\overline{n}|}} - C$$

Reducing to improper fraction gives

$$w = C \frac{ia_{\overline{n+x}|} - ia_{\overline{n}|}}{ia_{\overline{n}|}}$$

Substituting in numerator $\frac{1 - \frac{1}{(1+i)^n}}{i}$ for $a_{\overline{n}|}$, etc., gives

$$w = C \frac{1 - \frac{1}{(1+i)^{n+x}} - 1 + \frac{1}{(1+i)^n}}{ia_{\overline{n}|}}$$

Combining terms and substituting v^n for $\frac{1}{(1+i)^n}$, etc., gives

$$w = Cv^n \frac{\frac{1 - v^x}{i}}{a_{\overline{n}|}}$$

Substituting $a_{\overline{x}|}$ for $\frac{1 - v^x}{i}$ gives

$$w = \frac{Cv^n a_{\overline{x}|}}{a_{\overline{n}|}}$$

Substituting for $a_{\overline{n}|}$ its equivalent $v^n s_{\overline{n}|}$ and reducing to lower terms gives

$$w = \frac{Ca_{\overline{x}|}}{s_{\overline{n}|}}$$

$$a_{\overline{12}|} \text{ at } 5\% = 8.8632516$$

$$s_{\overline{8}|} \text{ at } 5\% = 9.5491089$$

$$\frac{150 \times \$8.8632516}{9.5491089} = \$139.23, \text{ additional cost}$$

$$\$150 + \$139.23 = \$289.23, \text{ cost of cement sidewalk}$$

Problems

11. A memorial building cost \$30,000. What endowment should be provided at the time of its erection to provide for its rebuilding every 40 years, assuming that the cost of construction remains constant and that money is worth 4% effective?

12. Assuming that the upkeep of the building mentioned in problem 11 is \$3500 a year, what endowment would be necessary at 5% interest to provide for its upkeep indefinitely?

13. A philanthropist provided for the erection of a college building at a cost of \$500,000. He endowed it with a sum of money sufficient to provide for a faculty and upkeep, estimated to cost \$100,000 a year, and for its reconstruction every 75 years. Allowing interest at $4\frac{1}{2}\%$ effective, find his total original outlay.

14. To guard a dangerous railroad crossing costs \$1500 a year. Allowing that money is worth 5% effective, is it or is it not advantageous, and by how much, to construct a bridge over the railroad at a cost of \$18,750, on the assumption that the bridge will need rebuilding every 35 years at the same cost?

15. To guard a given property requires a night watchman at a cost of \$1200 a year. Can the owner afford to install an electric burglar alarm and automatic water sprinkler system at a cost of \$12,000, if the system must be replaced every 25 years at the same cost, money being worth $5\frac{1}{2}\%$ effective, and the cost of maintaining the system \$100 a year?

16. A certain kind of roofing costs \$6.50 a square and is guaranteed to last 12 years. Another kind costs \$9 a square and is guaranteed for 20 years. If the cost of laying the roof is, in each case, \$3 a square, which kind of roofing is the more economical? (Allow interest at 6%.)

17. A small wooden bridge over a river costs for construction \$25,000, and requires rebuilding every 20 years. How much additional money can be profitably expended for the erection of a steel and concrete structure instead, if money is worth $4\frac{1}{2}\%$ effective and the service life of the bridge is extended to 35 years?

PURCHASING AN ANNUITY

The problems of determining the annual or periodic return from a given investment and of finding the periodic payment required to cancel a present debt are but two of many coming under the heading "Purchasing an Annuity."

The purchase price, or investment, is a present value. From the table, page 299, we obtain 4.2123638 as the present value of 1 per annum for 5 years at 6%. If in the place of 4.2123638, 1 is put as the present value of 5 annual payments at 6%, each payment will be equal to the quotient of $1 \div 4.2123638$, or .2373964. That is, 1 is the present value of 5 annual payments of .2373964, or, conversely, an annual payment of .2373964 will cancel a present debt of 1 in 5 years at 6%.

The truth of the preceding statement is clearly illustrated in the following schedule:

SCHEDULE OF AMORTIZATION

Showing Annual Payment Required to Cancel a Present Debt of 1, in 5
Annual Payments at 6%

YEARS	ANNUAL PAYMENT	INTEREST ON BALANCE	AMORTIZATION	PRINCIPAL UNPAID
0				\$1.
1	\$.2373964	\$.06	\$.1773964	.8226036
2	.2373964	.0493562	.1880402	.6345634
3	.2373964	.0380738	.1993226	.4352408
4	.2373964	.0261144	.2112820	.2239588
5	.2373964	.0134375	.2239589	.0000000

At the end of the first year the principal has earned \$.06. The annual payment is first applied to paying the accumulated interest, and the balance, \$.1773964, is used to reduce the principal, so that at the beginning of the second year the principal sum to be paid is \$.8226036. The interest on \$.8226036 for 1 year at 6% is \$.0493562. Again the payment of \$.2373964 is applied as before, and so on.

Each annual payment is composed, as it were, of two items: first, the interest on the unpaid principal; and second, a payment on account of the principal. The column headed "Amortization" contains the several payments on account of the principal. The total of the amortization column is equal to the principal of the debt (\$1) at the beginning.

The 4.2123638 mentioned in the preceding discussion is the present value of 1 per annum for 5 years at 6%. That is,

$$a_{\overline{n}|} = 4.2123638, \text{ when } n = 5 \text{ and } i = .06$$

The annual payment required to cancel a debt of 1 was found by dividing 1 by 4.2123638. Symbolically expressed, then, the quotient is indicated thus :

$$\frac{1}{a_{\overline{n}|i}} \quad (23)$$

This formula is generally referred to as representing "the annuity that 1 will purchase." For calculation purposes, with logarithms, the preceding formula should be used in its simplified form, thus:

$$\frac{1}{a_{\overline{n}|i}} = \frac{1}{\frac{1-v^n}{i}} = \frac{i}{1-v^n} = \frac{i}{1-\frac{1}{(1+i)^n}} = \frac{i(1+i)^n}{(1+i)^n-1}$$

Hence for use with logarithms, the form is

$$\frac{i(1+i)^n}{(1+i)^n-1} \quad (24)$$

The several steps necessary to obtain the simplified form have been shown, to make clear the equality of value of the two forms.

Since $A_{\overline{n}|i}$ = present value,

$$A_{\overline{n}|i} \frac{i(1+i)^n}{(1+i)^n-1} = R \quad (25)$$

For the application of this formula see example 4 on page 192.

EXAMPLES

1. An annuity for 25 years can be bought on a $3\frac{1}{2}\%$ basis. How large an annuity can be had for a payment of \$50,000?

\$1 will purchase an annuity of \$.0606740 for 25 years at $3\frac{1}{2}\%$. (Table, page 302.)

$$50,000 \times \$.0606740 = \$3033.70, \text{ amount of annuity}$$

2. What semiannual payment is required to cancel a present debt of \$15,000 in 20 years at 5% nominal interest?

In the table, page 301, find .0398362 (40 periods; $2\frac{1}{2}\%$) as the payment required to cancel a debt of 1.

$$15,000 \times \$.0398362 = \$597.54, \text{ semiannual payment required}$$

3. Find the annual payment made at the beginning of the year required to amortize a present debt of \$25,000 in 15 years if interest accumulates at 5.4% per annum.

$$\frac{25,000}{a_{\overline{15}|i}} = \frac{25,000}{1 + \frac{1 - \frac{1}{(1+i)^{n-1}}}{i}} = \frac{25,000}{1 + \frac{1 - \frac{1}{(1.054)^{14}}}{.054}}$$

$$\begin{aligned} \log 1 &= 10 & -10 \\ \log (1.054)^{14} &= .31976 \\ 9.68024 - 10 &= \log .4789 \end{aligned}$$

$$1 - .4789 = .5211$$

$$.5211 \div .054 = 9.65$$

$$1 + 9.65 = 10.65, \text{ value of denominator}$$

$$\log 25,000 = 4.39794$$

$$\begin{aligned} \log 10.65 &= 1.02735 \\ 3.37059 &= \log 2347.42 \end{aligned}$$

Therefore the annual payment is \$2347.42.

4. It is agreed that a debt of \$24,925 bearing interest at 5% shall be paid in equal annual installments of \$2000 each. In how many years will the debt be paid? (Reckon to nearest whole year.)

By formula 25

$$\frac{24,925 \times .05 \times (1.05)^n}{(1.05)^n - 1} = 2000$$

Multiplying and clearing of fractions gives

$$1246.25 \times (1.05)^n = 2000 \times (1.05)^n - 2000$$

Transposition gives

$$753.75 \times (1.05)^n = 2000$$

Dividing by 753.75 gives

$$(1.05)^n = \frac{2000}{753.75}$$

Hence $n \times \log 1.05 = \log 2000 - \log 753.75$

$$\text{Therefore } n = \frac{\log 2000 - \log 753.75}{\log 1.05} = \frac{3.30103 - 2.87723}{.02119} = \frac{.4238}{.02119}$$

That is, $n = 20$, the number of years required.

5. Show by a schedule that \$5000 will, at 5%, purchase an annuity of \$1154.87 for 5 years.

YEARS	ANNUAL PAYMENT	INTEREST ON BALANCE	AMORTIZATION	VALUE OF FUND AT END OF EACH YEAR
0				\$5000.
1	\$1154.87	\$250.	\$904.87	4095.13
2	1154.87	204.7565	950.1135	3145.0165
3	1154.87	157.2508	997.6192	2147.3973
4	1154.87	107.3698	1047.5002	1099.8971
5	1154.87	54.9948	1099.8752	.0219

Problems

1. A man owes a debt of \$12,500 and agrees to pay it in 8 equal annual installments. Find the amount of each payment, allowing for interest at 5% effective.

2. The Federal Farm Loan Bank loaned a farmer \$5000 at 5% interest, convertible semiannually. The agreement was that the farmer should repay principal and interest in equal semiannual installments covering a period of 20 years. Find the amount of each payment.

3. A house and lot were purchased for \$25,000, on the following terms: cash, \$5000; the balance with interest at 6% effective, payable in 15 equal annual installments. How much was the annual payment?

4. Two men each borrowed \$3000 at 5% effective, for a period of 6 years. The first man (A) agreed to repay principal and interest in 6 equal annual payments; the other man (B) agreed to return principal and accumulated interest in one sum at the expiration of the 6 years. To be sure of having the correct sum at maturity, B deposited annually a sum sufficient, with interest at 5%, to meet the total of his debt at the date of maturity. Which method, if either, was the more economical?

5. The beneficiary of a policy of insurance is offered a cash payment of \$30,000 or an annuity of \$2400 for 20 years certain, the first payment to be made 1 year hence. Allowing interest at $4\frac{1}{2}\%$ effective, which is the better option, and how much better per annum?

6. A man has been paying \$2500 a year for rent. He decides to buy a house costing \$30,000 on the following conditions: 30% cash, the balance with interest at 5% to be paid in annual installments. The amount of each installment is the difference between the former rental and carrying expenses of the property, which averages as follows: taxes \$1.95 per \$100 on a valuation of \$25,000; insurance 20¢ per \$100 on a policy of \$20,000; and other expenses amounting to \$400 a year. How long will it take to pay for the house?

7. Mr. Leetch borrowed from a Farm Loan Bank \$10,000, and agreed to repay principal and interest at 5%, in 10 equal annual installments. Find the annual payment and set up a schedule showing the gradual amortization of the debt.

Another problem closely related to the foregoing is that of finding what periodic payment will cancel a future debt. It has been shown (page 190) that the annuity required to pay a present debt, with interest accumulation (or what is the same problem, to find the annuity a given sum will purchase), is determined by dividing the present value of the debt by the present value of "1 per annum" for the given time and rate.

The proposition now is to find the annuity to cancel a future debt on which there is *no interest* accumulation. This situation often arises with a corporation having bonds outstanding which are to be paid off at maturity. Interest on the bonds is paid semiannually, but the face of the bonds remains to be paid at one time. To provide a sum large enough to redeem the bonds at maturity a definite sum is set aside each year to accumulate at compound interest. Such a fund is called a **sinking fund**.

In the table (page 295) we find that the amount of 1 per annum for 5 years at 5% is 5.5256312. That is, a payment of \$1 annually into a fund to accumulate at 5% will, in 5 years, amount to \$5.5256312. If it were desired to accumulate a fund of \$1 instead of \$5.5256312, the annual contribution would be found by dividing \$1 by 5.5256312, indicated by the expression $\frac{1}{5.5256312}$.

But $5.5256312 = s_{\overline{n}|}$, when $n = 5$ years, and $i = 5\%$. Therefore, symbolically, the annual contribution to a sinking fund is expressed as

$$\frac{1}{s_{\overline{n}|}} \quad (26)$$

$1 \div 5.5256312 = .1809748$, the contribution per annum that will, in 5 years, at 5% amount to 1.

That a payment of .1809748 for 5 years at 5% will amount to 1 is shown in a

SCHEDULE OF ACCUMULATION

YEARS	ANNUAL CONTRIBUTION	INTEREST ON FUND	TOTAL ANNUAL INCREMENT	VALUE OF FUND AT END OF EACH YEAR
1	.1809748			.1809748
2	.1809748	.0090487	.1900235	.3709983
3	.1809748	.0185499	.1995247	.5705230
4	.1809748	.0285262	.2095010	.7800240
5	.1809748	.0390012	.2199760	1.0000000

For use with logarithms, the foregoing formula needs to be written in its simpler form, which may be derived thus:

$$\frac{1}{s_{\overline{n}|}} = \frac{1}{\frac{(1+i)^n - 1}{i}} = \frac{i}{(1+i)^n - 1} \quad (27)$$

Calling the debt to be paid S and the annual payment R gives

$$\frac{Si}{(1+i)^n - 1} = R \quad (28)$$

It should be noted that the difference between $\frac{1}{a_{\overline{n}|}}$ and $\frac{1}{s_{\overline{n}|}}$ is i . This fact may be illustrated in this way. Assume that a debt of \$1000 bearing interest at 5% is to be paid in 4 years. For the purpose of comparison, assume two methods of payment, as follows:

1. Payment in 4 equal annual installments.
2. Payment of interest annually and of the principal sum at the end of 4 years.

By the first method there is required an annual payment of

$$1000 \times \$.2820118, \text{ or } \$282.01$$

By the second method, a payment of \$50 for interest is required annually. Also to provide for the payment of the \$1000 at the end of 4 years, there should be set aside annually a given sum which, with interest accumulation, will amount to \$1000. By formula 26, the amount required annually is found to be

$$\$1000 \div 4.310125, \text{ or } \$232.01$$

A comparison of these two results shows a difference of exactly \$50, which is the interest paid annually.

It is evident, then, that the cost to the debtor is the same whether he pays a debt in equal periodic installments or pays only the interest periodically and sets up a sinking fund to provide for the principal sum at maturity.

Since

$$\frac{1}{a_{\overline{n}|}} - i = \frac{1}{s_{\overline{n}|}}$$

it is necessary to provide but one table for these two formulas or functions. The table (page 300) showing "the annuity which 1 will buy" may also be used to show "the annuity that will produce 1"; thus (taking the foregoing illustration) the table shows that \$1 will, at 5%, purchase an annuity of \$.2820118 for 4 years, or, what is the same thing, a payment of \$.2820118 annually will in 4 years cancel a debt of \$1 and interest at 5%. To cancel a debt of \$1000 requires an annual payment of

$$1000 \times \$.2820118, \text{ or } \$282.0118$$

as shown above.

Since

$$\frac{1}{a_{\overline{n}|}} - i = \frac{1}{s_{\overline{n}|}},$$

$$\frac{1}{s_{\overline{n}|}} = .2820118 - .05 = .2320118$$

That is, an annual payment of \$.2320118 for 4 years will, at 5%, cancel a debt of \$1 due 4 years hence. To cancel a debt of \$1000 4 years hence will require an annual payment of

$$1000 \times \$.2320118, \text{ or } \$232.0118$$

Hence the table showing "the annuity which 1 will buy" may also be used to show "the annuity that will produce 1" by deducting i from the printed table value.

EXAMPLES

1. What annual contribution to a sinking fund with interest accumulation at 5% will in 25 years cancel a debt of \$1,000,000?

From the table (page 303) showing the annuity that 1 will purchase, take .0709525, and deduct .05. The result, .0209525, is the annuity that will produce 1 in 25 years at 5%. The annuity that will produce \$1,000,000 is $1,000,000 \times \$0.0209525$, or \$20,952.50.

2. A debt of \$1500 is due in 5 years. The debtor proposes to provide for its payment by contributing annually a given amount to a sinking fund that will accumulate at 4.8%. Find the necessary annual contribution, and prove its accuracy by setting up a schedule.

By formula 28,

$$R = \frac{Si}{(1+i)^n - 1} = \frac{\$1500 \times .048}{(1.048)^5 - 1} = \frac{\$72.00}{(1.048)^5 - 1}$$

$$\log (1.048)^5 = .10180 = \log 1.26414$$

$$1.26414 - 1 = .26414$$

$$\$72.00 \div .26414 = \$272.58, \text{ annual contribution}$$

SCHEDULE OF ACCUMULATION

YEAR	ANNUAL CONTRIBUTION	INTEREST ON FUND	TOTAL ANNUAL INCREMENT	TOTAL VALUE OF FUND	
1	\$272.58		\$272.58	\$ 272.58	
2	272.58	\$13.084	285.664	558.244	
3	272.58	26.796	299.376	857.620	
4	272.58	41.166	313.746	1171.366	
5	272.58	56.226	328.806	1500.172	

The slight discrepancy of \$.172 is due to the use of a five-place table of logarithms. Theoretically, the exact amount of the annual payment is \$272.54895.

Problems

1. A corporation issued \$5,000,000 of 30-year 6% bonds. To redeem the bonds at maturity a sinking fund was established, which would increase at $4\frac{1}{2}\%$ per annum. Find the annual contribution.
2. A bond issue of \$15,000,000 is due 40 years hence. What semi-annual contribution to a sinking fund, which will accumulate nominally at 5%, is necessary to cancel the bonds at maturity?
3. At the age of 20 a young man resolves that, when he becomes 60 years of age, he will have \$50,000 saved. If he invests his savings semiannually at 6% per annum nominally, what amount must he save semiannually to fulfill his resolution?
4. A corporation has outstanding a bond issue of \$12,500,000. To redeem the bonds at maturity, an annual contribution of \$70,026.25 is made to a sinking fund, which accumulates at $4\frac{1}{2}\%$ per annum. In how many years do the bonds mature?
5. A man borrowed \$3000 for 5 years, agreeing to pay interest annually at 6%, and to repay the principal sum at maturity. What annual contribution to a sinking fund earning $5\frac{1}{4}\%$ effective will be sufficient to amortize the debt at maturity? Prove the accuracy of your result by setting up a schedule.
6. An annual payment of \$597.09 will, with interest at 5% effective, cancel a debt of \$125,000 due in 50 years. Set up a schedule showing the amount of the sinking fund for the first ten years, and prove the schedule value for the tenth year by calculation.
7. At the birth of a son, a father makes a first payment into a fund which, at the son's 21st birthday, will amount to \$21,000. An amount equal to the first payment is set aside on each birthday to and including the 20th. Allowing for interest at $4\frac{1}{2}\%$, find the annual payment to the fund.
8. A city borrowed \$10,000,000 for a public improvement, and issued 30-year 5% bonds therefor. A sinking fund at 5% nominal was provided to redeem the bonds at maturity. Payments into the sink-

ing fund were made semiannually. What amount of tax was required each year to pay the interest and to provide for the sinking fund?

9. In problem 8, how much more or less would the cost to the city have been if, instead of setting up a sinking fund to amortize the bonds at maturity, the city had taken up its own bonds at par as fast as the funds were available?

10. In problem 8, how much more or less would it have cost the city to have taken up its own bonds at $102\frac{1}{2}$ at 5-year intervals, to the extent of the sinking fund then accumulated?

In the preceding problems it has been assumed that the sinking fund has been accumulated at the same rate of interest at which the loan was made. Such conditions do not always exist. Often a sinking fund must be accumulated at a lower rate than is paid on the loan.

Thus, a loan may be made under the agreement that the lender shall receive interest at, say, 6% on the full amount of the loan for the entire period of the loan. Repayments of principal made annually may be invested in a sinking fund at only 5% for the replacement of the principal at maturity. Since the sinking fund earns but 5%, and the borrower has agreed to pay 6% on the full amount of the loan, he must include in his annual payment an additional 1% (that is, $6\% - 5\%$) of the amount in the sinking fund at the close of the previous year, to make up the deficit.

The rate to be paid on the loan is called the **remunerative rate** and is designated by the letter i . The rate at which the sinking fund is accumulated is called the **accumulative rate**, and is designated by the letter j .

EXAMPLE

A loan of \$1000 is made for 5 years, under an agreement that principal and interest shall be returned in 5 equal annual payments. The interest rate is 6%. Repayments of principal are to be invested in a sinking fund to accumulate at 5%. The amount of the sinking fund at the end of the 5 years is to be \$1000. Find the annual payment, and show by a schedule the amortization of the debt and the accumulation of the sinking fund.

From the table showing the annuity that 1 will purchase, we find .2309748 under 5 years at 5%. This annuity will repay a principal of 1 and interest at 5% in 5 years. The problem calls for interest at 6%, hence .01 is added to the foregoing annuity, making the annual payment .2409748 on the basis of 1. The annual payment to redeem \$1000 is, obviously, \$240.9748.

SCHEDULE SHOWING DISTRIBUTION OF ANNUITY TO REDEEM A DEBT
OF \$1000 IN 5 YEARS, WHEN INTEREST IS PAID AT 6% AND
SINKING FUND IS ACCUMULATED AT 5%

YEAR	ANNUAL RENT OF ANNUITY	PRINCIPAL CONTAINED IN ANNUAL PAYMENT	TOTAL PRINCIPAL REPAID TO DATE	PRINCIPAL STILL OUT- STANDING	INT. AT 6% ON PRINCIPAL OUTSTAND- ING AT END OF PREVIOUS YEAR	INT. AT 5% EARNED ON REPAID PRINCIPAL AT END OF PREVIOUS YEAR	BALANCE OF INT. DUE ON REPAID PRINCIPAL AT END OF PREVIOUS YEAR (1%)
1	\$240.9748	\$180.9748	\$ 180.9748	\$819.0252	\$60.		
2	240.9748	190.0236	370.9984	629.0016	49.1415	\$ 9.0487	\$1.8097
3	240.9748	199.5247	570.5231	429.4769	37.7401	18.5500	3.7100
4	240.9748	209.5011	780.0242	219.9758	25.7685	28.5261	5.7052
5	240.9748	219.9761	1000.0003		13.1985	39.0012	7.8002

The first annual payment of \$240.9748 is distributed as follows: 1st, \$60 is required for interest on the outstanding principal of \$1000; 2d, \$180.9748 is put into the sinking fund to accumulate at 5%.

The second annual installment of \$240.9748 is distributed thus: 1st, \$49.1415 as interest at 6% on outstanding principal, and 2d, \$1.8097 as deficit of 1% on amount in sinking fund. (The creditor is to receive 6% interest on the full \$1000 for the entire 5 years. The sinking fund of \$180.9748 has earned only 5%, hence the debtor must pay an additional 1% of \$180.9748 to make the full 6%.) The balance of the payment [$\$240.9748 - (\$49.1415 + \$1.8097)$], or \$190.0236, is added to the sinking fund.

The third payment of \$240.9748 is distributed in a similar manner: 1st, \$37.7401 as 6% of outstanding principal, plus \$3.71 as a deficit of 1% on the amount in the sinking fund; and 2d, \$199.5247 to be added to the sinking fund, and so on, in the same way for each payment.

The formula for determining the annual payment is $\frac{1}{a_{\overline{n}|i}} + (i - j)$, $a_{\overline{n}|j}$ being found at rate j . (For rates and periods within the table showing $\frac{1}{a_{\overline{n}|i}}$, use values of this table and add thereto $(i - j)$.)

Problems

1. A merchant obtains a loan of \$15,000 from a bank on condition that he shall repay loan and interest thereon in 10 equal annual installments. The agreement is that interest at 6% shall be paid on the full amount of the loan for the entire period of 10 years, although the bank is able to reinvest the capital at only $4\frac{1}{2}\%$. Determine the annual payment and show, by a schedule similar to the preceding one, the distribution of the annual payments, the value of the sinking fund, and the amount of outstanding principal, annually.

2. An insurance company loaned \$25,000 for 10 years on condition that the loan and interest thereon at 5% on the full value of the loan would be returned in 10 equal annual payments. If the sinking fund can be accumulated at 4%, find the annual payment, and prove the work by a schedule of amortization.

Three problems arise in connection with the form of annuity discussed in the preceding paragraphs. They have to do with the outstanding principal under three different conditions:

(1) The case in which the lender desires to cancel the loan before maturity and asks for the return of the outstanding principal.

(2) The case in which the borrower wishes to return the outstanding principal in one payment before maturity.

(3) The case in which both borrower and lender are desirous of canceling the loan before maturity.

Case 1

The original terms of the contract are considered somewhat burdensome to the debtor. That being the case, he cannot be expected to fulfill the terms of the agreement if the creditor should desire to cancel the loan before maturity. Hence if the creditor wishes the principal outstanding to be returned in one amount before it is due, he cannot

expect the borrower to pay him interest at the higher (or remunerative) rate for the remainder of the term. The borrower, therefore, will be required to pay only the difference between the amount of the original loan and the amount of the sinking fund at the time the balance of the loan is repaid.

If A represents the amount of the loan, the annual contribution to a sinking fund to replace the loan in n payments at the accumulative rate j is indicated by $\frac{A}{*s'_{\overline{n}|}}$.

If the loan is canceled at the expiration of m years, the amount of the sinking fund at that time is indicated by $\frac{A}{s'_{\overline{n}|}} \cdot s'_{\overline{m}|}$. Hence the payment (called "redemption money") required at the end of m years to cancel the loan at the request of the lender is indicated by

$$A - \frac{A}{s'_{\overline{n}|}} \cdot s'_{\overline{m}|} \quad (29 a)$$

Applying this formula to the illustrative problem given on page 199 and assuming that the lender wishes the outstanding principal returned at the expiration of 3 years, we have

$$\begin{aligned} \$1000 - \left(\frac{\$1000}{5.5256312} \times 3.1525 \right) &= \$1000 - \$570.52 \\ &= \$429.48, \text{ redemption money} \end{aligned}$$

If one has a schedule prepared like that on page 200, the redemption money under this case is the same as the outstanding principal.

Case 2

In this problem it is assumed that the borrower wishes to cancel the contract and make settlement before the date of maturity. Equity requires that the lender shall not be the loser under this condition. The borrower must pay such a sum as will enable the lender to maintain the same income as if the contract were not canceled. The redemption money, therefore, will be the present value of the remaining annual payments, at the accumulative rate j , for $(n-m)$ years. Hence the redemption money is

$$Ra'_{\overline{n-m}|} \quad (29 b)$$

*The prime (') is used to indicate the use of the accumulative rate j as distinguished from the remunerative rate i .

R in this case represents the full amount of the annual payment made each year for m years.

Under this plan of settlement, the borrower in the problem on page 199 would, at the end of 3 years, be required to pay, in full settlement of outstanding principal,

$$240.9748 \times \$1.8594104 = \$448.07, \text{ redemption money}$$

Case 3

The third method of settlement has to do with the case in which both parties desire to cancel the loan before maturity. This results in what may be called a "compromise." The lender is entitled to the present value of the remaining payments. These payments, if made annually, could be invested only at the accumulative rate j , but the borrower is entitled to make settlement by paying their present worth at the remunerative rate i . Hence the redemption money in this case is equal to the present value at the rate i of the amount of the remaining payments accumulated at the rate j . The formula is

$$\frac{Rs'_{\overline{n-m}|}}{1 + is'_{\overline{n-m}|}} \quad (29c)$$

Using the data given in the illustration on page 200, we find that, at the expiration of 3 years,

$$240.9748 \times \frac{\$2.05}{1 + (.06 \times 2.05)} = \$439.89, \text{ redemption money}$$

TO THE STUDENT. The schedule on the following page will bear careful study. Particular attention should be given the calculation of the "redemption money" by each of the three methods. Do not be satisfied with merely looking over the schedule and seeing that it proves, but calculate for yourself the redemption money to make sure you understand the difference in the methods. Having done that, the following problems will cause no difficulty.

COMPLETE SCHEDULE SHOWING ANNUITY TO REPAY A DEBT OF \$1000 AT 6% INTEREST, AND TO
 PROVIDE SINKING FUND AT 5%, WITH REDEMPTION MONEY FOR EACH YEAR BY ALL THREE
 METHODS OF RECKONING

Year	ANNUAL RENT OF ANNUITY	PRINCIPAL CONTAINED IN ANNUAL PAYMENT	TOTAL PRINCIPAL REPAID TO DATE	PRINCIPAL STILL OUTSTANDING	INT. AT 6% ON PRINCIPAL AT END OF PREVIOUS YEAR	INT. AT 5% EARNED ON REPAID PRINCIPAL AT END OF PREVIOUS YEAR	BALANCE OF INT. DUE ON REPAID PRINCIPAL AT END OF PREVIOUS YEAR (1%)	REDEMPTION MONEY FORMULA 29 a	REDEMPTION MONEY FORMULA 29 b	REDEMPTION MONEY FORMULA 29 c
1	\$240.9748	\$180.9748	\$180.9748	\$819.0252	\$60.0000	\$00.0000	\$0.0000	\$819.0242	\$854.4847	\$825.2227
2	240.9748	190.0236	370.9984	629.0016	49.1415	9.0487	1.8097	629.0016	656.2341	638.8370
3	240.9748	199.5247	570.5231	429.4769	37.7401	18.5500	3.7100	429.4769	448.0710	439.8916
4	240.9748	209.5011	780.0242	219.9758	25.7685	28.5261	5.7052	219.9758	229.4998	227.3347
5	240.9748	219.9761	1000.0003	000.0000	13.1985	39.0012	7.8002	000.0000	000.0000	000.0000

Problems

1. An insurance company lent one of its policyholders \$10,000 for 10 years under an agreement to repay principal and interest at 6% in 10 equal annual payments. It was further agreed that the borrower should pay interest at 6% on the full amount of the loan for the whole 10-year period. Assuming that the insurance company obtained only $4\frac{1}{2}\%$ on sinking fund investments:

(a) Determine the annual payment required.

(b) Determine the redemption money to settle at the end of 6 years, under Case 1; under Case 2; under Case 3.

(c) Set up a schedule showing facts as indicated in the schedule on page 204.

2. A man borrows \$15,000 at a savings bank for 15 years at 6% interest, and agrees to repay principal and interest in 15 equal annual payments. The bank is able to invest repayments of principal at 4%. Find redemption money for each of the following conditions:

(a) Borrower is in funds and wishes to return unpaid principal at the end of the 1st year.

(b) At the end of 7 years there is a "run" on the bank and they call in the loan.

(c) At the end of 10 years it is mutually agreed to cancel the outstanding payments.

Why is redemption money in (a) greater than the amount of the original loan?

XV. BOND VALUATION

A **bond** is a formal, written promise, under seal, to pay to a specified party, or order, or to bearer, a certain sum of money at a fixed or determinable future time, interest thereon at a given rate being payable at periodic intervals.

Bonds may be issued by a government, a state, a city, a corporation, or an individual.

Interest on bonds may be payable annually, semiannually, or quarterly. The most common interest period is one half year. Unless otherwise specified, the half-yearly interest periods will be understood to apply to all problems in this text.

The **par value** of a bond is its face value. The most common par value is \$1000; other par values are \$50, \$100, \$500, \$5000, and \$10,000. Bond problems in this text, unless otherwise specified, will be based on \$1000 par value.

Bonds are bought and sold on the market, like other commodities, for what they will bring. The market value, or price, is rarely the par value. It may be above par value (at a premium) or below par value (at a discount).

Bond prices are expressed as per cents of their par value; hence the "price" of a \$100 bond would be the same as the "price" of a \$5000 bond. That is, the quotation 98 would mean 98% of the par value of the bond under consideration.

Mathematically speaking, the market price of a bond depends on three factors:* (1) the interest rate paid on the bond, (2) the interest rate desired by the purchaser on his investment, and (3) the length of time the bond has to run before it is redeemed.

A bond is redeemed when the debtor pays the promised amount and the bond is returned by the creditor.

Other factors, whose effect cannot be determined mathematically, enter into the determination of the price. The most important of

* This statement is based on the assumption that the bond is redeemed at par value. If the bond is redeemed at a price other than par, a fourth factor, the redemption price, must be considered.

such factors are: (1) the security offered by the debtor as a guaranty of payment of principal and interest and (2) the marketability of the bond. Bonds "listed" on the stock exchange have, as a rule, a more ready market than unlisted bonds.

Bonds are of two general classes — coupon and registered. Attached to a coupon bond there are small coupons or interest certificates which are cut from the bond as they become due and may be cashed or deposited in one's bank account. The coupons are payable to bearer and may be passed from hand to hand like money.

A **registered bond** takes its name from the fact that the name and address of the owner are registered on the books of the debtor or of his agent. Interest on a registered bond is paid by check issued by the debtor instead of by coupons. Frequently coupon bonds are registerable as to principal only.

The dates at which the interest on a bond is due and payable are called the **interest dates**. If interest is payable semiannually, the interest dates are indicated by the initial letters of the months in which the interest is payable. Thus, J-J signifies that the interest dates are January 1 and July 1. (The first day of the month is understood unless otherwise indicated.) F-A means February 1 and August 1; J-D means June 1 and December 1, etc. If interest is payable quarterly, the interest dates are indicated in a similar manner. Thus J-A-J-O means interest is payable January 1, April 1, July 1, and October 1.

Interest is due and payable at the end of the interest period. As soon as the interest is paid or becomes due for a given period, interest begins to accumulate for the next period. Hence when a bond is sold between the interest dates, it carries with it the interest accumulation since the date at which the interest last became due. The interest accumulation to the date of sale belongs to the seller, that is, the buyer must pay the seller for the accumulated interest.

The cost of a bond, therefore, consists of two parts: first, the present value of the bond and of all future interest accretions; second, the interest earned on the bond since the last interest date. This fact gives rise to the expression "and interest" in connection with bond quotations. Published quotations, however, do not, as a rule, contain the expression "and interest," its inclusion being understood.

EXAMPLE

Find the cost, May 18, of a 6% bond, face value \$1000, at $.97\frac{3}{4}$, if interest dates are January–July.

$97\frac{3}{4}\%$ of \$1000 = \$977.50, cost without interest

From Jan. 1 to May 18 is 4 mo. 17 da.

The interest on \$1000 for 4 mo. 17 da. at 6% is \$22.83

\$977.50 + \$22.83 = \$1000.33, cost

Accrued interest on bonds is reckoned on the 360-day basis. In counting the time allow 30 days for each month. “And interest” is understood to be a part of the quotation.

To find the price of a bond to yield a given rate on the investment

Bond tables have been prepared, and are extensively used by bond brokers, to show the yield on bonds for certain stipulated rates. Many problems and rates, however, are “outside” the tables.

The problem of finding the price of a bond to yield a given rate has several phases, as follows :

1. For whole interest periods
 - (a) When the yield rate is greater than the bond interest rate
 - (b) When the yield rate is less than the bond interest rate
2. For whole periods plus a fraction of a period
 - (a) When the yield rate is greater than the bond interest rate
 - (b) When the yield rate is less than the bond interest rate
3. For a fraction of a period only

If a bond paying 5% interest on its face value is bought at a price to yield 6% on its cost, manifestly its cost must be less than par, that is, the bond is at a discount. On the other hand, if it is bought to yield only 4% on its cost, its cost must be above par, that is, at a premium.

EXAMPLE

At what price must a 6% bond, due in 15 years, be purchased to yield 5% on the investment?

The price must be above par because the bond rate is greater than the yield rate (often called the basis rate).

The semiannual interest on a 6% bond is \$30. On a 5% basis \$1000 will purchase a 5% bond on which the semiannual interest is \$25.

On a 6% bond with coupon of \$30, the yield is 5% at a cost to be determined.

On a 5% bond with coupon of \$25, the yield is 5% at a cost of \$1000.

A 6% bond yields \$5 more interest every 6 months than a 5% bond. This additional \$5 every half year is an annuity the present value of which is the premium on the bond.

Therefore, since \$1000 will buy a 5% bond (coupon \$25) on a 5% basis, a 6% bond (coupon \$30) on a 5% basis will cost \$1000 plus the present value of an annuity of \$5 for 30 periods at 5% per annum, or $2\frac{1}{2}\%$ per period.

$$\text{If } n = 30 \text{ and } \frac{i}{2} = .025$$

$$a_{\overline{n}|} = 20.9302926$$

$$\text{and } A_{\overline{n}|} = 5 \times \$20.9302926 = \$104.65$$

Hence the cost of a 6% bond due in 15 years to yield 5% is \$1000 + \$104.65 = \$1104.65.

The market price or quotation would be 110.47.

In general terms let

F = face or par value of bond

n = number of interest periods

i = rate of interest on investment

r = rate of interest on bond

P = price, or market value of bond

Symbolically, the foregoing solution is represented thus :

$$P = F + \left(\frac{Fr - Fi}{2} \cdot a_{\overline{n}|} \text{ at rate } \frac{i}{2} \right) \quad (30)$$

If the basis or yield rate is greater than the bond rate, the market value of the bond is below par, and the formula is

$$P = F - \left(\frac{Fi - Fr}{2} \cdot a_{\overline{n}|} \text{ at rate } \frac{i}{2} \right) \quad (31)$$

That is, a 5% bond due in 15 years to yield 6% would cost less than \$1000,

A 6% bond on a 6% basis would cost \$1000. Hence a 5% bond on a 6% basis would cost \$1000 less the present value of an annuity of \$5 for 30 periods at 3% per period.

$$a_{\overline{n}|} = 19.6004414$$

$$A_{\overline{n}|} = 5 \times \$19.6004414 = \$98$$

$$P = \$1000 - \$98 = \$902 \text{ (quoted as 90.20)}$$

There are other methods of finding the price of a bond, but probably none of them are simpler or easier than the preceding illustrations.

One of the other methods is to find the present value of the face value of the bond and also the present value of all the interest payments, both at the basis rate, and add the two results.

Using the data in the last solution, we find that the present value of the par value of the bond would be 1000 times the present value of 1 for 30 periods at .03 per period.

$$v^{30} \text{ at } .03 = .4119868$$

$$1000 \times \$.4119868 = \$411.99$$

The present value of the interest payments would be 25 times the present value of "1 per annum" for 30 periods at .03 per period.

$$a_{\overline{30}|} \text{ at } .03 = 19.6004413$$

$$25 \times \$19.6004413 = \$490.01$$

$$P = \$411.99 + \$490.01 = \$902, \text{ cost of bond}$$

The formula for the foregoing is

$$P = \left(Fv^n + \frac{Fr}{2} \cdot a_{\overline{n}|} \right) \text{ both at rate } \frac{i}{2}$$

Another method is to find the present value of the sum of the face of the bond and the amount of all interest payments to maturity at the basis rate.

Again using the same data as before, we find that the face of the bond is \$1000. The sum of the interest payments is 25 times $s_{\overline{n}|}$ at .03.

$$s_{\overline{30}|} = 47.5754157$$

$$25 \times \$47.5754157 = \$1189.39$$

$$\$1000 + \$1189.39 = \$2189.39$$

The present value of 1 for 30 periods at .03 = .4119868

$$P = 2189.39 \times \$.4119868 = \$902, \text{ cost of bond}$$

In general terms the foregoing solution would be expressed thus :

$$P = \left(F + \frac{Fr}{2} \cdot s_n \right) v^n \left(\text{at rate } \frac{i}{2} \right) \quad (32)$$

Comparatively few sales of bonds are made having exact whole interest periods yet to run, because there are only 2, or at most 4, days in a year when such sales may be made.

Determining a price, therefore, at which a bond should be sold on a given date to yield a given rate on the investment requires in most cases an interpolation to arrive at a proper price.

EXAMPLE

Find, as of August 15, 1922, the price of a 6% bond due June 1, 1935, to yield 5%.

The time the bond has yet to run is found by compound subtraction.

$$\begin{array}{r} 1935-6-1 \\ 1922-8-15 \\ \hline 12-9-16, \text{ time the bond has to run} \end{array}$$

12 yr. 9 mo. 16 da. is equal to 25 interest periods and 3 mo. 16 da. over.

That is, the time is between 25 and 26 periods.

First find the price of the bond under question for 25 periods and for 26 periods.

\$1094.7531 = price of 6% bond due in 26 periods to yield 5%

1092.1219 = price of 6% bond due in 25 periods to yield 5%

\$ 2.6312 = change in price for 6 months

6)\$2.6312

30)\$4.385 = change in price for 1 mo.

\$0.0146 = change in price for 1 da.

3 × \$.4385 = \$1.3155, change in price for 3 mo.

16 × \$.0146 = .2336, change in price for 16 da.

\$1.5491, change in price for 3 mo. 16 da.

Adding the change in price for 3 mo. 16 da. to the price for 25 periods gives the price for 12 yr. 9 mo. 16 da.

\$1092.1219 + \$1.5491 = \$1093.6710, price for 12 yr. 9 mo. 16 da.,
quoted as 109.37

In finding the price of a bond on which the yield rate is greater than the bond rate for a number of whole interest periods and a fraction of a period the preceding plan of solution is followed. In adjusting the price for the fraction of a period, care should be exercised to note whether the change in price for the fractional period should be added or subtracted.

To find the price of a bond having only a part of a single period to run is a simple calculation in true discount by simple interest.

EXAMPLE

A 5% bond matures Oct. 1, 1923. Find its cost June 10, 1923, to yield 6%.

From June 10 to Oct. 1 is 3 mo. 21 da.

The amount of 1 at 6% for 3 mo. 21 da. is 1.0185

On Oct. 1 the holder of the bond will receive \$1000 for the bond and \$25 for the final coupon

Therefore

$$\$1025 \div 1.0185 = \$1006.38, \text{ cost}$$

This cost, \$1006.38, is the full cost of the bond and includes the "and interest" part of the cost. The quotation or market price of the bond is found by deducting from \$1006.38 the interest at 5% on \$1000 for 2 mo. 9 da., that is, for the time since the interest was last paid.

The interest on \$1000 at 5% for 2 mo. 9 da. is \$9.58

$$\$1006.38 - \$9.58 = \$996.80, \text{ market price}$$

To find the rate of return on a bond having only a part of a period to run

EXAMPLE

Find the annual rate of interest realized from an investment at 99½ in a quantity of 5% bonds, due December 1, if the bonds were bought August 13.

At maturity the holder will receive

For interest on one \$1000 bond	\$ 25.
For each \$1000 bond	<u>1000.</u>
Total.	\$1025.

The cost of one \$1000 bond on Aug. 13 is

99 $\frac{5}{8}$ % of \$1000	\$ 996.25
Plus accrued interest at 5% on \$1000, from June 1 (2 mo. 12 da.)	10.00
Total	<u>\$1006.25</u>

The amount to be received from the investment . . . \$1025.

The amount of the investment 1006.25

The net income \$ 18.75

From Aug. 13 to Dec. 1 is 108 da., counting 30 days to the month.

$\$18.75 \div 108 = \$.1736$, income per day

$360 \times \$.1736 = \$62.50$, income per annum

$\$62.50 \div \$1006.25 = .0621$; 6.21%, rate of return on investment

Serial bonds

Instead of providing a sinking fund for the redemption of bonds at maturity it is sometimes desirable to redeem them in installments. Bonds so redeemed are called **serial bonds**. They may be redeemed semiannually, annually, or at longer intervals of time. The time set for the redemption of the first lot of bonds may be at the first, second, third, or any other desired interest period. Frequently the bonds are redeemed at regular intervals of time and in nearly equal amounts, though neither of these provisions is compulsory. The cost, or the market value, of an issue of serial bonds is determined by finding the cost or market value of each separate series of bonds in the issue and adding the results.

EXAMPLE

A \$50,000 issue of 6% bonds to run 5 years is to be redeemed by paying off \$10,000 at the expiration of each year. If the bonds are issued on a 5% basis, find the amount received for the entire issue.

$10 \times 5 \times \$1.9274242 =$	\$96.37, premium on 1-year bonds
$10 \times 5 \times \$3.7619742 =$	\$188.10, premium on 2-year bonds
$10 \times 5 \times \$5.5081254 =$	\$275.41, premium on 3-year bonds
$10 \times 5 \times \$7.1701372 =$	\$358.51, premium on 4-year bonds
$10 \times 5 \times \$8.7520639 =$	<u>\$437.60</u> , premium on 5-year bonds
	\$1355.99, premium on entire issue

$\$50,000 + \$1355.99 = \$51,355.99$, cost, or market value

Problems

1. Find the price of a 6% bond due in 30 years to yield 5%.
2. A 5% bond having 25 years to run was bought on a 6% basis. Find the market quotation.
3. Mr. A purchases some railroad bonds paying $4\frac{1}{2}\%$ interest on a 6% basis, $7\frac{1}{2}$ years before they are due. At what price does he buy them?
4. Find the correct market quotation of $5\frac{1}{2}\%$ bonds due in 5 years if bought on a $6\frac{1}{2}\%$ basis.
5. On August 16, 1923 a man bought 5% bonds, due October 1, 1930, on a 6% basis. If interest dates were April 1 and October 1, what price did he pay?
6. A quantity of 6% bonds, due December 1, 1936, was bought May 10, 1923 on a 5% basis. Interest dates being June 1 and December 1, find the price paid.
7. On November 12, 1923 Oliver Dixon bought \$50,000 of 6% bonds, due August 1, 1938, on a $5\frac{1}{2}\%$ basis. Interest dates were February 1 and August 1. Find (a) the market price of the bonds November 12, 1923, and (b) the total cost of the bonds on that date.
8. J. D. Truxton was offered, on April 19, 1923, a 5% bond, due August 1, 1923, at a price to yield a net return of 6% on the investment. Find (a) the cost of the bond; (b) how much of the cost is accumulated interest; and (c) the "price" of the bond exclusive of interest accumulation.
9. If you buy on June 6 a 5% bond, due Nov. 1 of the same year, at 99.595 and interest, what nominal rate of interest do you receive on your investment?
10. The city of New York advertises for bids on \$45,000,000 of $4\frac{1}{4}\%$ bonds, due in 50 years. You are asked by a banking syndicate to quote a price on a 4.084% basis. What price shall you quote?

11. A real estate company plans the erection of a modern office building at a cost of approximately \$1,250,000. To provide itself with sufficient funds it issues \$750,000 of $5\frac{1}{2}\%$ bonds on a 6% basis. The conditions of the loan provide for the redemption of the bonds serially as follows :

\$100,000 at the expiration of	2 years
\$150,000 at the expiration of	4 years
\$250,000 at the expiration of	7 years
\$250,000 at the expiration of	10 years

Find the price received for each group of bonds at the date of issue, and the amount received for the entire lot of bonds.

12. \$500,000 of 6% bonds were issued as follows :

\$150,000 of them on a $5\frac{1}{2}\%$ basis to be redeemed in	2 years
\$150,000 of them on a 5.4% basis to be redeemed in	5 years
\$200,000 of them on a $5\frac{1}{4}\%$ basis to be redeemed in	8 years

Find the price of each group and the amount received for the entire issue.

AMORTIZATION AND ACCUMULATION

Bonds bought either above or below par, approach par, in price, as they approach maturity. If the income, or basis, rate continues unchanged and non-mathematical factors are disregarded, the approach to par will be gradual and nearly uniform.

It has been shown (page 209) that the price of a bond whose interest rate is greater than the basis rate is above par, and that the price of a bond whose interest rate is less than the basis rate is below par.

Thus, a 6% bond due in 5 years from May 1, 1922, on a 5% basis, will cost, without interest, \$1043.76. At maturity the bond will be redeemed at par, \$1000. There is, then, an apparent loss of \$43.76. The loss, however, is only apparent. The semiannual interest on the bond is \$30. The interest on the investment for 6 mo. at 5% per annum is $2\frac{1}{2}\%$ of \$1043.76, or \$26.094. The difference between the interest on the bond and the interest at 5% on the investment is \$3.906. This excess of \$3.906 is applied toward the reduction (called amortization) of the premium. Each semiannual interest payment will contain a small sum in excess of the amount required to pay the interest at the basis rate on the investment. These several excess

items are sufficient to cancel, or amortize, the entire amount of premium on the bond. Hence the premium, which appeared to be a loss, is not a loss at all.

The necessity for some method of amortizing the premium on bonds bought above par is evident when one considers the purpose and nature of many bond investments. Assume a trust fund of \$250,000 in the hands of a trustee to be invested for the benefit of a dependent. The income is to be paid to the dependent for life, after which the principal sum becomes the property of another. The principal sum must be kept intact and not be allowed to depreciate. If the fund of \$250,000 is invested in 6% bonds due in 20 years, on a 5% basis, the face value of the bonds that can be bought is \$222,000 (in nearest whole \$1000). To pay to the dependent beneficiary all the interest received on the bonds would result in a depreciation of the principal sum from \$250,000 to \$222,000. The difference of \$28,000 is the premium paid on the bonds, and must be "amortized" by taking a portion of each interest payment for that purpose.

Two methods of amortization are in use — the "scientific" method and the "straight line method." The scientific method is based on the principles of compound interest, and shows correctly the semi-annual "book value" of the bonds. By this method the amount of the premium amortized semiannually increases gradually as the bond approaches maturity, owing to the fact that as the bond approaches maturity the book value becomes less and less. Hence interest on book value becomes less, and a greater portion of each payment of bond interest is applicable to the amortization of the premium.

By the straight line method the premium is divided by the number of interest periods over which the bond runs, and a uniform amount is applied to the reduction of the premium semiannually. By this method too large a sum is amortized in the early life of the bond and too small a sum in the latter periods of the bond. The result is that the schedule does not show a correct semiannual "book value" of the bond. The variation is slight on a bond having but few interest periods. Its use by savings banks in New York State is permitted. Its only advantage is in its ease of operation. The following schedules illustrate the two methods of amortization of the premium on a 6% bond due in 5 years from May 1, 1922, on a 5% basis.

SCHEDULE OF AMORTIZATION (SCIENTIFIC METHOD)

DATES	SEMIANNUAL INTEREST ON BOND	INTEREST EARNED ON 5% BASIS	AMORTIZATION	BOOK VALUE
May 1, 1922	<i>Cost</i>	<i>of Bond</i>		\$1043.760
Nov. 1, 1922	\$30	\$26.094	\$3.906	1039.854
May 1, 1923	30	25.996	4.004	1035.850
Nov. 1, 1923	30	25.896	4.104	1031.746
May 1, 1924	30	25.794	4.206	1027.540
Nov. 1, 1924	30	25.689	4.311	1023.229
May 1, 1925	30	25.581	4.419	1018.810
Nov. 1, 1925	30	25.470	4.530	1014.290
May 1, 1926	30	25.357	4.643	1009.637
Nov. 1, 1926	30	25.241	4.759	1004.878
May 1, 1927	30	25.122	4.878	1000.000

By the straight line method the premium of \$43.76 is divided by 10, the number of interest periods, giving \$4.37 to be amortized each period, with \$.06 over, which is added to the first amount.

SCHEDULE OF AMORTIZATION (STRAIGHT LINE METHOD)

DATES	SEMIANNUAL INTEREST ON BOND	AMORTIZATION	BOOK VALUE
May 1, 1922	<i>Cost of</i>	<i>Bond</i>	\$1043.76
Nov. 1, 1922	\$30	\$4.43	1039.33
May 1, 1923	30	4.37	1034.96
Nov. 1, 1923	30	4.37	1030.59
May 1, 1924	30	4.37	1026.22
Nov. 1, 1924	30	4.37	1021.85
May 1, 1925	30	4.37	1017.48
Nov. 1, 1925	30	4.37	1013.11
May 1, 1926	30	4.37	1008.74
Nov. 1, 1926	30	4.37	1004.37
May 1, 1927	30	4.37	1000.00

Compare the foregoing schedule with the scientific schedule.

Bonds bought below par increase in value as they approach maturity. There is, then, an apparent profit on such bonds. Thus a 4% bond due in 5 years from May 1, 1922, on a 5% basis, costs, without interest, \$956.24. At maturity \$1000 will be received for the bond. The difference of \$43.76 is the discount and is the amount of the apparent profit. The semiannual interest on the bond is \$20. On a 5% basis, the holder of the bond is entitled to $2\frac{1}{2}\%$ of \$956.24, or \$23.906, income the first 6 months he owns the bond. Since he received but \$20, the difference, \$3.906, is added to the value of the bond, so that, should he sell the bond after holding it 6 months, he would receive \$3.906 more than he gave for it.

The following schedules illustrate the two methods of showing the "accumulation" of the discount.

SCHEDULE OF ACCUMULATION
(SCIENTIFIC METHOD)

DATES	SEMIANNUAL INTEREST ON BOND	INTEREST EARNED ON 5% BASIS	ACCUMULATION	BOOK VALUE
May 1, 1922	<i>Cost of</i>	<i>Bond</i>		\$ 956.24
Nov. 1, 1922	\$20	\$23.906	\$3.906	960.146
May 1, 1923	20	24.004	4.004	964.150
Nov. 1, 1923	20	24.104	4.104	968.254
May 1, 1924	20	24.206	4.206	972.460
Nov. 1, 1924	20	24.311	4.311	976.771
May 1, 1925	20	24.419	4.419	981.190
Nov. 1, 1925	20	24.530	4.530	985.720
May 1, 1926	20	24.643	4.643	990.363
Nov. 1, 1926	20	24.759	4.759	995.122
May 1, 1927	20	24.878	4.878	1000.000

In comparing the foregoing schedule of accumulation with the scientific schedule of amortization on page 217, it will be observed that the discount in one case is the same as the premium in the other. This is so because the *basis* or *yield* rate is the same in both problems.

Is the premium on a 6% bond due in 10 years on a 5% basis equal to the discount on a 5% bond due in 10 years on a 6% basis?

SCHEDULE OF ACCUMULATION
(STRAIGHT LINE METHOD)

DATES	SEMIANNUAL INTEREST ON BOND	ACCUMULATION	BOOK VALUE
May 1, 1922	<i>Cost of</i>	<i>Bond</i>	\$956.24
Nov. 1, 1922	\$20	\$4.43	960.67
May 1, 1923	20	4.37	965.04
Nov. 1, 1923	20	4.37	969.41
May 1, 1924	20	4.37	973.78
Nov. 1, 1924	20	4.37	978.15
May 1, 1925	20	4.37	982.52
Nov. 1, 1925	20	4.37	986.89
May 1, 1926	20	4.37	991.26
Nov. 1, 1926	20	4.37	995.63
May 1, 1927	20	4.37	1000.00

The following is a schedule showing the amortization of both premium and the face value of an issue of serial bonds. The data are taken from the example on page 213.

PERIODS	BOOK VALUE OF BONDS	SEMIANNUAL INTEREST ON BONDS	INTEREST EARNED ON 5% BASIS	AMORTIZATION OF PREMIUM	AMORTIZA- TION OF PRINCIPAL
1.	\$51355.99	\$1500.00	\$1283.8998	\$216.1002	
2.	51139.8898	1500.00	1278.4972	221.5028	\$10,000
3.	40918.3870	1200.00	1022.9597	177.0403	
4.	40741.3467	1200.00	1018.5336	181.4664	10,000
5.	30559.8803	900.00	763.9970	136.0030	
6.	30423.8773	900.00	760.5969	139.4031	10,000
7.	20284.4742	600.00	507.1118	92.8982	
8.	20191.5760	600.00	504.7894	95.2106	10,000
9.	10096.3654	300.00	252.4091	47.5909	
10.	10048.7745	300.00	251.2194	48.7806	10,000

Problems

1. Find the price of a $4\frac{1}{2}\%$ bond maturing in 3 years, on a 5% basis. Show by a scientific schedule the accumulation of the discount.

2. A 5% bond due in $3\frac{1}{2}$ years was bought to yield $4\frac{1}{2}\%$. Find its cost, and show by the straight line method the gradual amortization of the premium.

3. A 6% bond was sold to yield 7.4%. Find the selling price if it was due in $1\frac{1}{2}$ years. Prove the accuracy of your result by a scientific schedule of accumulation.

4. The interest dates on a 5% bond are April and October. The date of maturity is Oct. 1, 1945. Find, July 13, 1923, the price at which the bond will yield 6%.

5. A company issued, March 1, 1923, \$500,000 of notes (short-term bonds are often called notes) paying $6\frac{1}{2}\%$ interest, to run 5 years. They were sold at date of issue on a 7% basis. Determine the selling price, and set up a scientific schedule to show the semiannual accumulations and book values.

6. A trust company has \$150,000 to invest for a client, with instructions to pay to a designated beneficiary the income from the fund for a period of 5 years, and, at the expiration of the 5 years, to pay to the beneficiary the principal sum. The trust company invests the money in 6% bonds, having 5 years to run on a $5\frac{1}{2}\%$ basis. Disregarding commission for handling the business:

(a) Find the price of the bonds bought.

(b) Find the quantity of bonds bought.

(c) Set up a scientific schedule showing that the principal sum will be kept intact. (Make reckoning in nearest whole \$1000 of bonds bought.)

7. On August 22, a man bought a quantity of 6% bonds, due Nov. 1 of the same year, at 99.70 and interest. What rate of income will he receive on the investment?

8. Find the market quotation on a 5% bond, due Dec. 1, if bought July 27, the same year, on a $7\frac{1}{2}\%$ basis.

9. Prepare a bid on a $5\frac{1}{2}\%$ basis on a new issue of $4\frac{3}{4}\%$ bonds, running 40 years from June 1, 1923.

10. Using the price found in problem 9, find the cost, including interest, on \$250,000 of the bonds (in problem 9) October 11, 1923.

11. You have been offered \$25,000 worth of $4\frac{1}{2}\%$ bonds due in $3\frac{1}{2}$ years at $95\frac{1}{2}$, which, you are assured, will give you a 6% return on your investment. Prove by a schedule the accuracy or falsity of the claim. If the claim is false, determine the correct price of the bonds to yield 6%.

XVI. DEPRECIATION

Depreciation is the term used to indicate a decrease in value. Many forms of property, as buildings, machinery, automobiles, live stock, etc., depreciate in value from natural wear and tear, from old age, or from other causes.

Every business concern owning depreciable property, in order to know its correct financial status, must take into consideration the extent to which its property is decreasing in value.

Many methods of calculation are employed in determining the amount of depreciation to be written off annually. The most common methods are known as:

1. The Straight Line Method.
2. The Sinking Fund Method.
3. The Fixed Percentage on Diminishing Value Method.

Other methods are Working Hours Method, Service Output Method, Sum of Expected Life Periods Method, Annuity Method, Unit Cost Method, Maintenance Method, etc. For an explanation of these latter methods, the student is referred to any good textbook on accounting.

In general, a piece of machinery, a building, a truck, or other form of depreciable property does not decrease in value to the point at which it becomes wholly worthless. It may become unsuited for the use for which it was originally intended, and still have a value for some other purpose. Thus, a truck may no longer be useful as a truck, but certain parts of it may be practically as good as new and of value in the repair shop in replacing broken parts of other cars. Hence, in most instances, depreciable property has a residual or scrap value. The total depreciation, then, is the difference between the original value and the scrap value.

The actual amount of depreciation in any given year (assuming that the life of the article is more than one year) is not known, but is merely an intelligent guess or estimate. Good accounting requires that something shall be written off annually.

METHODS OF RECKONING DEPRECIATION

The straight line method is probably the simplest and easiest method in use. For that reason it is extensively used. By this method the total depreciation is distributed pro rata over the life of the article. Thus, if a gas engine cost \$1000, June, 1923, and at the expiration of 10 years its scrap value is \$100, the total depreciation is \$900. The annual amount of depreciation is $\frac{1}{10}$ of \$900, or \$90.

A depreciation schedule will show the annual depreciation, the total depreciation, and the book value of the article at the expiration of each period.

SCHEDULE OF DEPRECIATION
(STRAIGHT LINE METHOD)

DATES	ANNUAL DEPRECIATION	TOTAL DEPRECIATION	BOOK VALUE OF ARTICLE
June 1, 1923	<i>Cost of</i>	<i>Article</i>	\$1000
June 1, 1924	\$90	\$ 90	910
June 1, 1925	90	180	820
June 1, 1926	90	270	730
June 1, 1927	90	360	640
June 1, 1928	90	450	550
June 1, 1929	90	540	460
June 1, 1930	90	630	370
June 1, 1931	90	720	280
June 1, 1932	90	810	190
June 1, 1933	90	900	100

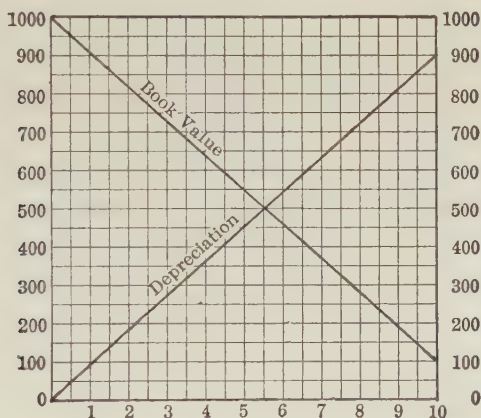
Graphically represented, this schedule would appear as shown on the next page.

Exercise

1. Set up a schedule similar to the above and show the facts graphically for a printing press costing \$62,500, if it is estimated to last 12 years and to have a residual value of \$5500.

2. Assume that at the expiration of 7 years a revised estimate is made of the schedule in exercise 1, which shows that the press will become obsolete in 3 years from date and that its residual value then will be but \$4250. Revise your schedule and graph to show the facts for the 8th, 9th, and 10th years.

GRAPHIC REPRESENTATION OF DEPRECIATION BY THE STRAIGHT LINE METHOD



The **sinking fund method** derives its name from the fact that the amount of depreciation on a given article is provided for by means of a sinking fund.

For the method of determining the annual contribution to a sinking fund see pages 194 and 195.

EXAMPLE

Using the same data for a gas engine as in the preceding exercise (page 222), find the annual contribution to a sinking fund, which will accumulate at 5%, to provide for the depreciation, set up a schedule of depreciation, and show the results graphically.

The amount of depreciation is \$900

The time is 10 years

The rate is 5%

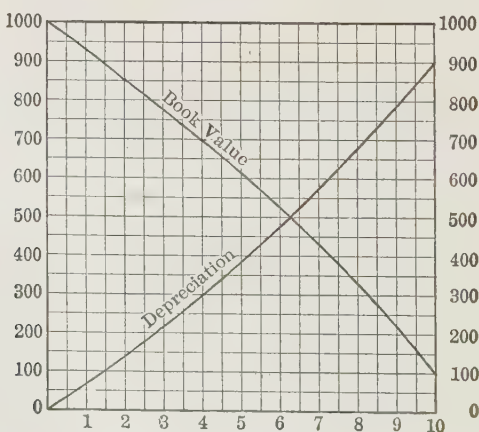
Therefore, $\frac{1}{s_{\overline{n}|i}} = .0795046$

$900 \times .0795046 = \$71.55$, annual contribution

SCHEDULE OF DEPRECIATION (SINKING FUND METHOD)

DATES	ANNUAL CONTRIBUTION	INTEREST ON FUND	TOTAL ANNUAL INCREASE ON FUND	TOTAL DEPRECIATION	BOOK VALUE OF ARTICLE
June 1, 1923	<i>Original</i>	<i>Cost</i>			\$1000.00
June 1, 1924	\$71.55		\$ 71.55	\$ 71.55	928.45
June 1, 1925	71.55	\$ 3.5775	75.1275	146.6775	853.3225
June 1, 1926	71.55	7.3339	78.8839	225.5614	774.4386
June 1, 1927	71.55	11.2781	82.8281	308.3895	691.6105
June 1, 1928	71.55	15.4195	86.9695	395.3590	604.6410
June 1, 1929	71.55	19.7679	91.3179	486.6769	513.3231
June 1, 1930	71.55	24.3338	95.8838	582.5607	417.4393
June 1, 1931	71.55	29.1280	100.6780	683.2387	316.7613
June 1, 1932	71.55	34.1619	105.7119	788.9506	211.0494
June 1, 1933	71.55	39.4475	110.9975	899.9481	100.0519

GRAPHIC REPRESENTATION OF DEPRECIATION BY SINKING FUND METHOD



By the **fixed percentage on diminishing value method** a given per cent of the book value as it appeared at the end of the preceding year is deducted annually. The problem in this method is to find the rate of depreciation to be deducted annually. For that purpose the following nomenclature will be used.

V = cost or present value of asset

R = residual or scrap value after n periods

n = number of periods

r = rate of depreciation

It is evident from the foregoing that

$V(1 - r)$ = value of asset at end of 1 period

and $V(1 - r)(1 - r)$ = value of asset at end of 2 periods

Also

$V(1 - r)(1 - r)(1 - r)(1 - r) \dots (1 - r)$ = value at end of n periods

Hence $V(1 - r)^n = R$

$$(1 - r)^n = \frac{R}{V} \quad (\text{by dividing by } V)$$

$$1 - r = \sqrt[n]{\frac{R}{V}} \quad (\text{by taking } n\text{th root})$$

$$\text{Therefore} \quad r = 1 - \sqrt[n]{\frac{R}{V}} \quad (33)$$

EXAMPLE

An engine costs \$1575 February 1, 1923, and is estimated to last 12 years, at which time its residual value will be \$125. What rate of depreciation on diminishing values should be written off annually?

$$r = 1 - \sqrt[12]{\frac{125}{1575}}$$

$$\log 125 = 2.09691 - 10$$

$$\log 1575 = 3.19728$$

$$\hline 8.89963 - 10$$

$$\text{Add } 110 \quad - 110$$

$$12)118.89963 - 120$$

$$\hline 9.90830 - 10$$

$$\text{Antilog } 9.90830 - 10 = .80966$$

$$1 - .80966 = .19034 = 19.034\%, \text{ rate of depreciation}$$

The annual depreciation, total depreciation, and book values are shown in the following:

SCHEDULE OF DEPRECIATION
(FIXED PERCENTAGE ON DIMINISHING VALUE METHOD)

DATES	ANNUAL DEPRECIATION	TOTAL DEPRECIATION	BOOK VALUE
Feb. 1, 1923	<i>Cost of</i>	<i>Engine</i>	\$1575.
Feb. 1, 1924	\$299.7855	\$ 299.7855	1275.2145
Feb. 1, 1925	242.7243	542.5098	1032.4902
Feb. 1, 1926	196.5242	739.0340	835.9660
Feb. 1, 1927	159.1178	898.1518	676.8482
Feb. 1, 1928	128.8313	1026.9831	548.0169
Feb. 1, 1929	104.3095	1131.2926	443.7074
Feb. 1, 1930	84.4553	1215.7479	359.2521
Feb. 1, 1931	68.3800	1284.1279	290.8721
Feb. 1, 1932	55.3646	1339.4925	235.5075
Feb. 1, 1933	44.8265	1384.3190	190.6810
Feb. 1, 1934	36.2942	1420.6132	154.3868
Feb. 1, 1935	29.3860	1449.9992	125.0008

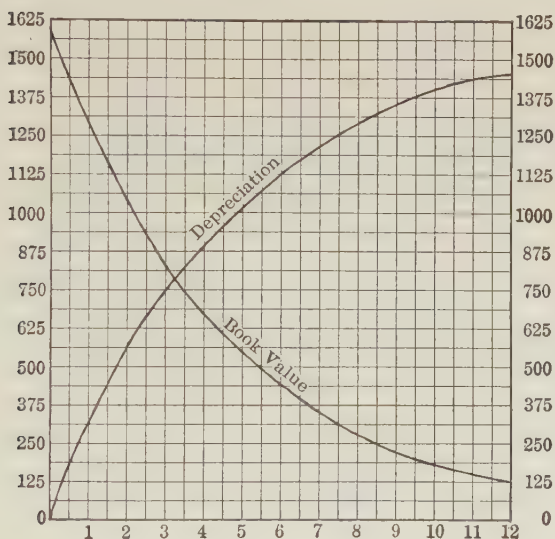
The first annual depreciation charge of \$299.7855 is found by taking 19.034% of \$1575. The book value at the end of the first year is the difference between \$1575 and \$299.7855, or \$1275.2145. The second annual depreciation charge is found by taking 19.034% of \$1275.2145. The book value at the end of the second year is the difference between \$1275.2145 and \$242.7243, or \$1032.4902, etc.

Exercises

1. Show, by a schedule similar to the one on page 224, that 10 annual payments of \$75.87 into a sinking fund earning 6% effective will provide for the depreciation on a piece of machinery costing \$1250 if its scrap value is \$250.

2. A motor truck costing \$1600 has been in use 6 years. Find, by a schedule similar to the one on this page, its present "book value" if depreciation at the rate of 25% per annum has been written off.

GRAPHIC REPRESENTATION OF DEPRECIATION BY THE FIXED PERCENTAGE ON DIMINISHING VALUE METHOD



Relative merits of the three methods

The *straight line method*, because of its simplicity, is readily understood by all, and for assets of small value and short life serves very satisfactorily. The same sum is deducted from earnings each year by this method.

By the *sinking fund method* the smallest amount is taken from earnings the first year, unless a sinking fund is actually established. If the "fund" is merely a book record, each succeeding year will require a constantly increasing amount to be written into the account to provide for the interest increase in the fund contemplated by the method. Depreciation and repairs (as the asset grows older) become an increasingly heavier burden on earnings. If the sinking fund is actually established, the interest accumulation is thereby provided for and only the fixed sum need be contributed from earnings.

By the *fixed percentage method* the largest amount is written into the reserve for depreciation account the first year, the amount growing less and less each year. By this plan the combined cost of depreciation and repairs is more evenly distributed over the life of the asset. When the depreciation charge is heavy, the repair bill (assumedly) is light, and vice versa.

Mining properties

An exhausted mine is considered as having no residual or scrap value. A depreciation fund, then, by whatsoever method used, must provide for the full cost of the mine.

A more important problem in connection with mining properties is to determine what price should be paid for a mine to provide for a depreciation fund and income on investment, having given the net annual yield of the mine and the number of years over which this yield will continue.

The sinking fund method is generally used in providing depreciation funds for mining properties. The rate of interest earned on such funds is often lower than the income rate desired on the investment. The income rate is the remunerative rate (i) and the sinking fund rate is the accumulative rate (j). (See page 199.)

On pages 199-201 the problem was to find the annuity. Here it is to find the present value of an annuity, that is, the price that may be paid for a mine yielding a given amount annually.

The annual payment is the product of the present value of the debt multiplied by the value of $\left[\frac{1}{a_{\overline{n}|}} + (i - j)\right]$. (See page 201.)

That is, since

$$R = A_{\overline{n}|} \left[\frac{1}{a_{\overline{n}|}} + (i - j) \right]$$

$$A_{\overline{n}|} = \frac{R}{\frac{1}{a_{\overline{n}|}} + (i - j)} \quad (34)$$

EXAMPLE

A mine yields \$25,000 net per annum, and it is estimated that it will continue that yield for 35 years. What price can one afford to pay for it to have an 8% return on investment for the entire period,

if a sinking fund to provide for its depreciation can be accumulated at $4\frac{1}{2}\%$?

$$\frac{1}{a_{\overline{n}|i-j}} + (i - j) = .0572704 + .035 = .0922704$$

$$\$25,000 \div .0922704 = \$270,942.79, \text{ cost of mine}$$

Problems

1. The X Company have installed a freight elevator in their plant at a cost of \$7500. They estimate that it will need replacing at the expiration of 9 years. If the residual value is \$750, what amount should be charged to earnings annually:

(a) By the straight line method?

(b) By the sinking fund method, interest at $4\frac{1}{2}\%$?

2. The Star Publishing Company install new presses May 1, 1923, at a cost of \$25,000. It is estimated that the presses will be either worn out or obsolete in 8 years. Scrap value is reckoned at \$3000. A depreciation fund is provided for by the fixed percentage on diminishing value method. Determine the annual rate of depreciation and set up a schedule to show depreciation and book value at the end of each year.

3. At the expiration of 6 years a new estimate is made on the probable life of the presses (in problem 2). It is now regarded as certain that the presses will be usable for 2 years longer than the original estimate. What rate of depreciation should be charged for the remaining 4 years?

4. Show graphically the book values and the depreciation for problems 2 and 3 combined.

5. A copper mine has been yielding a net annual income of \$80,000. A careful estimate indicates that the mine will continue to yield an annual income as stated for the next 15 years, at which time it will be exhausted. The mine is for sale. How much can a prospective buyer afford to offer for it if he reckons on a constant nominal return of 10% on the full amount of his investment, and can invest the sinking fund at 5% (interest to be converted semiannually in each case)?

Composite life of a plant

In a manufacturing plant there are a number of distinct and separate parts, such as building, engine, boiler, machinery, etc. Obviously the several parts are of different initial and scrap values, and will depreciate at different rates. Hence some parts will last longer than others. The **composite or average life of a plant**, then, is the time required for the total depreciation charge for the plant to amount, at a given rate of interest, to the total wearing value of the plant.

The wearing value of the plant, or of any part of the plant, is measured by the difference between its initial and residual values. If an engine cost \$1000 and its residual value is \$100, its wearing value is \$900.

The rate of depreciation is expressed as the ratio of the total annual depreciation to the total wearing value.

Let W = total wearing value of the plant
 D = total annual depreciation charge
 d = rate of depreciation
 n = the composite life of the plant

$$\text{Since } W = D \bar{s}_{\overline{n}|i} = \frac{D [(1+i)^n - 1]}{i} \quad (1)$$

$$Wi = D[(1+i)^n - 1] \quad (2)$$

$$\text{and } \frac{Wi}{D} = (1+i)^n - 1 \quad (3)$$

$$\text{By transposition } (1+i)^n = 1 + \frac{Wi}{D} \quad (4)$$

Since by definition $d = \frac{D}{W}$, by substituting d for $\frac{D}{W}$ in equation (4), the equation becomes

$$(1+i)^n = 1 + \frac{i}{d} \quad (5)$$

Solving equation (5) for n gives

$$n = \frac{\log \left(1 + \frac{i}{d}\right)}{\log (1+i)} \quad (6)$$

EXAMPLES

1. A plant consisting of three parts, A, B, and C, has initial costs, probable life of parts, and residual values as in the following schedule. Allowing interest at $4\frac{1}{2}\%$, find:

- (a) The rate of depreciation.
 (b) The composite life of the plant.

PART	INITIAL VALUE	RESIDUAL VALUE	PROBABLE LIFE
A	\$75,000	\$10,000	40 years
B	25,000	2,500	15 years
C	7,500	500	10 years

Wearing value of A is \$65,000

Wearing value of B is 22,500

Wearing value of C is 7,000

Wearing value of plant is \$94,500

$$\frac{1}{s_{\overline{40}|}} = .0093431$$

65,000 $\times$ \$.0093431 = \$ 607.30, annual depreciation on A

$$\frac{1}{s_{\overline{15}|}} = .0481138$$

22,500 $\times$ \$.0481138 = \$1082.56, annual depreciation on B

$$\frac{1}{s_{\overline{10}|}} = .0813788$$

7000 $\times$ \$.0813788 = \$ 569.65, annual depreciation on C

\$2259.51, total annual depreciation

(a) 2259.51 $\div$ 94,500 = .02391, rate of depreciation

$$(b) n = \frac{\log \left(1 + \frac{i}{d}\right)}{\log (1 + i)} = \frac{\log 2.882}{\log 1.045} = \frac{.45969}{.01912} = 24, \text{ no. of years}$$

The wearing value of an asset after the expiration of a given number of years (say t years) is equal to the difference between its initial wearing value and the compound amount of the depreciation charges for the given number of years.

If W = initial wearing value
 and W_t = wearing value at the end of t years
 then $W_t = W - W \cdot \frac{1}{s_{\overline{n}|}} \cdot s_{\overline{t}|}$

2. An electric motor cost \$4500. It is estimated to last 10 years, at which time its scrap value will be \$250. At 4% per annum find the wearing value of the motor at the expiration of 5 years.

$$\$4500 - \$250 = \$4250, \text{ initial wearing value}$$

$$W_t = W - W \cdot \frac{1}{s_{\overline{n}|}} \cdot s_{\overline{t}|}$$

$$\frac{1}{s_{\overline{10}|}} = .0832909$$

$$4250 \times \$0.0832909 = \$353.99, \text{ annual depreciation}$$

$$s_{\overline{5}|} = 5.4163226$$

$$353.99 \times \$5.4163226 = \$1917.32, \text{ amount of depreciation at 4\%}$$

$$\$4250 - \$1917.32 = \$2332.68, \text{ value at end of 5 years}$$

Problems

1. Allowing interest at 5%, find the composite life of the following fixed assets of the Weston Manufacturing Company.

ASSETS	INITIAL COST	ESTIMATED SCRAP VALUE	PROBABLE LIFE
Building	\$100,000	\$35,000	20 years
Machinery	70,000	25,000	15 years
Tools	20,000	5,000	10 years
Patterns	10,000		8 years

2. A gas engine costs \$1500. Its estimated life is 12 years, at which time it is assumed that it will be worth \$100. Allowing interest at 4%, find its wearing value at the end of 9 years.

3. A public utility consists of several parts whose wearing value and probable life are as given in the following schedule. Find, on a 4% basis:

- The total annual depreciation charge.
- The rate of depreciation.
- The composite life of the plant.

PART	WEARING VALUE	PROBABLE LIFE
A	\$2,250,000	15 years
B	4,125,000	40 years
C	1,750,000	20 years
D	500,000	8 years
E	900,000	12 years

XVII. PARTNERSHIP SETTLEMENTS

Partnership agreements should stipulate the ratio in which the profits and losses of the business are to be apportioned among the partners, as well as the plan of distributing the assets in case of winding up the business.

There are several standard methods of distributing the profits and losses. Among the more common methods are the following:

1. *By agreement*

- (a) Equally, or in any other fixed ratio.
- (b) In proportion to net capital.
- (c) In proportion to average investment.
- (d) By allowing interest on each partner's investment, and (or) paying each partner a salary, and then dividing the remaining profits or losses in some fixed ratio. Or,
- (e) By a combination of two or more of the foregoing plans.

2. *By law*

In the absence of agreement, profits and losses should be divided equally.

The distribution of assets, in case of liquidation, is not, of necessity, made in the same ratio as the distribution of profits and losses. Absence of agreement as to the distribution of assets has caused much discussion and a number of lawsuits. There seems to be no uniform rule by which such distribution shall be made. The problem grows out of those cases in which there has been a net loss, or in which the assets, upon liquidation, do not yield their estimated value, and the available assets are not sufficient to make full return of capital invested.

A common method is to distribute the profits or losses (whether on operation or liquidation) in the profit and loss ratio, and then distribute the remaining assets in proportion to net capital.

A difficulty arises in those cases in which the remaining assets, upon liquidation, are paid in several installments, and in which the capital ratios differ from the profit and loss ratios. In such cases the earlier

installments should be so distributed as to reduce, as soon as possible, the capital ratios to that of the profit and loss ratios. All future installments may then be distributed in the profit and loss ratios.

EXAMPLES

1. F and G enter into copartnership on Jan. 1, 1923. At that time F invests \$45,000; but he withdraws \$5000 July 1. G invests \$50,000 on Jan. 1; but he withdraws \$3000 on May 1, and \$4000 on Aug. 1. Profits and losses are to be shared in proportion to average investment. Dec. 31, 1923, the ledger shows the following assets and liabilities: cash, \$42,000; merchandise, \$65,000; accounts receivable, \$24,000; notes receivable, \$18,000; notes payable, \$16,000; and accounts payable, \$11,000. An allowance of 5% for bad debts is made on accounts receivable and notes receivable. Determine the present worth of each at the time of closing.

STATEMENT OF ASSETS AND LIABILITIES

Cash		42,000	
Merchandise		65,000	
Accounts Receivable	24,000		
Less 5%	1,200	22,800	
Notes Receivable	18,000		
Less 5%	900	17,100	
Notes Payable			16,000
Accounts Payable			11,000
PRESENT WORTH			119,900
		146,900	146,900

$\$119,900 - (\$40,000 + \$43,000) = \$36,900$, firm's net gain

F invests:

\$45,000 for 6 months = \$270,000 for 1 month
 40,000 for 6 months = 240,000 for 1 month
 A total of \$510,000 for 1 month

G invests:

\$50,000 for 4 months = \$200,000 for 1 month
 47,000 for 3 months = 141,000 for 1 month
 43,000 for 5 months = 215,000 for 1 month
 A total of \$556,000 for 1 month

\$510,000 + \$556,000 = \$1,066,000, firm's total investment for
1 month

F's share of the gain = $\frac{510}{1066} \times \$36,900 = \$17,653.85$

G's share of the gain = $\frac{556}{1066} \times \$36,900 = \$19,246.15$

F

1923			1923		
July	1		Jan.	1	45,000
Dec.	31	Pres. Worth	Dec.	31	Net Gain
					17,653 85
					62,653 85
1924			1924		
Jan.	1	Pres. Worth	Jan.	1	Pres. Worth
					57,653 85

G

1923			1923		
May	1		Jan.	1	50,000
Aug.	1		Dec.	31	Net Gain
Dec.	31	Pres. Worth			19,246 15
					69,246 15
1924			1924		
Jan.	1	Pres. Worth	Jan.	1	Pres. Worth
					62,246 15

In this problem the question might be raised as to whether the withdrawals are to be charged against capital or against profits. The wording of the problem would seem to indicate that they are to be charged against capital. Otherwise the expression "average investment" would have no meaning in this problem.

2. The balance of each account in Dolson and Company's ledger, and the inventories, Dec. 31, 1923, are as follows:

John W. Dolson (Capital)	20,910.67
John W. Dolson (Private) (Debit)	2,040
H. M. Dawson (Capital)	10,000
H. M. Dawson (Private) (Debit)	1,000
Merchandise Inventory (Jan. 1, 1923)	21,201.29
Purchases	27,499.26
Return Purchases	1,150

Discount on Purchases	525
Sales	37,400.60
Return Sales	900
Discount on Sales	250
Notes Receivable	297.40
Accounts Receivable	7,030.30
Accounts Payable	8,646.45
Notes Payable	2,075.75
Real Estate	7,500
Furniture and Fixtures	1,210
Traveling Expenses	662.50
Salesmen's Salaries	3,200
Salaries and Wages	4,720
Interest and Discount (Earned)	75
Advertising	525.73
General Expense	1,799.27
Cash	947.72

INVENTORIES

Merchandise on Hand	\$26,660.27
Real Estate	7,500
Due J. M. Hart (bill for blank books and stationery)	67.30
Allow 10% depreciation on Furniture and Fixtures and 5% for bad debts on Accounts Receivable.	

The partnership agreement provides that Dolson is to have a salary of \$4000, and Dawson a salary of \$2000 a year. After salaries are paid, the remaining net profit (or loss) is to be shared equally. In case of dissolution of the business, the assets are to be distributed in proportion to net worth just prior to dissolution.

At the conclusion of business Dec. 31, 1923, after statements and ledger have been closed, it is decided to dissolve the partnership. On Jan. 2, 1924, the partners accept as consideration for the business as shown by the books Dec. 31, 1923, an offer of \$10,000 cash, and two notes of \$10,500 each, due in 3 and 6 months respectively. The purchasers are to take over all the assets and assume responsibility for all the liabilities.

Dolson and Dawson have the notes discounted at once at a bank, and then divide the entire cash proceeds of the sale in the ratio agreed upon in the articles on copartnership.

(a) Prepare a statement of trading and profit and loss, and a balance sheet, as at the close of business Dec. 31, 1923.

(b) How much of the cash should each partner receive?

(a) STATEMENT OF TRADING AND PROFIT AND LOSS

DOLSON AND COMPANY

DECEMBER 31, 1923

Sales		37,400	60		
Less Return Sales		900		36,500	60
<i>Cost of Goods Sold</i>					
Inventory, Jan. 1, 1923		21,201	29		
Purchases	27,499.26				
Less Return Purchases	1,150.	26,349	26		
Total Cost of Stock		47,550	55		
Less Inventory, Dec. 31, 1923		26,660	27		
Cost of Goods Sold				20,890	28
Gross Profit on Trading				15,610	32
<i>Selling Expenses</i>					
Advertising		525	73		
Traveling Expenses		662	50		
Salesmen's Salaries		3,200	00	4,388	23
Net Selling Profit				11,222	09
<i>Other Income</i>					
Discount on Purchases		525			
Interest and Discount		75		600	00
Total Income				11,822	09
<i>Deductions from Income</i>					
Discount on Sales		250	00		
Depreciation on Furniture and Fix- tures, 10% of \$1210		121	00		
Salaries and Wages		4,720	00		
General Expense, Charged	1,799.27				
Bill due J. M. Hart	67.30	1,866	57		
Bad Debts, 5% of \$7030.30		351	52		
Salaries to Partners:					
John W. Dolson	4,000.00				
H. M. Dawson	2,000.00	6,000	00	13,309	09
Dolson and Company, Net Loss				1,487	00

BALANCE SHEET
DOLSON AND DAWSON
DECEMBER 31, 1923

ASSETS		LIABILITIES	
<i>Current Assets</i>		<i>Current Liabilities</i>	
Cash	947 72	Notes Payable	2,075 75
Notes Receivable	297 40	Accounts Payable	8,646 45
Accounts Rec. 7,030 30		Due J. M. Hart	67 30
Less 5% 351 52	6,678 78	Total Liabilities	10,789 50
Mdse. Inventory	26,660 27	<i>Capital Accounts</i>	
Total Current Assets	34,584 17	John W. Dolson :	
<i>Fixed Assets</i>		Inv. 20,910 67	
Real Estate 7,500 00		Sal. 4,000.00	
Fur., Fixt. 1,210		Draw. 2,040	
Less 10% 121 1,089 00	8,589 00	Loss 743.50	
Total Fixed Assets	43,173 17	2,783.50	
Total Assets		Net Increase 1,216 50	
		Present Worth 22,127 17	
		H. M. Dawson :	
		Inv. 10,000 00	
		Sal. 2,000.00	
		Draw. 1,000	
		Loss 743.50	
		1,743.50	
		Net Increase 256 50	
		Present Worth 10,256 50	
	43,173 17	43,173 17	

(b)

Jan. 2, 1924 + 3 months = April 2, 1924, due date of 3 months' note

From Jan. 2, 1924, to April 2, 1924 = 91 days (1924 is a leap year)

The bank discount on \$10,500 for 91 days = \$159.25

\$10,500 - \$159.25 = \$10,340.75, proceeds of 3 months' note

Jan. 2, 1924 + 6 months = July 2, 1924

From Jan. 2, 1924, to July 2, 1924 = 182 days

The bank discount on \$10,500 for 182 days is \$318.50

\$10,500 - \$318.50 = \$10,181.50, proceeds of 6 months' note

$\$10,000 + \$10,340.75 + \$10,181.50 = \$30,522.25$, net cash proceeds of the sale

Dolson receives $\frac{22,127.17}{32,383.67} \times \$30,522.25 = \$20,855.30$

Dawson receives $\frac{10,256.50}{32,383.67} \times \$30,522.25 = \$9,666.95$

Problems

1. J. B. Altgeld and L. M. Goodman, partners, shared losses and gains in the ratio of $\frac{2}{3}$ and $\frac{1}{3}$ respectively. Prepare a trial balance, a trading and profit and loss account, and a balance sheet from the following data: On Jan. 1, 1923, J. B. Altgeld invested \$20,079.16; but on Aug. 1 he withdrew \$1750. On Jan. 1, 1923, L. M. Goodman invested \$10,039.58; but on June 1 he withdrew \$1260.50. Other ledger balances were as follows: cash, \$12,711.30; notes receivable, \$3974.18; notes payable, \$8250.50; accounts receivable, \$6356.45; accounts payable, \$10,976.72; merchandise sales, \$26,119.18; raw material, cost, \$8416.75; plant and machinery, \$10,500; real estate, \$14,750; interest income, \$658.61; insurance, \$125; general expense, \$5505.71; advertising, \$1375; salesmen's salaries, \$5000; traveling expenses, \$1908.61; office salaries, \$1250; commissions paid, \$1240.25. The inventories at the close of the year were: Merchandise, \$1615.80; raw materials, \$1250; plant and machinery, \$9500; real estate, \$16,000.

2. On June 30, 1923 Chester Mack and Walter Fox had inventories as follows: merchandise, \$7400; real estate, \$9500; blank books and stationery, \$28.50. The cash balance as shown by the ledger was \$4360. Footings of other ledger accounts were: Debits — Chester Mack (private), \$1960; Walter Fox (private), \$550; purchases, \$18,200; First National Bank, \$4608.25; real estate, \$9000; notes receivable, \$6420; interest and discount, \$127.40; general expense, \$425.60; J. Wilson, \$3872.75; insurance, \$112.50; repairs, \$87.50; office salaries, \$2500; salesman's salary, \$1800; traveling expenses, \$937.60; bonds (cost), \$10,500; freight (inward), \$231.50; freight (outward), \$318.50; return sales, \$297.50. Credits — Chester Mack (capital), \$15,000; Walter Fox (capital), \$15,317.58; sales, \$18,700; First National Bank, \$2500; rent, \$250; accounts payable,

\$4825; notes payable, \$3265; interest and discount, \$192.80; interest on bonds, \$500; purchase returns, \$347.50; J. Wilson (to be supplied by the student). Set up a trial balance, a statement of trading and profit and loss, and a balance sheet.

3. Arnold and Jenkins, operating separately, decided to enter into partnership on July 1, 1915. Their respective balance sheets as of that date were as follows:

Arnold's Assets

Furniture and Fixtures	\$ 2,250	
Accounts Receivable	7,500	
Merchandise	7,650	
Cash	<u>600</u>	\$18,000

Arnold's Liabilities

Accounts Payable	\$ 3,000	
Capital Account	<u>15,000</u>	\$18,000

Jenkins's Assets

Furniture and Fixtures	\$ 1,800	
Accounts Receivable	4,500	
Merchandise	6,000	
Cash	<u>1,200</u>	\$13,500

Jenkins's Liabilities

Accounts Payable	\$ 4,500	
Capital Account	<u>9,000</u>	\$13,500

It was agreed that Arnold and Jenkins should put in their accounts receivable at 10% less than face value, these amounts to be charged against their capital accounts and carried on the partnership books as a reserve for bad debts. Of Jenkins's furniture only \$750 was to be taken over by the partnership. With the above exceptions, the assets and liabilities of the parties were to be taken over by the partnership at the above figures, except that Jenkins was to invest in the partnership, in cash, an amount which, after making the adjustments above referred to, would make his capital account the same as that of Arnold.

You are asked to draw off a balance sheet of Arnold and Jenkins's partnership July 1, 1915, showing effect of above provisions.

(Arkansas, C. P. A.)

4. A and B buy merchandise to the amount of \$4000, A contributing \$2500 and B \$1500. They sell to C a one-third interest in the business for \$2000. How much shall A and B receive respectively in order to make A, B, and C equally interested? Draft entries to show solution. (South Carolina, C. P. A.)

5. By partnership agreement existing between Brown and Gray, Brown has $\frac{2}{3}$ of the profits and Gray $\frac{1}{3}$. Brown's capital account stands credited \$50,000; Gray's capital account stands credited \$40,000.

The assets of the partnership are as follows:

Factory and Machinery	\$75,000
Stock as per Inventories	30,000
Accounts Receivable	15,500

The liabilities are as follows:

Accounts Payable	\$15,000
Notes Payable	10,000
Overdraft Bank	5,500

Gray takes over the liabilities, as above mentioned, and the assets at the following agreed valuation:

Factory and Machinery	\$70,000
Stock as per Inventories	26,000
Accounts Receivable	14,000

An arrangement is made whereby Gray receives \$500 from Brown for accepting sole liability for discounted bills receivable.

Make up a statement showing the amount that Brown should receive, it being understood that losses on capital are borne by the partners in the proportion in which the profits are divided.

(South Carolina, C. P. A.)

XVIII. REVIEW PROBLEMS

1. A wholesale hardware company desires a net profit of 5% on the cost of its goods. It employs agents to sell its merchandise and pays them 5%. An allowance of $2\frac{1}{2}\%$ is made for bad debts, and $3\frac{1}{3}\%$ of sales for other expenses. If a trade discount of 20% and 10% is allowed to customers, at what per cent above cost should goods be marked in the catalogue?

2. In a manufacturing concern insurance was carried as follows: Department A, \$30,000; Department B, \$48,000; Department C, \$15,000; Department D, \$35,000; Department E, \$60,000. The total premium for one year was \$1527.50. The rates of insurance on Departments A and B were the same; the rate on Department C was $1\frac{1}{2}$ times the rate on Department A. The rate on D was double the rate on A, and the rate on E was $2\frac{1}{4}$ times the rate on A. Find the amount of insurance chargeable to each department.

3. A fire destroyed the entire stock of merchandise of the Johnson Furniture Company. The ledger, which was saved, gave the following information: Inventory at last closing, \$36,000; purchases to date of the fire, \$156,000; sales, \$168,000. Investigation revealed the fact that a sale of \$18,000, made the day before the fire, had not been recorded on the books. This last sale was made at a profit of $33\frac{1}{3}\%$ of the cost of sales. The average rate of profits (on sales other than this last sale) had been 40% of the cost of sales. If the stock of merchandise was fully covered by insurance, what amount should be claimed as fire loss?

4. A business block has been insured since May, 1910, for \$250,000 at \$1.10 a hundred for each 5-year period. The block is worth \$375,000. The policy is the New York Standard (80%) Average Clause Policy. In May, 1923, a fire occurs, causing a damage of \$150,000 to the building. Assuming the premiums to be worth 4% compound interest to the company, determine the company's net loss.

5. John Andrews opened a grocery business with a cash capital of \$25,000. At the end of the first year his ledger showed the following accounts: cash, \$7500; accounts receivable, \$2350; notes receivable, \$8750; notes payable, \$1500; merchandise, debit balance, \$7650; accounts payable, \$5750; general expense, \$3500; furniture and fixtures, \$2500. He allowed 10% depreciation on furniture and fixtures. His total loss was \$5250. Set up a statement of assets and liabilities.

6. On the night of Aug. 1, Fred Sewall's store was struck by lightning and the store and contents were burned. Sewall was able to rescue only his books and the other contents of the safe. His claim for insurance amounted to \$18,762.50. An investigation into his records showed the following: inventory the preceding Jan. 1, \$13,425.60; purchases, \$63,750.68; sales, \$72,472.50; return sales, \$586.40; interest and discount, credit, \$75.25; discount on purchases, \$2875.22; advertising, \$250; traveling expenses, \$1375; salesman's salary, \$1400; general expense, \$375; insurance, \$112.50; salaries and wages, \$3076.20; repairs, \$156.40; net profit for the period, \$6407.79. Set up a statement to prove or disprove the claim for insurance.

7. Artemas Warner and Theodore Branson have been competing grocers. On January 1, 1923, they agree to unite their two businesses under the firm name of Branson and Warner Grocery Company. Each partner is to have a half interest in the new concern. It is agreed that each partner shall contribute to the new enterprise the assets of his old business and that the business liabilities of each shall be assumed by the new company.

Artemas Warner's ledger and inventory record show the following:

ASSETS	
Cash	\$ 7,568.13
Notes Receivable	3,785.60
Accounts Receivable	4,595.72
Stock of Merchandise	18,978.50
Fuel	150.00
Store Fixtures	2,590.00
Office Furniture	1,500.00
Delivery Equipment	1,875.00
Real Estate	15,000.00

LIABILITIES	
Notes Payable	\$ 2,750.75
Accounts Payable	5,972.25
Expense (Sundry unpaid bills)	50.00
Mortgage Payable	6,000.00

Theodore Branson's business assets and liabilities are as follows :

ASSETS	
Cash	\$ 4,763.80
Notes Receivable	7,564.75
Accounts Receivable	6,943.64
Stock of Merchandise	21,494.50
Fuel	300.00
Store Fixtures	2,500.00
Insurance	75.00
Office Furniture	1,000.00
Delivery Equipment	1,500.00

LIABILITIES	
Notes Payable	3,641.50
Accounts Payable	6,958.30

Branson has a lease of his present store, which expires Dec. 31, 1925, at an annual rental of \$3000, payable quarterly in advance. The new firm agrees to take over the lease. They pay the rent for the first quarter of 1923. It is decided to sell the lease from April, 1923. If the lease cannot be disposed of at its face value, Branson agrees to be personally responsible for any deficit.

The partnership agreement provides that an allowance of 5% shall be made on accounts receivable for bad debts, and that notes receivable and notes payable shall be reckoned at their cash value January 1, 1923, interest and discount being at 6%. Interest on the mortgage at 5% is payable semiannually April 10 and October 10. Accrued interest to date is to be charged against Warner and assumed by the firm.

Warner's bill book shows the following data regarding unpaid notes :

NOTES RECEIVABLE				
	FACE	TIME	DATE	INTEREST
# 18	\$1392.80	3 mo.	Dec. 13, 1922	—
# 21	1176.60	90 da.	Dec. 19, 1922	6%
# 22	1216.20	6 mo.	Dec. 20, 1922	6%

NOTES PAYABLE				
	FACE	TIME	DATE	INTEREST
# 31	\$1191.50	60 da.	Nov. 16, 1922	—
# 51	1559.25	4 mo.	Dec. 19, 1922	6%

Branson's bill book shows the following :

NOTES RECEIVABLE				
	FACE	TIME	DATE	INTEREST
# 103	\$5000	8 mo.	Sept. 1, 1922	6%
# 111	2564.75	90 da.	Oct. 11, 1922	—

NOTES PAYABLE				
# 63	\$1875	60 da.	Nov. 27, 1922	—
# 67	1766.50	4 mo.	Dec. 10, 1922	6%

Prepare a statement of assets and liabilities for each of the old concerns, and for the new firm, it being understood that either partner having the lesser net worth shall contribute a sufficient amount of cash to make his net investment equal to that of the other partner.

8. Owing to depreciated property values, the store formerly occupied by Mr. Branson (see problem 7) can be rented from April 1, 1923, at only \$2500 a year. The Owen Drug Company agrees to assume the old lease at \$3000 a year upon being paid, April 1, 1923, the present value of the difference between \$3000 a year and \$2500 a year. Allowing interest at 5% nominal, and reckoning rentals as being paid quarterly in advance, find the amount Branson must pay the Owen Drug Company to take over the lease.

9. The Branson and Warner Grocery Company finds itself overstocked and decides on a 10-day sale as follows: All prices (which, on the average, are $33\frac{1}{3}\%$ above cost) are to remain unchanged. On all sales under \$5 each a cash discount of 10% is to be allowed; on all sales of \$5 or over and under \$10, an additional dollar's worth of merchandise is to be given the purchaser; and on all sales of \$10 and over, additional merchandise worth \$2.50 is to be given the purchaser. During the sale the gross amount of sales less than \$5 each amounts to \$3472.56. There are 586 separate sales averaging \$5.53 each, and 403 sales averaging \$11.62 each.

Determine (a) the rate of profit, based on cost, on each group of sales; and (b) the rate of profit, based on cost, on the entire sale.

10. A man has two sons and a daughter, aged 9, 12, and 14 years, respectively. He takes \$50,000 and invests it for them in such a way that, with interest accumulation at $4\frac{1}{2}\%$ effective, each will receive the same amount upon becoming 21 years of age. How much money should be credited to each child at the present time?

11. A young man has bequeathed to him by his father an annuity of \$5000 a year for a period of 20 years from May 1, 1922, the first payment being due May 1, 1923. On May 1, 1924, after receiving his second payment, the young man agrees to sell the remainder of the annuity on a 5% basis. How much should he receive for it?

12. At the birth of a son, Sept. 10, 1921, a father decides to provide for his college education by setting aside in trust for the son a certain sum annually for that purpose. It is estimated that his college expenses will be \$1200 a year and that they will begin when the son is 17 years old. What sum should the father deposit annually in the fund to accumulate at $4\frac{1}{2}\%$ effective, the first deposit being made on the date of the son's birth and continuing until 21 payments have been made, so that the son may withdraw from the fund \$1200 annually for four years for his college expenses, beginning with his 17th birthday?

13. A philanthropist left a perpetual trust fund of \$60,000, the income from which was to be divided between two institutions as follows: One of them was to receive the entire income from the fund until it had received its half. The total income from that time on was to pass to the other institution forever. For how long a time did the first institution receive the income if money was worth 5%? 6%?

14. To erect a college laboratory cost \$150,000. It required an expenditure of \$20,000 annually, payable at the end of the year for maintenance. What sum of money was required at the time it was first built to provide for the first construction, for rebuilding every 60 years at the same cost, and for continual maintenance, if interest was at 5%?

15. A memorial hall cost \$400,000. The annual repair bill was estimated to be \$1500, with an additional outlay of \$15,000 every 12 years, for extended repairs and renovation. It was further estimated that at the expiration of 60 years a new building must be erected to take its place. Allowing that the materials in the old building would have a value of \$20,000, and that money was worth 5%:

(a) Find the annual contribution required to keep the building in constant repair, and to provide for an indefinite number of reërections every 60 years, assuming the cost of construction to remain constant.

(b) If, instead of making annual contributions for the foregoing costs, it had been decided to provide for them by an endowment fund, what sum earning 5% effective should have been set aside after the building was first erected, the income from which would cover all expenses and reconstruction costs for an indefinite period?

16. A policyholder desires to borrow \$3000 from an insurance company for a period of 5 years. The only terms on which the company will make the loan are that the rate of interest shall be 6% a year on the entire loan throughout the whole period of 5 years, and that the loan, including interest, shall be repaid in 10 equal, semi-annual installments. It is understood that repayments of principal will accumulate at 5% nominal, convertible semiannually. Find the semiannual payment, and set up a schedule to show the condition of the loan at each half year, including the redemption money required at any period, by each of the three methods.

17. What uniform semiannual contribution to a sinking fund will cancel \$50,000 worth of bonds due in 10 years, if the sinking fund earns 6% nominal for the first 5 years and 5% nominal for the remaining 5 years?

18. Prove problem 17 by setting up a schedule of accumulation.

19. For the construction of a system of water supply a city borrowed \$5,000,000 on 30-year $4\frac{3}{4}\%$ bonds dated May 1, 1923. The bonds were sold on a 5% basis. For their redemption the city set up a sinking fund to accumulate at $4\frac{3}{4}\%$ nominally, interest convertible semiannually. Find the semiannual cost to the city for interest and sinking fund charges.

20. A jitney company has an income of 18% a year on its invested capital, but sets nothing aside to replace the jitneys when they are worn out. Assuming that the jitneys have a service life of 8 years and have a scrap value of 15% of their original cost, find the net rate of income the company receives after providing a sinking fund to accumulate at 5%. Prove the accuracy of the sinking fund provision by setting up a schedule of accumulation. (Show the schedule in terms of per cent to 8 places of decimals.)

21. A printing press costing \$25,000 was assumed to have a life of 10 years, and to have a scrap value at the end of that time of \$3000.

For its replacement a sinking fund was established into which were put equal semiannual payments which, at 5% nominal, would be sufficient to replace the press at the end of the 10 years. When the press had been in use $7\frac{1}{2}$ years a fire occurred, completely ruining it. It was sold as junk for \$100. The purchase of a new press for \$25,000 required how much money in addition to the junk value and the amount of the sinking fund?

22. Using the data in problem 21, assume that at the end of 6 years a revised estimate of the life of the press was made as follows: The press would need replacing at the end of 8 years instead of 10 years, and its scrap value at that time would be \$2000 instead of \$3000. Determine the additional contribution to the sinking fund for the next 2 years to provide for the replacement at the end of 8 years.

23. Beginning with the seventh year, set up a schedule of depreciation for problem 22, for the seventh and eighth years.

24. A piece of machinery costs \$11,350.29. It is expected to last 20 years when, as junk, it is estimated to have a value of \$786. Depreciation is reckoned on the fixed percentage on diminishing value method.

(a) Find the rate of depreciation.

(b) Assume that at the end of 10 years a revaluation of the machine is made with the result that only 5 years more of service are expected from it. Its junk value is also revalued at \$710. If the same plan of writing off depreciation is continued, what will be the rate for the last 5 years? (Use the nearest whole "round" rate.) Show the amounts charged to depreciation for the eleventh, twelfth, thirteenth, fourteenth, and fifteenth years.

25. A mining property is yielding a net income of \$125,000 a year. It is estimated that the same rate of income will continue for the next 40 years, when the mine will be exhausted. A prospective purchaser reckons on a net annual return of 9% on his investment, after providing for a sinking fund to accumulate at 5%, to amount to the purchase price in 40 years. What price can he afford to offer for the mine?

26. A company issued \$150,000 of 6% bonds as follows: The bonds were dated March 1, 1923, and were to be redeemed serially, beginning March 1, 1925, at which time \$25,000 of the bonds were to be paid

off, and a like sum every two years thereafter until the bonds were all redeemed. If the bonds were issued on a 5% basis, find the amount received for them.

Set up a schedule to show the gradual amortization of both the premium and the face value of the bonds.

27. Assume that the company in problem 26 had issued 5% bonds on a 6% basis, and had made provision for paying them off serially as indicated. Find the amount received for them, and set up a schedule showing the accumulation of the discount, and the amortization of the face of the bonds.

28. A manufacturing company issued \$15,000 of 6% serial bonds redeemable in equal annual installments of their face value over a period of 5 years from April 1, 1923. The bonds were sold as follows:

1. Bonds maturing April 1, 1924, on a $5\frac{1}{2}\%$ basis.
2. Bonds maturing April 1, 1925, on a $5\frac{1}{4}\%$ basis.
3. Bonds maturing April 1, 1926, on a 5% basis.
4. Bonds maturing April 1, 1927, on a 5% basis.
5. Bonds maturing April 1, 1928, on a $4\frac{3}{4}\%$ basis.

Find the price of each lot and the total amount received for all.

29. An estate of \$450,000 is left in trust to two brothers for a period of 5 years, when it is to be divided equally between them. During the five years they are to receive only the income from the estate. The annual cost of administration is $\frac{1}{4}\%$ of the face value of the estate. The funds are invested in $4\frac{1}{2}\%$ bonds due in 5 years, on a 5% basis. Interest and administration fees are payable semiannually. Prepare a statement showing the semiannual amounts payable to the brothers. Any fractional part of the price of a \$1000 bond is to be deposited in a savings bank paying 4% interest convertible semiannually.

30. A corporation has a capital stock of \$5,000,000 common, and \$2,500,000 7% cumulative preferred. There are outstanding \$3,000,000 of 6% bonds due in 30 years from Jan. 1, 1922. To redeem the bonds at maturity a sinking fund earning $4\frac{1}{2}\%$ effective is established.

Gross earnings for the year ended Dec. 31, 1922 were \$24,896,400, and expenses were \$24,425,000.

(a) Find the price, on a 5% basis, of the bonds on the date of issue.

(b) Find the cost on Jan. 25, 1922, of \$100,000 worth of the bonds at the price found in (a).

(c) Find the annual contribution to the sinking fund.

(d) Distribute the earnings for the year.

31. A wooden stairway costs \$50 and will last 12 years, when it must be completely rebuilt. How much can be profitably expended to erect instead a steel and concrete stairway which will last 30 years before reconstruction is necessary, allowing interest at 6%?

The following problems have been taken from C. P. A. examinations of the several states indicated.

32. A was the owner and operator of a stone quarry which, owing to weather conditions, could not be operated between December 1 and February 28. B caused damages to the quarry which delayed the commencement of operations till April 15th, from which date the quarry was worked until November 30th and produced 71,000 cu. yd. at a quarry cost of 29¢ a cubic yard. The entire product sold at 77¢ a cubic yard. Overhead expense for the year was \$10,000. B repaired the quarry at his own expense. You are required by the lawyer for A to indicate the measure of consequential damage as a basis for action. In your answer illustrate your method. (Illinois)

33. If a principal of \$1 will amount to \$3.3863549 in 25 years, at 5% per annum, what would be the present value of an annuity of \$1,250 for 30 years at the same rate? (Ohio)

34. A company purchased a piece of real estate for \$100,000. The terms of payment agreed upon were \$10,000 each year thereafter without interest, until the whole \$100,000 was paid. At the end of the fourth year, 12 months before the fifth payment was due, the company found itself possessed of considerable cash, and decided to relieve itself of the liability specified by depositing one amount sufficient to pay the annual installments as they matured. The bank agreed to pay 5% per annum on all money deposited for that purpose.

What is the single amount that the company must deposit, at 5%, to pay the yearly installments as they mature? (Florida)

35. A contractor proposes to build a bridge to Belle Isle and accept the city's 4% 20-year bonds to the amount of \$2,000,000 in payment. He advocates as a means of retiring the bonds the establishing of a

toll system on foot passengers and automobiles at the respective rates of 1 cent and 5 cents each. Assuming the ratio of foot passengers to automobiles to be ten to one, how many of each will be necessary to pay the interest annually and create a fund which placed at the same rate of interest will be sufficient to retire the bonds at maturity?

NOTE. \$1 at 4% for 20 years = \$2.19112314 (Michigan)

36. The present value of an annuity of \$1 for four periods at 2% is \$3.80772870.

What is the value on January 1, 1914, of a 5% per annum bond issue of \$100,000 bought on a 4% per annum basis (semiannual coupons), due January 1, 1916?

Prepare amortization table as follows :

DATE	TOTAL INTEREST 2½%	INCOME	AMORTI- ZATION	BOOK VALUE	PAR
Jan. 1, 1914					\$100,000
July 1, 1914					
Jan. 1, 1915					
July 1, 1915					
Jan. 1, 1916					

Insert values to the nearest cent under the various heads.

(Kansas and Missouri)

37. The Graham Mercantile Company had a fire. It was ascertained that at the time of the fire their merchandise inventory was \$10,000, on which they sustained a loss of \$5000. Their merchandise was insured for \$7000, subject to the co-insurance clause of 80%. What should the insurance company offer them in settlement of this loss?

(Arkansas)

38. A in London, in current account with B of New York, engages an accountant to prepare a statement to be mailed to B from the following data :

DEBITS		CREDITS	
1914		1914	
May 12	£750	June 10	£500
May 30	117	June 30	300
June 12	340		
July 1	150		
Total Debits	£1,357		
Total Credits	800		
Balance	£557		

Find the average due date of the account and the interest at 5% to July 1, 365 days to the year. (New York)

39. A trust company assumes the responsibility of a sinking fund on which it guarantees to pay 3%. The trustee receives \$103,000 deposited to the credit of the sinking fund, and on Jan. 1, 1911, invests on a 3% basis in 100 Clair County Light, Heat, and Power Company 5% \$1000 first mortgage gold bonds, due July 1, 1912, interest payable semiannually. Prepare the accounts of the trust company, showing the invested amount and the amortization of the premiums at the several interest periods. (New York)

40. The duly appraised inventory, dated Jan. 1, 1910, of the estate of John D. Hamilton, deceased, contained the following item:

50 Shares Plumbers' National Bank \$100 each par value, at \$550
 \$27,500

The bank for a period of years had consistently paid semiannual dividends of 12%.

On Feb. 1, 1910, the bank declared its usual 12% semiannual dividend. Wishing to increase its capitalization it paid the dividend in cash and gave its stockholders the option of buying new stock at par to the amount of the dividend.

The executors indorsed and returned the dividend check, receiving in exchange the six shares of new stock.

Thereafter they sold stock as follows:

March	1,	2 shares at \$500 each
	3,	2 shares at \$495 each
	15,	4 shares at \$480 each

The income of the estate was payable to two sisters of the decedent, Helen H. Winter and Sarah H. Samuels, during their lives, and a brother, William B. Hamilton, was the residuary legatee.

Show the effect of the above transactions upon the interest of each of the three beneficiaries. Give your calculations in full.

(Pennsylvania)

41. Three partners contribute capital as follows: X, \$90,000; Y, \$45,000; and Z, \$15,000. They share profits in the proportion of X, 50%; Y, 30%; Z, 20%. X's salary is \$5000, Y's salary is \$3000, and Z's salary is \$2000. At the end of their fiscal period Z

dies. The books are closed and the net assets are ascertained to be \$152,500. X and Y liquidate the firm's assets and distribute the firm's assets quarterly as follows:

First quarter		\$42,410.20	
Second quarter		74,622.30	
Third quarter		31,967.50	\$149,000

Prepare statement of the partners' accounts, showing distribution of the assets and the apportionment of the loss. (New York)

42. A coöperative assessment association begins writing policies on a 5-year endowment plan in 1909, the membership being entitled to participation in the surplus at the end of said period.

The following certificates maturing in 1914 are in force:

MONTHLY ASSESSMENT	CERTIFICATES IN FORCE
\$.50	10
1.00	360
2.00	1515

The net amount to be distributed is \$136,620.60

Determine the dividend due each certificate. (New York)

43. A and B, equal partners in a manufacturing business, admit their factory superintendent, C, as an equal partner with them in the profits, without his furnishing any capital, A and B reserving for themselves any good will which may have accrued to the business.

On Dec. 31, 1912, a balance sheet is drafted and approved by all concerned as follows:

Assets

Real Estate and Plant	\$ 90,000	
Merchandise Inventory	35,000	
Accounts Receivable	25,000	
Bills Receivable	15,000	
Cash and Bank Funds	<u>18,000</u>	\$183,000

Liabilities

Bills Payable	\$10,000	
Accounts Payable	12,500	
A's Account	\$ 4500	
B's Account	4000	
C's Account	<u>2000</u>	10,500

Capital Accounts

A	\$75,000		
B	<u>75,000</u>	<u>150,000</u>	\$183,000

Later the business is sold as a "going" concern and the partnership is dissolved. The purchaser assumes all outside liabilities and pays the sum of \$225,000 cash, of which the real estate and plant is valued at \$120,000. Draft the settlement accounts as between the partners. (Illinois)

44. A and B enter into partnership and will share profits in the proportions indicated by their investments. A furnishes \$25,000 and B furnishes \$15,000, which is invested in lands and buildings \$10,000, merchandise \$30,000. However, before they have actually commenced business, C, realizing that A and B have a promising venture, offers to buy a one-third interest in the business for \$20,000. A agrees to sell provided B will consent to pay him a bonus of \$4000 out of his (B's) share. This B agrees to do and consents to the sale. How should the \$20,000 be divided between A and B, so that the interest of all partners will be equal? (Illinois)

45. Two concerns fail, owing each other money, the amounts of which are included in their respective accounts payable accounts. A summary balance sheet of X, which is accepted as correct, is as follows:

BALANCE SHEET OF FIRM X

Due from Y	\$ 10,000	Due to Y	\$ 40,000
All other assets	180,000	All other liabilities	200,000
Deficit	50,000		
	<u>\$240,000</u>		<u>\$240,000</u>

A summary balance sheet of Y, which is accepted as correct, is as follows:

BALANCE SHEET OF FIRM Y

Due from X	\$ 40,000	Due to X	\$ 10,000
All other assets	160,000	All other liabilities	270,000
Deficit	80,000		
	<u>\$280,000</u>		<u>\$280,000</u>

The court holds that it is unfair to other creditors to allow such firms to strike a balance between their respective accounts and then to settle with the outside creditors on a percentage basis. Accord-

ingly it is necessary to obtain the "ratio of solvency," that is, the percentage (how many cents on the dollar) each concern will be able to pay on the basis of the respective balance sheets. Determine the ratio of solvency for each firm. (New York)

46. A cloak manufacturing concern, turning out but one grade of cloaks, was robbed on the night of April 16, 1923 and forthwith filed a claim under a burglary insurance policy it was carrying.

The proof of the loss filed by the assured contained two items — 300 cloaks valued at \$12,000 and 1000 yards of silk valued at \$1500.

The insurance company, being notified of the loss, immediately ordered an inventory to be taken, which was done on the morning of April 17, 1923 and disclosed the following :

1250 cloaks
6250 yards of cloth
2500 yards of silk

On the same day you were called in by the insurance company to examine the books for the purpose of proving or disproving the claim, when the following information was obtained :

1. That a complete inventory had been taken on January 1, 1923 of the cloaks, cloth, and silk on hand at that date, the inventory sheets of which had subsequently been lost or destroyed. However, the books showed that the total valuation was \$32,675 and the firm's representative assured you that this was correct and that the inventory had been properly valued at cost prices which had not fluctuated since.

2. That the cloth and silk purchases subsequent to January, 1923 had amounted to 18,750 yards of cloth and 5000 yards of silk at average prices of \$.50 and \$1 per yard of each fabric respectively.

3. That 3000 cloaks had been manufactured during the same period, which had consumed 20,000 yards of cloth and 5000 yards of silk, while 4500 cloaks had been sold.

You were further informed that the manufacturing cost was as follows :

Material	\$5.00 per garment
Labor and other expenses	<u>3.50</u> per garment
Total	\$8.50 per garment

Give the gist of your report to your client ; and state what, if any, different case you could have made out for the firm had you been employed by it instead of by the insurance company.

The following problems are taken from examinations given by the *American Institute of Accountants*, and are used by permission of the Institute.

47. A dealer in foreign exchange finds that he has had the following transactions in London exchange during a particular month:

Exchange bought in local market

Jan. 1, 30-day bill, payable in London £300 at 4.75

Jan. 15, Bill due at sight in London £2500 at 4.76

Exchange sold in local market

Jan. 5, Bill due in London at sight £1000 at 4.77

Jan. 20, Cable transfer £2000 at 4.78

Foreign correspondent's draft honored and paid

Jan. 20, Bill at 30 days after sight accepted Dec. 21st £500 at 4.78

State how the balance of the account stands at the close of the month and how much profit or loss has been derived from the transactions. (On January 31 the rate for cable transfers is 4.80.) Is the profit or loss so stated final?

48. A lease has 5 years to run at \$1000 a year payable at the end of each year, with an extension for a further 5 years at \$1200 a year. On a 6% basis what sum should be paid now in lieu of the 10 years' rent? ($v^5 = .7473$)

49. The maximum sum insured under the soldiers' insurance with the government is payable \$57.50 a month for 20 years certain after death or disability. How can the equivalent sum payable in cash be found? Give the correct formula and explain why $12 \times 57.50 \times a_{\overline{20}|}$ differs from the equivalent cash sum. Given $a_{\overline{20}|}$ at $3\frac{1}{2}\% = 14.21$.

50. A corporation wants to retire a debt of \$105,000 bearing 5% interest payable annually. The tenth payment, including interest, is to be \$15,000. The other nine periodical payments are all to include interest and to be of the same amount. Required the amount of each such nine payments. ($1.05^9 = 1.551328$.)

51. You are called upon to state what is the annual sinking fund necessary to redeem a principal sum of \$1,000,000 due 30 years hence, it being assumed that the annual sums set aside are invested at compound interest at 5%. State what computation you would make to arrive at the result desired. You need not work out the computation.

52. In auditing the books of a corporation you find that, in order to provide a sum to redeem a mortgage of \$100,000 falling due at the end of 10 years, a reserve of \$8000 a year has been set aside annually for 3 years, but that, contrary to intention, the company has failed to accumulate interest thereon. Assuming interest at 4% (convertible annually), what should have been the total accumulations to date, and what should now be set aside annually for the next 7 years in order to complete the sinking fund? ($1.04^7 = 1.31593$)

53. A company is issuing \$100,000 of 4% 20-year bonds, which it wishes to pay at maturity by means of a sinking fund, in which equal annual deposits are to be made. The board of directors wishes to assume that this fund will earn $5\frac{1}{2}\%$ interest for the first 5 years, 5% for the next 5 years, and 4% for the last 10 years. What is the annual deposit required?

GIVEN	5½%	5%	4%
$s_{\overline{5} }$	5.581	5.526	5.416
$s_{\overline{10} }$	12.875	12.578	12.006
$(1+i)^5$	1.307	1.276	1.217
$(1+i)^{10}$	1.708	1.629	1.480

54. A \$10,000 5% semiannual coupon bond was bought on a 4% basis, due $1\frac{1}{2}$ years hence. How much did it cost?

55. A bond, bearing interest at 5% a year, payable annually, and repayable in 5 years, with bonus of 10%, is for sale. What price can a purchaser pay who desires to realize 6% on his investment? (v^5 at 6% = .7473)

56. How much could a purchaser who wished to realize 3% on his investment pay for a bond of \$10,000 which had 4 years to run at 5% interest payable yearly, and which was repayable thereafter with a bonus of 10%?

57. A machine costing \$81 is estimated to have a life of 4 years, with a residual value of \$16. Prepare a statement showing the annual charge for depreciation according to each of the following methods:

- (a) Straight line method.
- (b) Constant percentage of diminishing value method.
- (c) Annuity method.

(For convenience in arithmetical calculation assume the rate of interest to be 10%.)

Discuss the significance of each of the methods.

58. A company has acquired machinery, which cost \$100,000, and which it expects to be able to use for 10 years. The scrap value at the end of that time is estimated at \$25,000. A bond issue of \$75,000 due in 10 years, bearing 6% interest and secured by a mortgage on the machinery, was floated at 98 soon after the purchase of the machinery. The trust indenture requires that at the end of each year, before the payment of dividends, a sum shall be set aside and charged against earnings sufficient to provide a sinking fund on a 5% basis for the redemption of the bonds at maturity.

The president of the company is in favor of providing a reserve for depreciation on the machinery by the sinking fund method, using 5% as a basis, although he does not advocate creating a replacement fund for the machinery as well as a sinking fund for the bonds. The treasurer contends that it is not necessary to provide any reserve for depreciation, asserting as his reason that the creation of a sinking fund reserve and a reserve for depreciation would involve a double charge against profits, and, further, that as the sinking fund reserve is obligatory, the depreciation reserve is not required.

(a) Compute the amount of the annual contribution to the sinking fund for the redemption of the bonds.

(b) Set up a table showing the accumulation of the fund, on the assumption that it earned exactly 5%. Also indicate the annual entries for the sinking fund and the sinking fund reserve.

(c) Give your opinion as to whether or not there should be a reserve for depreciation as well as a sinking fund reserve. If the sinking fund is sufficient, what disposition will eventually be made of it? If two reserves are necessary, when and in what manner will they be closed out? ($1.05^{10} = 1.62889463$)

59. A company was organized with \$1,000,000 capital stock, which it placed at par, and \$1,000,000 5% bonds, which it sold at 90, this being a 6% basis. It paid to contractors, etc. for construction \$1,800,000, and this amount of investment ran, on the average, for 1 year before the property was ready for operation. When operation began, the company had therefore paid 1 year's interest on the

issue of bonds. No dividends were paid on the stock. In addition to the sum named above, the company also paid \$10,000 for legal expenses in connection with incorporation and \$5000 for franchise and other fees.

How should the accounts appear when the property is ready for operation?

60. A corporation, having issued its capital stock at par, buys 1000 shares at 95. It later sells 500 of these shares at 98, 300 shares at 85, and 200 shares at 101. Give the journal entries covering these transactions.

How should the items appear on the balance sheet immediately after the purchase of the stock, and immediately after each of the sales?

61. In examining the affairs of a manufacturing concern you find, among the assets, finished goods inventory of \$175,798 and ascertain that included in the above total is the sum of \$50,000 covering goods deposited as collateral to secure notes which are included in the notes payable account. How should you treat this in a statement prepared for credit purposes? Explain why.

62. The Federal estate tax imposes "a tax equal to the sum of the following percentages of the value of the net estate:

"1% of the value of the net estate not in excess of \$50,000;

"2% of the amount by which the net estate exceeds \$50,000 and does not exceed \$150,000;

"3% of the amount by which the net estate exceeds \$150,000 and does not exceed \$250,000."

The rates are further graduated up to 25%.

The net estate is computed by determining the gross estate and making deductions for liabilities, funeral and administration expenses, and an exemption of \$50,000.

The regulations provide that when the decedent was entitled to receive an annuity of a definite amount during a specified number of years and the right constitutes an asset of his estate, the present worth of the annuity, at the time of the decedent's death, shall be computed on a 4% basis and be included in the gross estate.

The regulations further provide that when the decedent possessed a remainder interest in property subject to an estate for a term of

years, and such interest constituted an asset of his estate, the present worth of the remainder interest at the time of his death must be computed on a 4% basis and be included in the gross estate.

George Smith died on January 1, 1921. His estate consisted of the following:

1. Sundry real and personal property valued at \$215,000.
2. An annuity of \$10,000 a year, payable on December 31 for a period of 20 years, the tenth payment of which was received December 31, 1920.
3. A remainder interest in a fund of \$40,000, subject to an estate for 15 years. This estate will terminate on December 31, 1930.
4. Liabilities and expenses totaled \$35,000.

Present value at 4% of \$1 due in 10 years = \$.675564.

Compute the estate tax.

63. A corporation has an issue of \$100,000 of cumulative preferred stock bearing 6% dividends. No dividends have been paid for 2 years. How could you disclose the facts in a balance sheet dated December 31, 1920:

(a) If a dividend of \$12,000 on the preferred stock was declared on December 27, payable January 15, and there is a remaining surplus of \$5000?

(b) If no dividends have been declared, and there is a surplus of \$17,000?

(c) If no dividends have been declared, and there is a surplus of \$4000?

(d) If no dividends have been declared, and there is a deficit of \$7000?

64. A music house sells pianos on the installment plan, receiving monthly payments. Pianos cost \$600 and are sold for \$900. Interest at 6% a year, or $\frac{1}{2}\%$ a month, is charged on unpaid balances of the purchase price. Income each year is computed by taking such a proportion of the total profit as the collections on principal during the year bear to the \$900 selling price. In addition, income is credited with interest collected and accrued during the year.

A piano is sold August 16, 1921. An initial payment of \$50 is received and the purchaser agrees to pay the balance in 30 equal monthly installments, payable on the 16th of each month, beginning September 16, 1921. These equal monthly payments are to be ap-

plied first to the interest accrued during the month and second to the principal. As the interest will decrease each month, the payment will increase.

(a) Compute the monthly payment to be made. (The present value of \$1 due 30 periods hence at $\frac{1}{2}\%$ a period is \$.8610297.)

(b) Set up a table with the following columns:

DATE	PAYMENTS			BALANCE OF PRINCIPAL
	Total	Interest	Principal	

Enter the payments received in August, September, October, November, and December, and show the balance unpaid December 16, 1921.

(c) State the total credit to income during 1921, including profit on sale and interest.

65. In a certain department of a large dry-goods house the purchases for a year were \$30,000. They were in the first place marked up for "selling" purposes to \$45,000. Later additional markups amounting to \$2000 were made and markdowns were also recorded aggregating \$5000. At the end of the fiscal period there were found to be on hand goods of the marked selling value of \$10,000. State how you would arrive at their inventory value for the purpose of closing the books and calculate the amount. Explain fully.

66. A, B, and C formed a partnership. A agreed to furnish \$10,000; B and C each, \$7000. A was to manage the business and receive $\frac{1}{2}$ of the profits; B and C were each to receive $\frac{1}{4}$ of the profits. A supplied merchandise worth \$8500, but no additional cash. B turned over to A, as managing partner, \$9000 cash, and C turned over \$5500. The business was conducted by A for some time, but without keeping exact books. While managing the business, A purchased additional merchandise amounting to \$75,000 and made sales of \$100,000. The cash received and paid out for the partnership was not kept separate from A's personal cash. In order to straighten out matters, B took over the management. He found receivables amounting to \$20,000 and of these he collected \$4500. The merchandise still on hand he sold for \$500. These receipts he deposited in a bank to the credit of the firm. The remainder of the accounts proved worthless. The outstanding accounts payable

amounted to \$2000, of which \$1500 had been incurred in purchasing merchandise and \$500 for expenses. These accounts he paid. A presented vouchers showing that during his management he had paid other expenses of \$2400. By mutual agreement B was held to be entitled to \$100 on account of interest on excess capital contributed and A and C were to be charged \$75 each for shortage in contribution of capital.

(a) Prepare trading and profit and loss accounts and accounts of each of the partners, indicating the final adjustment to be made in closing up the partnership.

(b) Show how the above final adjustment would be modified if A proved to have neither assets nor liabilities outside of the partnership.

67. A, B, and C were in partnership, A's capital being \$90,000 B's \$50,000, and C's \$50,000. Their agreement was to share profits in the following ratio: A, 60%; B, 15%; C, 25%. During the year C withdrew \$10,000. Net losses on the business during the year were \$15,000, and it was decided to close out the business. It was uncertain how much the assets would ultimately yield, although none of them were known to be bad. The partners therefore mutually agreed that as the assets were liquidated, distribution of cash on hand should be made monthly in such a manner as to avoid, as far as feasible, the possibility of paying to one partner cash which he might later have to repay to another. Collections were made as follows: May, \$15,000; June, \$13,000; July, \$52,000. After this no more could be collected. Show the partners' accounts, indicating how the cash was distributed in each installment, the essential feature in the distribution being to observe the agreement given above.

68. Smith, Hill, and Davis form a partnership under an agreement that Smith is to have a salary of \$200, Hill \$150, and Davis \$100 a month respectively. The profits are to be divided in proportion to the amount of business secured by each partner. The partners are to be individually responsible for any direct losses arising from their own business.

They are in business 9 months, at the end of which time their books state as follows;

Smith's sales	\$5310.00
Hill's sales	3100.00
Davis's sales	3200.00
Net profits	2468.50

They then decide to rescind the salary agreement, treating any salary drawn as an advance, but otherwise to divide the profits according to the original agreement.

You find errors during the 9 months' period, namely :

Office furniture charged to operation	\$ 65.00
Funds lent by Davis credited to his salary account	400.00

and open items not entered on the books as follows :

Smith's salary (ninth month)	\$200.00
Hill's salary (ninth month)	150.00
Advertising	27.50
Clerk hire	120.00
Telephone	6.00
Rent	50.00
Stationery	15.00
Accounts receivable, Smith's business, uncollectible . . .	210.00

You find that the sales represent a gross profit of 100% over cost of merchandise.

State the journal entries necessary to readjust the accounts, and prepare a corrected profit and loss account and a statement of the distribution of the profits.

TABLE I

THE COMMON OR BRIGGS

LOGARITHMS

OF THE NATURAL NUMBERS

FROM 1 TO 10000

1-100

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
0	—	20	1.30 103	40	1.60 206	60	1.77 815	80	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	30	1.47 712	50	1.69 897	70	1.84 510	90	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
20	1.30 103	40	1.60 206	60	1.77 815	80	1.90 309	100	2.00 000

1-100

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
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130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
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133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
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136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
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145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
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152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
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163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
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171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
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175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
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177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
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179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
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181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
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183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
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187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
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203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
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214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
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219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
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223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
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229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
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245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
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252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
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256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
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258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
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264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
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269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
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271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
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276	44 091	44 107	44 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
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281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
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283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
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285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
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287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
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293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
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299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
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319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
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326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
327	51 455	51 468	51 481	51 495	51 508	51 521	51 534	51 548	51 561	51 574
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335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
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337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
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348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
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356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
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366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
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392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
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402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
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404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
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407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
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409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
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414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
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419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
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423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
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438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
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442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
444	64 738	64 748	64 758	64 768	64 777	64 787	64 797	64 807	64 816	64 826
445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
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454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
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487	68 753	68 762	68 771	68 780	68 789	68 797	68 806	68 815	68 824	68 833
488	68 842	68 851	68 860	68 869	68 878	68 886	68 895	68 904	68 913	68 922
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499	69 810	69 819	69 827	69 836	69 845	69 854	69 862	69 871	69 880	69 888
No.	0	1	2	3	4	5	6	7	8	9

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527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
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533	72 673	72 681	72 689	72 697	72 705	72 713	72 722	72 730	72 738	72 746
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535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
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537	72 997	73 006	73 014	73 022	73 030	73 038	73 046	73 054	73 062	73 070
538	73 078	73 086	73 094	73 102	73 111	73 119	73 127	73 135	73 143	73 151
539	73 159	73 167	73 175	73 183	73 191	73 199	73 207	73 215	73 223	73 231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	73 320	73 328	73 336	73 344	73 352	73 360	73 368	73 376	73 384	73 392
542	73 400	73 408	73 416	73 424	73 432	73 440	73 448	73 456	73 464	73 472
543	73 480	73 488	73 496	73 504	73 512	73 520	73 528	73 536	73 544	73 552
544	73 560	73 568	73 576	73 584	73 592	73 600	73 608	73 616	73 624	73 632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
546	73 719	73 727	73 735	73 743	73 751	73 759	73 767	73 775	73 783	73 791
547	73 799	73 807	73 815	73 823	73 830	73 838	73 846	73 854	73 862	73 870
548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
549	73 957	73 965	73 973	73 981	73 989	73 997	74 005	74 013	74 020	74 028
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No.	0	1	2	3	4	5	6	7	8	9
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551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	75 282	75 289	75 297	75 305	75 312	75 320	75 328	75 335	75 343	75 351
567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
570	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
574	75 891	75 899	75 906	75 914	75 921	75 929	75 937	75 944	75 952	75 959
575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
580	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
585	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
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590	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
598	77 670	77 677	77 685	77 692	77 699	77 706	77 714	77 721	77 728	77 735
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No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	77 887	77 895	77 902	77 909	77 916	77 924	77 931	77 938	77 945	77 952
602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
609	78 462	78 469	78 476	78 483	78 490	78 497	78 504	78 512	78 519	78 526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	79 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
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631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
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633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
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635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
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649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
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653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
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657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
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660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
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665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
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669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 263	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	89 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
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833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
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851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
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860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	93 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
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901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914*	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
No.	0	1	2	3	4	5	6	7	8	9

COMPOUND INTEREST TABLES

TABLE I. AMOUNT OF 1

$$s = (1 + i)^n$$

<i>n</i>	$\frac{1}{2}$ per cent	1 per cent	$1\frac{1}{4}$ per cent	$1\frac{1}{2}$ per cent	$1\frac{3}{4}$ per cent
1	1.005 0000	1.010 0000	1.012 5000	1.015 0000	1.017 5000
2	1.010 0250	1.020 1000	1.025 1562	1.030 2250	1.035 3062
3	1.015 0751	1.030 3010	1.037 9707	1.045 6784	1.053 4241
4	1.020 1505	1.040 6040	1.050 9453	1.061 3636	1.071 8590
5	1.025 2512	1.051 0100	1.064 0822	1.077 2840	1.090 6166
6	1.030 3775	1.061 5202	1.077 3832	1.093 4433	1.109 7024
7	1.035 5294	1.072 1354	1.090 8505	1.109 8449	1.129 1222
8	1.040 7070	1.082 8567	1.104 4861	1.126 4926	1.148 8818
9	1.045 9106	1.093 6853	1.118 2922	1.143 3900	1.168 9872
10	1.051 1401	1.104 6221	1.132 2708	1.160 5408	1.189 4445
11	1.056 3958	1.115 6684	1.146 4242	1.177 9489	1.210 2598
12	1.061 6778	1.126 8250	1.160 7545	1.195 6182	1.231 4393
13	1.066 9862	1.138 0933	1.175 2640	1.213 5524	1.252 9895
14	1.072 3211	1.149 4742	1.189 9548	1.231 7557	1.274 9168
15	1.077 6827	1.160 9690	1.204 8292	1.250 2321	1.297 2279
16	1.083 0712	1.172 5786	1.219 8896	1.268 9856	1.319 9294
17	1.088 4865	1.184 3044	1.235 1382	1.288 0203	1.343 0281
18	1.093 9289	1.196 1475	1.250 5774	1.307 3406	1.366 5311
19	1.099 3986	1.208 1090	1.266 2096	1.326 9508	1.390 4454
20	1.104 8956	1.220 1900	1.282 0372	1.346 8550	1.414 7782
21	1.110 4201	1.232 3919	1.298 0627	1.367 0578	1.439 5368
22	1.115 9722	1.244 7159	1.314 2885	1.387 5637	1.464 7287
23	1.121 5520	1.257 1630	1.330 7171	1.408 3772	1.490 3615
24	1.127 1598	1.269 7346	1.347 3510	1.429 5028	1.516 4428
25	1.132 7956	1.282 4320	1.364 1929	1.450 9454	1.542 9805
26	1.138 4596	1.295 2563	1.381 2454	1.472 7095	1.569 9827
27	1.144 1518	1.308 2089	1.398 5109	1.494 8002	1.597 4574
28	1.149 8726	1.321 2910	1.415 9923	1.517 2222	1.625 4129
29	1.155 6220	1.334 5039	1.433 6922	1.539 9805	1.653 8576
30	1.161 4001	1.347 8489	1.451 6134	1.563 0802	1.682 8001
31	1.167 2071	1.361 3274	1.469 7585	1.586 5264	1.712 2491
32	1.173 0431	1.374 9407	1.488 1305	1.610 3243	1.742 2135
33	1.178 9083	1.388 6901	1.506 7321	1.634 4792	1.772 7022
34	1.184 8029	1.402 5770	1.525 5663	1.658 9964	1.803 7245
35	1.190 7269	1.416 6028	1.544 6359	1.683 8813	1.835 2897
36	1.196 6805	1.430 7688	1.563 9438	1.709 1395	1.867 4073
37	1.202 6639	1.445 0765	1.583 4931	1.734 7766	1.900 0899
38	1.208 6772	1.459 5272	1.603 2868	1.760 7983	1.933 3384
39	1.214 7206	1.474 1225	1.623 3279	1.787 2102	1.967 1718
40	1.220 7942	1.488 8637	1.643 6195	1.814 0184	2.001 5973
41	1.226 8982	1.503 7524	1.664 1647	1.841 2287	2.036 6253
42	1.233 0327	1.518 7899	1.684 9668	1.868 8471	2.072 2662
43	1.239 1979	1.533 9778	1.706 0288	1.896 8798	2.108 5309
44	1.245 3938	1.549 3176	1.727 3542	1.925 3330	2.145 4302
45	1.251 6208	1.564 8108	1.748 9461	1.954 2130	2.182 9752
46	1.257 8789	1.580 4588	1.770 8080	1.983 5262	2.221 1773
47	1.264 1683	1.596 2634	1.792 9431	2.013 2791	2.260 0479
48	1.270 4892	1.612 2261	1.815 3548	2.043 4783	2.299 5987
49	1.276 8416	1.628 3483	1.838 0468	2.074 1305	2.339 8417
50	1.283 2258	1.644 6318	1.861 0224	2.105 2424	2.380 7889
60	1.348 8502	1.816 6967	2.107 1814	2.443 2198	2.831 8163
70	1.417 8305	2.006 7634	2.385 9000	2.835 4563	3.368 2883
80	1.490 3386	2.216 7152	2.701 4849	3.290 6628	4.006 3919
90	1.566 5547	2.448 6327	3.058 8126	3.818 9485	4.765 3808
100	1.646 6685	2.704 8138	3.463 4043	4.432 0456	5.668 1559

TABLE I. AMOUNT OF 1—*Continued*

$$s = (1 + i)^n$$

<i>n</i>	2 per cent	2½ per cent	2¾ per cent	2⅔ per cent	2½ per cent
1	1.020 0000	1.021 2500	1.022 5000	1.023 7500	1.025 0000
2	1.040 4000	1.042 9516	1.045 5062	1.048 0641	1.050 6250
3	1.061 2080	1.065 1143	1.069 0301	1.072 9556	1.076 8906
4	1.082 4322	1.087 7480	1.093 0833	1.098 4383	1.103 8129
5	1.104 0808	1.110 8626	1.117 6777	1.124 5262	1.131 4082
6	1.126 1624	1.134 4684	1.142 8254	1.151 2337	1.159 6934
7	1.148 6857	1.158 5759	1.168 5390	1.178 5755	1.188 6858
8	1.171 6594	1.183 1956	1.194 8311	1.206 5666	1.218 4029
9	1.195 0926	1.208 3385	1.221 7148	1.235 2226	1.248 8630
10	1.218 9944	1.234 0157	1.249 2034	1.264 5592	1.280 0845
11	1.243 3743	1.260 2386	1.277 3105	1.294 5924	1.312 0867
12	1.268 2418	1.287 0186	1.306 0500	1.325 3390	1.344 8888
13	1.293 0066	1.314 3678	1.335 4361	1.356 8158	1.378 5110
14	1.319 4788	1.342 2981	1.365 4834	1.389 0402	1.412 9738
15	1.345 8683	1.370 8219	1.396 2068	1.422 0299	1.448 2982
16	1.372 7857	1.399 9519	1.427 6215	1.455 8031	1.484 5056
17	1.400 2414	1.429 7009	1.459 7429	1.490 3784	1.521 6183
18	1.428 2462	1.460 0820	1.492 5872	1.525 7749	1.559 6587
19	1.456 8112	1.491 1088	1.526 1704	1.562 0120	1.598 6502
20	1.485 9474	1.522 7948	1.560 5092	1.599 1098	1.638 6164
21	1.515 6663	1.555 1542	1.595 6207	1.637 0887	1.679 5818
22	1.545 9797	1.588 2012	1.631 5221	1.675 9696	1.721 5714
23	1.576 8993	1.621 9505	1.668 2314	1.715 7738	1.764 6107
24	1.608 4372	1.656 4170	1.705 7666	1.756 5235	1.808 7260
25	1.640 6060	1.691 6158	1.744 1463	1.798 2409	1.853 9441
26	1.673 4181	1.727 5627	1.783 3896	1.840 9491	1.900 2927
27	1.706 8865	1.764 2734	1.823 5159	1.884 6716	1.947 8000
28	1.741 0242	1.801 7642	1.864 5450	1.929 4326	1.996 4950
29	1.775 8447	1.840 0517	1.906 4972	1.975 2566	2.046 4074
30	1.811 3616	1.879 1528	1.949 3934	2.022 1690	2.097 5676
31	1.847 5888	1.919 0848	1.993 2548	2.070 1955	2.150 0068
32	1.884 5406	1.959 8653	2.038 1030	2.119 3626	2.203 7569
33	1.922 2314	2.001 5124	2.083 9603	2.169 6975	2.258 8509
34	1.960 6760	2.044 0446	2.130 8494	2.221 2278	2.315 3221
35	1.999 8896	2.087 4805	2.178 7936	2.273 9820	2.373 2052
36	2.039 8873	2.131 8395	2.227 8164	2.327 9890	2.432 5353
37	2.080 6851	2.177 1411	2.277 9423	2.383 2788	2.493 3487
38	2.122 2988	2.223 4053	2.329 1960	2.439 8816	2.555 6824
39	2.164 7448	2.270 6527	2.381 6029	2.497 8288	2.619 5745
40	2.208 0397	2.318 9041	2.435 1890	2.557 1523	2.685 0638
41	2.252 2005	2.368 1808	2.489 9807	2.617 8846	2.752 1904
42	2.297 2445	2.418 5046	2.546 0053	2.680 0594	2.820 9952
43	2.343 1894	2.469 8978	2.603 2904	2.743 7108	2.891 5201
44	2.390 0531	2.522 3832	2.661 8644	2.808 8740	2.963 8081
45	2.437 8542	2.575 9838	2.721 7564	2.875 5847	3.037 9033
46	2.486 6113	2.630 7235	2.782 9959	2.943 8798	3.113 8509
47	2.536 3435	2.686 6263	2.845 6133	3.013 7970	3.191 6971
48	2.587 0704	2.743 7172	2.909 6396	3.085 3747	3.271 4896
49	2.638 8118	2.802 0211	2.975 1065	3.158 6523	3.353 2768
50	2.691 5880	2.861 5641	3.042 0464	3.233 6703	3.437 1087
60	3.281 0308	3.531 2151	3.800 1348	4.089 1674	4.399 7898
70	3.999 5582	4.357 5750	4.747 1414	5.170 9940	5.632 1029
80	4.875 4392	5.377 3161	5.930 1453	6.539 0278	7.209 5678
90	5.943 1331	6.635 6926	7.407 9578	8.268 9874	9.228 8563
100	7.244 6461	8.188 5490	9.254 0463	10.456 6236	11.813 7164

TABLE I. AMOUNT OF 1—Continued

$$s = (1 + i)^n$$

<i>n</i>	3 per cent	3½ per cent	4 per cent	4¼ per cent	4½ per cent
1	1.030 0000	1.035 0000	1.040 0000	1.042 5000	1.045 0000
2	1.060 9000	1.071 2250	1.081 6000	1.086 8062	1.092 0250
3	1.092 7270	1.108 7179	1.124 8640	1.132 9955	1.141 1661
4	1.125 5088	1.147 5230	1.169 8586	1.181 1478	1.192 5186
5	1.159 2741	1.187 6863	1.216 6529	1.231 3466	1.246 1819
6	1.194 0523	1.229 2553	1.265 3190	1.283 6788	1.302 2601
7	1.229 8739	1.272 2793	1.315 9318	1.338 2352	1.360 8618
8	1.266 7701	1.316 8090	1.368 5690	1.395 1102	1.422 1006
9	1.304 7732	1.362 8974	1.423 3118	1.454 4024	1.486 0951
10	1.343 9164	1.410 5988	1.480 2443	1.516 2145	1.552 9694
11	1.384 2339	1.459 9697	1.539 4541	1.580 6536	1.622 8530
12	1.425 7609	1.511 0687	1.601 0322	1.647 8314	1.695 8814
13	1.468 5337	1.563 9561	1.665 0735	1.717 8642	1.772 1961
14	1.512 5897	1.618 6945	1.731 6764	1.790 8734	1.851 9449
15	1.557 9674	1.675 3488	1.800 9435	1.866 9855	1.935 2824
16	1.604 7064	1.733 9860	1.872 9812	1.946 3324	2.022 3702
17	1.652 8476	1.794 6756	1.947 9005	2.029 0516	2.113 3768
18	1.702 4331	1.857 4892	2.025 8165	2.115 2862	2.203 4788
19	1.753 5060	1.922 5013	2.106 8492	2.205 1859	2.307 8603
20	1.806 1112	1.989 7889	2.191 1231	2.298 9063	2.411 7140
21	1.860 2946	2.059 4315	2.278 7681	2.396 6098	2.520 2412
22	1.916 1034	2.131 5116	2.369 9188	2.498 4658	2.633 6520
23	1.973 5865	2.206 1145	2.464 7155	2.604 6505	2.752 1664
24	2.032 7941	2.283 3285	2.563 3042	2.715 3482	2.876 0138
25	2.093 7779	2.363 2450	2.665 8363	2.830 7505	3.005 4345
26	2.156 5913	2.445 9586	2.772 4698	2.951 0574	3.140 6790
27	2.221 2890	2.531 5671	2.883 3686	3.076 4773	3.282 0096
28	2.287 9277	2.620 1720	2.998 7033	3.207 2276	3.429 7000
29	2.356 5655	2.711 8780	3.118 6514	3.343 5348	3.584 0365
30	2.427 2625	2.806 7937	3.243 3975	3.485 6350	3.745 3181
31	2.500 0804	2.905 0315	3.373 1334	3.633 7745	3.913 8574
32	2.575 0828	3.006 7076	3.508 0588	3.788 2099	4.089 9810
33	2.652 3352	3.111 9424	3.648 3811	3.949 2088	4.274 0302
34	2.731 9053	3.220 8603	3.794 3163	4.117 0502	4.466 3615
35	2.813 8624	3.333 5904	3.946 0890	4.292 0243	4.667 3478
36	2.898 2783	3.450 2661	4.103 9326	4.474 4359	4.877 3785
37	2.985 2267	3.571 0254	4.268 0899	4.664 5994	5.096 8605
38	3.074 7835	3.696 0113	4.438 8134	4.862 8449	5.326 2192
39	3.167 0270	3.825 3717	4.616 3660	5.069 5158	5.565 8991
40	3.262 0378	3.959 2597	4.801 0206	5.284 9702	5.816 3645
41	3.359 9899	4.097 8338	4.993 0614	5.509 5815	6.078 1009
42	3.460 6959	4.241 2580	5.192 7839	5.743 7387	6.351 6155
43	3.564 5168	4.389 7020	5.400 4953	5.987 8476	6.637 4382
44	3.671 4523	4.543 3416	5.616 5151	6.242 3311	6.936 1229
45	3.781 5958	4.702 3586	5.841 1757	6.507 6302	7.248 2484
46	3.895 0437	4.866 9411	6.074 8227	6.784 2044	7.574 4196
47	4.011 8950	5.037 2840	6.317 8156	7.072 5331	7.915 2685
48	4.132 2519	5.213 5890	6.570 5282	7.373 1158	8.271 4556
49	4.256 2194	5.396 0646	6.833 3494	7.686 4732	8.643 6711
50	4.383 9060	5.584 9269	7.106 6834	8.013 1483	9.032 6363
60	5.891 6031	7.878 0909	10.519 6274	12.149 6514	14.027 4079
70	7.917 8219	11.112 8253	15.571 6184	18.421 4773	21.784 1356
80	10.640 8906	15.675 7375	23.049 7991	27.930 9104	33.830 0964
90	14.300 4671	22.112 1760	34.119 3333	42.349 2505	52.537 1053
100	19.218 6320	31.191 4080	50.504 9482	64.210 5462	81.588 5180

TABLE I. AMOUNT OF 1—*Continued*

$$s = (1 + i)^n$$

<i>n</i>	4¾ per cent	5 per cent	6 per cent	7 per cent	8 per cent
1	1.047 5000	1.050 0000	1.060 0000	1.070 0000	1.080 0000
2	1.097 2562	1.102 5000	1.123 6000	1.144 9000	1.166 4000
3	1.149 3759	1.157 6250	1.191 0160	1.225 0430	1.259 7120
4	1.203 9713	1.215 5062	1.262 4770	1.310 7960	1.360 4890
5	1.261 1599	1.276 2816	1.338 2256	1.402 5517	1.469 3281
6	1.321 0650	1.340 0956	1.418 5191	1.500 7304	1.586 8743
7	1.383 8156	1.407 1004	1.503 6303	1.605 7815	1.713 8243
8	1.449 5468	1.477 4554	1.593 8481	1.718 1862	1.850 9302
9	1.518 4003	1.551 3282	1.689 4790	1.838 4592	1.999 0046
10	1.590 5243	1.628 8946	1.790 8477	1.967 1514	2.158 9250
11	1.666 0742	1.710 3394	1.898 2986	2.104 8520	2.331 6390
12	1.745 2128	1.795 8563	2.012 1965	2.252 1916	2.518 1701
13	1.828 1104	1.885 6491	2.132 9283	2.409 8450	2.719 6237
14	1.914 9456	1.979 9316	2.260 9040	2.578 5342	2.937 1936
15	2.005 9055	2.078 9282	2.396 5582	2.759 0315	3.172 1691
16	2.101 1860	2.182 8746	2.540 3517	2.952 1638	3.425 9426
17	2.200 9924	2.292 0183	2.692 7728	3.158 8152	3.700 0180
18	2.305 5395	2.406 6192	2.854 3392	3.379 9323	3.996 0195
19	2.415 0526	2.526 9502	3.025 5995	3.616 5275	4.315 7011
20	2.529 7676	2.653 2977	3.207 1355	3.869 6845	4.660 9571
21	2.649 9316	2.785 9626	3.399 5636	4.140 5624	5.033 8337
22	2.775 8034	2.925 2607	3.603 5374	4.430 4017	5.436 5404
23	2.907 6540	3.071 5238	3.819 7497	4.740 5299	5.871 4636
24	3.045 7676	3.225 0999	4.048 9346	5.072 3670	6.341 1807
25	3.190 4415	3.386 3549	4.291 8707	5.427 4326	6.848 4752
26	3.341 9875	3.555 6727	4.549 3830	5.807 3529	7.396 3532
27	3.500 7319	3.733 4563	4.822 3459	6.213 8676	7.988 0615
28	3.667 0167	3.920 1291	5.111 6867	6.648 8384	8.627 1064
29	3.841 2000	4.116 1356	5.418 3879	7.114 2570	9.317 2749
30	4.023 6570	4.321 9424	5.743 4912	7.612 2550	10.062 6569
31	4.214 7807	4.538 0395	6.088 1006	8.145 1129	10.867 6694
32	4.414 9828	4.764 9415	6.453 3867	8.715 2708	11.737 0830
33	4.624 6944	5.003 1885	6.840 5899	9.325 3398	12.676 0496
34	4.844 3674	5.253 3480	7.251 0253	9.978 1135	13.690 1336
35	5.074 4749	5.516 0154	7.686 0868	10.676 5815	14.785 3443
36	5.315 5124	5.791 8161	8.147 2520	11.423 9422	15.968 1718
37	5.567 9993	6.081 4069	8.636 0871	12.223 6181	17.245 6256
38	5.832 4792	6.385 4773	9.154 2524	13.079 2714	18.625 2756
39	6.109 5220	6.704 7512	9.703 5075	13.994 8204	20.115 2977
40	6.399 7243	7.039 9887	10.285 7179	14.974 4578	21.724 5215
41	6.703 7112	7.391 9882	10.902 8610	16.022 6699	23.462 4832
42	7.022 1375	7.761 5876	11.557 0327	17.144 2568	25.339 4819
43	7.355 6890	8.149 6669	12.250 4546	18.344 3548	27.366 6404
44	7.705 0843	8.557 1503	12.985 4819	19.628 4596	29.555 9717
45	8.071 0758	8.985 0078	13.764 6108	21.002 4518	31.920 4494
46	8.454 4519	9.434 2582	14.590 4875	22.472 6234	34.474 0853
47	8.856 0383	9.905 9711	15.465 9167	24.045 7070	37.232 0122
48	9.276 7001	10.401 2696	16.393 8717	25.728 9065	40.210 5731
49	9.717 3434	10.921 3331	17.377 5040	27.529 9300	43.427 4190
50	10.178 9172	11.467 3998	18.420 1543	29.457 0251	46.901 6125
60	16.189 8154	18.679 1859	32.987 6908	57.946 4268	101.257 0637
70	25.750 2954	30.426 4255	59.075 9302	113.989 3922	218.606 4059
80	40.956 4712	49.561 4411	105.795 9935	224.234 3876	471.954 8343
90	65.142 2639	80.730 3650	189.464 5112	441.102 9799	1018.915 0893
100	103.610 3555	131.501 2578	339.302 0835	867.716 3256	2199.761 2563

TABLE II. PRESENT VALUE OF 1

$$v^n = (1 + i)^{-n}$$

<i>n</i>	$\frac{1}{2}$ per cent	1 per cent	$1\frac{1}{4}$ per cent	$1\frac{1}{2}$ per cent	$1\frac{3}{4}$ per cent
1	0.995 0249	0.990 0990	0.987 6543	0.985 2217	0.982 8010
2	0.990 0745	0.980 2960	0.975 4611	0.970 6618	0.965 8978
3	0.985 1488	0.970 5902	0.963 4183	0.956 3170	0.949 2853
4	0.980 2475	0.960 9803	0.951 5243	0.942 1842	0.932 9585
5	0.975 3707	0.951 4657	0.939 7771	0.928 2603	0.916 9125
6	0.970 5181	0.942 0452	0.928 1749	0.914 5422	0.901 1425
7	0.965 6896	0.932 7180	0.916 7159	0.901 0268	0.885 6438
8	0.960 8852	0.923 4832	0.905 3984	0.887 7111	0.870 4116
9	0.956 1047	0.914 3398	0.894 2207	0.874 5922	0.855 4414
10	0.951 3479	0.905 2870	0.883 1809	0.861 6672	0.840 7286
11	0.946 6149	0.896 3237	0.872 2775	0.848 9332	0.826 2689
12	0.941 9053	0.887 4492	0.861 5086	0.836 3874	0.812 0579
13	0.937 2192	0.878 6626	0.850 8727	0.824 0270	0.798 0913
14	0.932 5565	0.869 9630	0.840 3681	0.811 8493	0.784 3649
15	0.927 9169	0.861 3495	0.829 9932	0.799 8515	0.770 8746
16	0.923 3004	0.852 8213	0.819 7464	0.788 0310	0.757 6163
17	0.918 7068	0.844 3775	0.809 6260	0.776 3853	0.744 5860
18	0.914 1362	0.836 0173	0.799 6306	0.764 9116	0.731 7799
19	0.909 5882	0.827 7399	0.789 7587	0.753 6075	0.719 1940
20	0.905 0629	0.819 5445	0.780 0086	0.742 4704	0.706 8246
21	0.900 5601	0.811 4302	0.770 3788	0.731 4980	0.694 6679
22	0.896 0797	0.803 3962	0.760 8680	0.720 6876	0.682 7203
23	0.891 6216	0.795 4418	0.751 4745	0.710 0371	0.670 9782
24	0.887 1857	0.787 5661	0.742 1971	0.699 5439	0.659 4380
25	0.882 7718	0.779 7684	0.733 0341	0.689 2058	0.648 0963
26	0.878 3799	0.772 0480	0.723 9843	0.679 0205	0.636 9497
27	0.874 0099	0.764 4039	0.715 0463	0.668 9857	0.625 9948
28	0.869 6616	0.756 8356	0.706 2185	0.659 0992	0.615 2283
29	0.865 3349	0.749 3422	0.697 4998	0.649 3589	0.604 6470
30	0.861 0297	0.741 9229	0.688 8887	0.639 7624	0.594 2476
31	0.856 7460	0.734 5772	0.680 3839	0.630 3078	0.584 0272
32	0.852 4836	0.727 3041	0.671 9841	0.620 9929	0.573 9825
33	0.848 2424	0.720 1031	0.663 6880	0.611 8157	0.564 1105
34	0.844 0223	0.712 9733	0.655 4943	0.602 7741	0.554 4084
35	0.839 8231	0.705 9142	0.647 4018	0.593 8661	0.544 8731
36	0.835 6449	0.698 9250	0.639 4092	0.585 0897	0.535 5018
37	0.831 4875	0.692 0049	0.631 5152	0.576 4431	0.526 2917
38	0.827 3507	0.685 1534	0.623 7187	0.567 9242	0.517 2400
39	0.823 2346	0.678 3697	0.616 0185	0.559 5313	0.508 3440
40	0.819 1389	0.671 6531	0.608 4133	0.551 2623	0.499 6010
41	0.815 0635	0.665 0031	0.600 9021	0.543 1156	0.491 0083
42	0.811 0085	0.658 4189	0.593 4835	0.535 0892	0.482 5635
43	0.806 9736	0.651 8999	0.586 1566	0.527 1815	0.474 2639
44	0.802 9588	0.645 4455	0.578 9201	0.519 3907	0.466 1070
45	0.798 9640	0.639 0549	0.571 7729	0.511 7149	0.458 0904
46	0.794 9891	0.632 7276	0.564 7140	0.504 1526	0.450 2117
47	0.791 0339	0.626 4630	0.557 7422	0.496 7021	0.442 4685
48	0.787 0984	0.620 2604	0.550 8565	0.489 3617	0.434 8585
49	0.783 1825	0.614 1192	0.544 0558	0.482 1298	0.427 3793
50	0.779 2861	0.608 0388	0.537 3390	0.475 0047	0.420 0288
60	0.741 3722	0.550 4496	0.474 5676	0.409 2960	0.353 1302
70	0.705 3029	0.498 3149	0.419 1290	0.352 6769	0.296 8867
80	0.670 9885	0.451 1179	0.370 1668	0.303 8902	0.249 6011
90	0.638 3435	0.408 3912	0.326 9242	0.261 8522	0.209 8468
100	0.607 2868	0.369 7112	0.288 7333	0.225 6294	0.176 4242

TABLE II. PRESENT VALUE OF 1—Continued

$$v^n = (1 + i)^{-n}$$

<i>n</i>	2 per cent	2½ per cent	2¾ per cent	3 per cent	3½ per cent
1	0.980 3922	0.979 1922	0.977 9951	0.976 8010	0.975 6098
2	0.961 1688	0.958 8173	0.956 4744	0.954 1402	0.951 8144
3	0.942 3223	0.938 8664	0.935 4273	0.932 0050	0.928 5994
4	0.923 8454	0.919 3306	0.914 8434	0.910 3834	0.905 9506
5	0.905 7308	0.900 2013	0.894 7123	0.889 2634	0.883 8543
6	0.887 9714	0.881 4701	0.875 0243	0.868 6334	0.862 2969
7	0.870 5602	0.863 1286	0.855 7695	0.848 4819	0.841 2652
8	0.853 4904	0.845 1688	0.836 9384	0.828 7980	0.820 7466
9	0.836 7553	0.827 5826	0.818 5216	0.809 5707	0.800 7284
10	0.820 3483	0.810 3624	0.800 5101	0.790 7894	0.781 1984
11	0.804 2630	0.793 5006	0.782 8950	0.772 4439	0.762 1448
12	0.788 4932	0.776 9895	0.765 6675	0.754 5239	0.743 5559
13	0.773 0325	0.760 8221	0.748 8190	0.737 0197	0.725 4204
14	0.757 8750	0.744 9910	0.732 3414	0.719 9216	0.707 7272
15	0.743 0147	0.729 4894	0.716 2263	0.703 2201	0.690 4656
16	0.728 4458	0.714 3103	0.700 4658	0.686 9061	0.673 6249
17	0.714 1626	0.699 4470	0.685 0521	0.670 9705	0.657 1951
18	0.700 1594	0.684 8930	0.669 9776	0.655 4047	0.641 1659
19	0.686 4308	0.670 6419	0.655 2348	0.640 1999	0.625 5277
20	0.672 9713	0.656 6873	0.640 8165	0.625 3479	0.610 2709
21	0.659 7758	0.643 0230	0.626 7154	0.610 8404	0.595 3863
22	0.646 8390	0.629 6431	0.612 9246	0.596 6696	0.580 8647
23	0.634 1559	0.616 5416	0.599 4372	0.582 8274	0.566 6972
24	0.621 7215	0.603 7127	0.586 2467	0.569 3064	0.552 8754
25	0.609 5309	0.591 1508	0.573 3464	0.556 0990	0.539 3906
26	0.597 5793	0.578 8502	0.560 7300	0.543 1981	0.526 2347
27	0.585 8620	0.566 8056	0.548 3912	0.530 5964	0.513 3997
28	0.574 3746	0.555 0116	0.536 3239	0.518 2871	0.500 8778
29	0.563 1123	0.543 4630	0.524 5221	0.506 2633	0.488 6612
30	0.552 0709	0.532 1547	0.512 9801	0.494 5185	0.476 7427
31	0.541 2460	0.521 0817	0.501 6920	0.483 0462	0.465 1148
32	0.530 6333	0.510 2392	0.490 6523	0.471 8400	0.453 7706
33	0.520 2287	0.499 6222	0.479 8556	0.460 8937	0.442 7030
34	0.510 0282	0.489 2261	0.469 2964	0.450 2015	0.431 9053
35	0.500 0276	0.479 0464	0.458 9696	0.439 7572	0.421 3711
36	0.490 2232	0.469 0785	0.448 8700	0.429 5553	0.411 0937
37	0.480 6109	0.459 3180	0.438 9927	0.419 5900	0.401 0670
38	0.471 1872	0.449 7606	0.429 3327	0.409 8559	0.391 2849
39	0.461 9482	0.440 4020	0.419 8853	0.400 3477	0.381 7414
40	0.452 8904	0.431 2382	0.410 6458	0.391 0600	0.372 4306
41	0.444 0102	0.422 2651	0.401 6095	0.381 9878	0.363 3470
42	0.435 3041	0.413 4786	0.392 7722	0.373 1261	0.354 4848
43	0.426 7688	0.404 8750	0.384 1292	0.364 4699	0.345 8389
44	0.418 4007	0.396 4505	0.375 6765	0.356 0146	0.337 4038
45	0.410 1968	0.388 2012	0.367 4098	0.347 7554	0.329 1744
46	0.402 1537	0.380 1236	0.359 3250	0.339 6878	0.321 1458
47	0.394 2684	0.372 2140	0.351 4181	0.331 8074	0.313 3129
48	0.386 5376	0.364 4691	0.343 6852	0.324 1098	0.305 6712
49	0.378 9584	0.356 8852	0.336 1224	0.316 5907	0.298 2158
50	0.371 5279	0.349 4592	0.328 7261	0.309 2461	0.290 9422
60	0.304 7823	0.283 1886	0.263 1486	0.244 5486	0.227 2836
70	0.250 0276	0.229 4854	0.210 6531	0.193 3864	0.177 5536
80	0.205 1097	0.185 9664	0.168 6299	0.152 9279	0.138 7046
90	0.168 2614	0.150 7002	0.134 9900	0.120 9338	0.108 3558
100	0.138 0330	0.122 1218	0.108 0608	0.095 6332	0.084 6474

TABLE II. PRESENT VALUE OF 1—Continued

$$v^n = (1+i)^{-n}$$

<i>n</i>	3 per cent	3½ per cent	4 per cent	4¼ per cent	4 ½ per cent
1	0.970 8738	0.966 1836	0.961 5385	0.959 2326	0.956 9378
2	0.942 5959	0.933 5107	0.924 5562	0.920 1272	0.915 7300
3	0.915 1417	0.901 9427	0.888 9964	0.882 6160	0.876 2966
4	0.888 4870	0.871 4422	0.854 8042	0.846 6341	0.838 5613
5	0.862 6088	0.841 9732	0.821 9271	0.812 1190	0.802 4510
6	0.837 4843	0.813 5006	0.790 3145	0.779 0110	0.767 8957
7	0.813 0915	0.785 9910	0.759 9178	0.747 2528	0.734 8285
8	0.789 4092	0.759 4116	0.730 6902	0.716 7893	0.703 1851
9	0.766 4167	0.733 7310	0.702 5867	0.687 5676	0.672 9044
10	0.744 0939	0.708 9188	0.675 5642	0.659 5373	0.643 9277
11	0.722 4213	0.684 9457	0.649 5809	0.632 6497	0.616 1987
12	0.701 3799	0.661 7833	0.624 5970	0.606 8582	0.589 6639
13	0.680 9513	0.639 4042	0.600 5741	0.582 1182	0.564 2716
14	0.661 1178	0.617 7818	0.577 4751	0.558 3868	0.539 9729
15	0.641 8620	0.596 8906	0.555 2645	0.535 6228	0.516 7204
16	0.623 1669	0.576 7059	0.533 9082	0.513 7868	0.494 4693
17	0.605 0164	0.557 2038	0.513 3732	0.492 8411	0.473 1764
18	0.587 3946	0.538 3611	0.493 6281	0.472 7493	0.452 8004
19	0.570 2860	0.520 1557	0.474 6424	0.453 4765	0.433 3018
20	0.553 6758	0.502 5659	0.456 3870	0.434 9894	0.414 6429
21	0.537 5493	0.485 5709	0.438 8336	0.417 2561	0.396 7874
22	0.521 8925	0.469 1506	0.421 9554	0.400 2456	0.379 7009
23	0.506 6918	0.453 2856	0.405 7263	0.383 9287	0.363 3501
24	0.491 9337	0.437 9571	0.390 1215	0.368 2769	0.347 7035
25	0.477 6056	0.423 1470	0.375 1168	0.353 2632	0.332 7306
26	0.463 6947	0.408 8377	0.360 6892	0.338 8616	0.318 4025
27	0.450 1891	0.395 0122	0.346 8166	0.325 0471	0.304 6914
28	0.437 0768	0.381 6543	0.333 4775	0.311 7958	0.291 5707
29	0.424 3464	0.368 7482	0.320 6514	0.299 0847	0.279 0150
30	0.411 9868	0.356 2784	0.308 3187	0.286 8918	0.267 0000
31	0.399 9872	0.344 2304	0.296 4603	0.275 1959	0.255 5024
32	0.388 3370	0.332 5897	0.285 0579	0.263 9769	0.244 4999
33	0.377 0262	0.321 3427	0.274 0942	0.253 2153	0.233 9712
34	0.366 0449	0.310 4760	0.263 5521	0.242 8924	0.223 8959
35	0.355 3834	0.299 9769	0.253 4155	0.232 9903	0.214 2544
36	0.345 0324	0.289 8327	0.243 6687	0.223 4919	0.205 0282
37	0.334 9829	0.280 0316	0.234 2968	0.214 3807	0.196 1992
38	0.325 2262	0.270 5619	0.225 2854	0.205 6409	0.187 7504
39	0.315 7536	0.261 4125	0.216 6206	0.197 2575	0.179 6655
40	0.306 5568	0.252 5725	0.208 2890	0.189 2158	0.171 9287
41	0.297 6280	0.244 0314	0.200 2779	0.181 5020	0.164 5251
42	0.288 9592	0.235 7791	0.192 5749	0.174 1026	0.157 4403
43	0.280 5429	0.227 8059	0.185 1682	0.167 0049	0.150 6605
44	0.272 3718	0.220 1023	0.178 0464	0.160 1966	0.144 1728
45	0.264 4386	0.212 6592	0.171 1984	0.153 6658	0.137 9644
46	0.256 7365	0.205 4679	0.164 6139	0.147 4012	0.132 0233
47	0.249 2588	0.198 5197	0.158 2826	0.141 3921	0.126 3381
48	0.241 9988	0.191 8064	0.152 1948	0.135 6279	0.120 8977
49	0.234 9503	0.185 3202	0.146 3411	0.130 0987	0.115 6916
50	0.228 1071	0.179 0534	0.140 7126	0.124 7949	0.110 7096
60	0.169 7331	0.126 9343	0.095 0604	0.082 3069	0.071 2890
70	0.126 2974	0.089 9861	0.064 2194	0.054 2845	0.045 9050
80	0.093 9771	0.063 7928	0.043 3843	0.035 8026	0.029 5595
90	0.069 9278	0.045 2240	0.029 3089	0.023 6132	0.019 0342
100	0.052 0328	0.032 0601	0.019 8000	0.015 5738	0.012 2566

TABLE II. PRESENT VALUE OF 1—*Continued*

$$v^n = (1 + i)^{-n}$$

<i>n</i>	4¾ per cent	5 per cent	6 per cent	7 per cent	8 per cent
1	0.954 6539	0.952 3810	0.943 3962	0.934 5794	0.925 9259
2	0.911 3641	0.907 0295	0.889 9964	0.873 4387	0.857 3388
3	0.870 0374	0.863 8376	0.839 6193	0.816 2979	0.793 8322
4	0.830 5846	0.822 7025	0.792 0937	0.762 8952	0.735 0298
5	0.792 9209	0.783 5262	0.747 2582	0.712 9862	0.680 5832
6	0.756 9650	0.746 2154	0.704 9605	0.666 3422	0.630 1696
7	0.722 6396	0.710 6813	0.665 0571	0.622 7497	0.583 4904
8	0.689 8708	0.676 8394	0.627 4124	0.582 0091	0.540 2689
9	0.658 5878	0.644 6089	0.591 8985	0.543 9337	0.500 2490
10	0.628 7235	0.613 9132	0.558 3948	0.508 3493	0.463 1935
11	0.600 2134	0.584 6793	0.526 7875	0.475 0928	0.428 8829
12	0.572 9960	0.556 8374	0.496 9694	0.444 0120	0.397 1138
13	0.547 0129	0.530 3214	0.468 8390	0.414 9644	0.367 6979
14	0.522 2080	0.505 0680	0.442 3010	0.387 8172	0.340 4610
15	0.498 5280	0.481 0171	0.417 2651	0.362 4460	0.315 2417
16	0.475 9217	0.458 1115	0.393 6463	0.338 7346	0.291 8905
17	0.454 3405	0.436 2967	0.371 3644	0.316 5744	0.270 2690
18	0.433 7380	0.415 5206	0.350 3438	0.295 8639	0.250 2490
19	0.414 0696	0.395 7340	0.330 5130	0.276 5083	0.231 7121
20	0.395 2932	0.376 8895	0.311 8047	0.258 4190	0.214 5482
21	0.377 3682	0.358 9424	0.294 1554	0.241 5131	0.198 6558
22	0.360 2561	0.341 8499	0.277 5051	0.225 7132	0.183 9405
23	0.343 9199	0.325 5713	0.261 7973	0.210 9469	0.170 3153
24	0.328 3245	0.310 0679	0.246 9786	0.197 1466	0.157 6993
25	0.313 4362	0.295 3028	0.232 9986	0.184 2492	0.146 0179
26	0.299 2231	0.281 2407	0.219 8100	0.172 1955	0.135 2018
27	0.285 6546	0.267 8483	0.207 3680	0.160 9304	0.125 1868
28	0.272 7012	0.255 0936	0.195 6301	0.150 4022	0.115 9137
29	0.260 3353	0.242 9463	0.184 5567	0.140 5628	0.107 3275
30	0.248 5301	0.231 3774	0.174 1101	0.131 3671	0.099 3773
31	0.237 2603	0.220 3595	0.164 2548	0.122 7730	0.092 0160
32	0.226 5014	0.209 8662	0.154 9574	0.114 7411	0.085 2000
33	0.216 2305	0.199 8725	0.146 1862	0.107 2347	0.078 8889
34	0.206 4253	0.190 3548	0.137 9115	0.100 2193	0.073 0453
35	0.197 0647	0.181 2903	0.130 1052	0.093 6629	0.067 6345
36	0.188 1286	0.172 6574	0.122 7408	0.087 5355	0.062 6246
37	0.179 5977	0.164 4356	0.115 7932	0.081 8088	0.057 9857
38	0.171 4537	0.156 6054	0.109 2388	0.076 4569	0.053 6905
39	0.163 6789	0.149 1480	0.103 0555	0.071 4550	0.049 7134
40	0.156 2567	0.142 0457	0.097 2222	0.066 7804	0.046 0309
41	0.149 1711	0.135 2816	0.091 7190	0.062 4116	0.042 6212
42	0.142 4068	0.128 8396	0.086 5274	0.058 3286	0.039 4641
43	0.135 9492	0.122 7044	0.081 6296	0.054 5127	0.036 5408
44	0.129 7844	0.116 8613	0.077 0091	0.050 9464	0.033 8341
45	0.123 8992	0.111 2965	0.072 6501	0.047 6135	0.031 3279
46	0.118 2809	0.105 9967	0.068 5378	0.044 4986	0.029 0073
47	0.112 9173	0.100 9492	0.064 6583	0.041 5875	0.026 8586
48	0.107 7970	0.096 1421	0.060 9984	0.038 8668	0.024 8691
49	0.102 9088	0.091 5639	0.057 5457	0.036 3241	0.023 0269
50	0.098 2423	0.087 2037	0.054 2884	0.033 9478	0.021 3212
60	0.061 7672	0.053 5355	0.030 3143	0.017 2573	0.009 8758
70	0.038 8345	0.032 8662	0.016 9274	0.008 7728	0.004 5744
80	0.024 4162	0.020 1770	0.009 4522	0.004 4596	0.002 1188
90	0.015 3510	0.012 3869	0.005 2780	0.002 2670	0.000 9814
100	0.009 6515	0.007 6045	0.002 9472	0.001 1524	0.000 4546

TABLE III. AMOUNT OF 1 PER ANNUM

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

<i>n</i>	½ per cent	1 per cent	1¼ per cent	1½ per cent	1¾ per cent
1	1.000 0000	1.000 0000	1.000 0000	1.000 0000	1.000 0000
2	2.005 0000	2.010 0000	2.012 5000	2.015 0000	2.017 5000
3	3.015 0250	3.030 1000	3.037 6562	3.045 2250	3.052 8062
4	4.030 1001	4.060 4010	4.075 6270	4.090 9034	4.106 2304
5	5.050 2506	5.101 0050	5.126 5723	5.152 2669	5.178 0894
6	6.075 5019	6.152 0151	6.190 6544	6.229 5509	6.268 7060
7	7.105 8794	7.213 5352	7.268 0376	7.322 9942	7.378 4083
8	8.141 4088	8.285 6706	8.358 8881	8.432 8391	8.507 5304
9	9.182 1158	9.368 5273	9.463 3742	9.559 3317	9.656 4122
10	10.228 0264	10.462 2125	10.581 6664	10.702 7217	10.825 3994
11	11.279 1665	11.566 8347	11.713 9372	11.863 2625	12.014 8439
12	12.335 5624	12.682 5030	12.860 3614	13.041 2114	13.225 1037
13	13.397 2402	13.809 3280	14.021 1159	14.236 8296	14.456 5430
14	14.464 2264	14.947 4213	15.196 3799	15.450 3820	15.709 5325
15	15.536 5475	16.096 8955	16.386 3346	16.682 1378	16.984 4494
16	16.614 2303	17.257 8645	17.591 1638	17.932 3698	18.281 6772
17	17.697 3014	18.430 4431	18.811 0534	19.201 3554	19.601 6066
18	18.785 7879	19.614 7476	20.046 1915	20.489 3757	20.944 6347
19	19.879 7168	20.810 8950	21.296 7689	21.796 7164	22.311 1658
20	20.979 1154	22.019 0040	22.562 9785	23.123 6671	23.701 6112
21	22.084 0110	23.239 1940	23.845 0158	24.470 5221	25.116 3894
22	23.194 4311	24.471 5860	25.143 0785	25.837 5799	26.555 9262
23	24.310 4032	25.716 3018	26.457 3670	27.225 1436	28.020 6549
24	25.431 9552	26.973 4648	27.788 0840	28.633 5208	29.511 0164
25	26.559 1150	28.243 1995	29.135 4351	30.063 0236	31.027 4592
26	27.691 9106	29.525 6315	30.499 6280	31.513 9690	32.570 4397
27	28.830 3702	30.820 8878	31.880 8734	32.986 6785	34.140 4224
28	29.974 5220	32.129 0967	33.279 3843	34.481 4787	35.737 8798
29	31.124 3946	33.450 3877	34.695 3766	35.998 7008	37.363 2927
30	32.280 0166	34.784 8915	36.129 0688	37.538 6814	39.017 1503
31	33.441 4167	36.132 7404	37.580 6822	39.101 7616	40.699 9504
32	34.608 6238	37.494 0678	39.050 4407	40.688 2880	42.412 1996
33	35.781 6669	38.869 0085	40.538 5712	42.298 6123	44.154 4130
34	36.960 5752	40.257 6986	42.045 3033	43.933 0915	45.927 1153
35	38.145 3781	41.660 2756	43.570 8696	45.592 0879	47.730 8398
36	39.336 1050	43.076 8784	45.115 5055	47.275 9692	49.566 1295
37	40.532 7855	44.507 6471	46.679 4493	48.985 1087	51.433 5368
38	41.735 4494	45.952 7236	48.262 9424	50.719 8854	53.333 6236
39	42.944 1267	47.412 2508	49.866 2292	52.480 6837	55.266 9621
40	44.158 8473	48.886 3734	51.489 5571	54.267 8939	57.234 1339
41	45.379 6415	50.375 2371	53.133 1765	56.081 9123	59.235 7312
42	46.606 5397	51.878 9895	54.797 3412	57.923 1410	61.272 3565
43	47.839 5724	53.397 7794	56.482 3080	59.791 9881	63.344 6228
44	49.078 7703	54.931 7572	58.188 3369	61.688 8679	65.453 1537
45	50.324 1642	56.481 0747	59.915 6911	63.614 2010	67.598 5839
46	51.575 7850	58.045 8855	61.664 6372	65.568 4140	69.781 5591
47	52.833 6639	59.626 3443	63.435 4452	67.551 9402	72.002 7364
48	54.097 8322	61.222 6078	65.228 3882	69.565 2193	74.262 7842
49	55.368 8214	62.834 8338	67.043 7431	71.608 6976	76.562 3830
50	56.645 1630	64.463 1822	68.881 7899	73.682 8280	78.902 2247
60	69.770 0305	81.669 6699	88.574 5078	96.214 6517	104.075 2159
70	83.566 1055	100.676 3368	110.871 9978	122.363 7530	135.330 7583
80	98.067 7136	121.671 5217	136.118 7953	152.710 8525	171.793 8242
90	113.310 9358	144.863 2675	164.705 0076	187.929 9004	215.164 6172
100	129.333 6984	170.481 3829	197.072 3420	228.803 0433	266.751 7679

TABLE III. AMOUNT OF 1 PER ANNUM—*Continued*

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

<i>n</i>	2 per cent	2½ per cent	2¾ per cent	3 per cent	3½ per cent
1	1.000 0000	1.000 0000	1.000 0000	1.000 0000	1.000 0000
2	2.020 0000	2.021 2500	2.022 5000	2.023 7500	2.025 0000
3	3.060 4000	3.064 2016	3.068 0062	3.071 8141	3.075 6250
4	4.121 6080	4.129 3158	4.137 0364	4.144 7696	4.152 5156
5	5.204 0402	5.217 0638	5.230 1197	5.243 2079	5.256 3285
6	6.308 1210	6.327 9264	6.347 7974	6.367 7341	6.387 7367
7	7.434 2834	7.462 3948	7.490 6228	7.518 9678	7.547 4302
8	8.582 9690	8.620 9707	8.659 1619	8.697 5433	8.736 1159
9	9.754 6284	9.804 1664	9.853 9930	9.904 1099	9.954 5188
10	10.949 7210	11.012 5049	11.075 7078	11.139 3326	11.203 3818
11	12.168 7154	12.246 5206	12.324 9113	12.403 8917	12.483 4663
12	13.412 0897	13.506 7592	13.602 2218	13.698 4841	13.795 5530
13	14.680 3315	14.793 7778	14.908 2718	15.023 8231	15.140 4418
14	15.973 9382	16.108 1456	16.243 7079	16.380 6389	16.518 9528
15	17.293 4169	17.450 4437	17.609 1913	17.769 6791	17.931 9267
16	18.639 2852	18.821 2656	19.005 3981	19.191 7090	19.380 2248
17	20.012 0710	20.221 2175	20.433 0196	20.647 5121	20.864 7304
18	21.412 3124	21.650 9184	21.892 7625	22.137 8905	22.386 3487
19	22.840 5586	23.111 0004	23.385 3497	23.663 6654	23.946 0074
20	24.297 3698	24.602 1092	24.911 5200	25.225 6774	25.544 6576
21	25.783 3172	26.124 9040	26.472 0292	26.824 7873	27.183 2740
22	27.298 9835	27.680 0582	28.067 6499	28.461 8760	28.862 8559
23	28.844 9632	29.268 2594	29.699 1720	30.137 8455	30.581 4273
24	30.421 8625	30.890 2100	31.367 4034	31.853 6194	32.349 0380
25	32.030 2997	32.546 6269	33.073 1700	33.610 1428	34.157 7639
26	33.670 9057	34.238 2427	34.817 3163	35.408 3837	36.011 7080
27	35.344 3238	35.965 8054	36.600 7059	37.249 3328	37.912 0007
28	37.051 2103	37.730 0788	38.424 2218	39.134 0045	39.859 8008
29	38.792 2345	39.531 8429	40.288 7668	41.063 4371	41.856 2958
30	40.568 0792	41.371 8946	42.195 2640	43.038 6937	43.902 7032
31	42.379 4408	43.251 0474	44.144 6575	45.060 8627	46.000 2707
32	44.227 0296	45.170 1321	46.137 9123	47.131 0582	48.150 2775
33	46.111 5702	47.129 9974	48.176 0153	49.250 4208	50.354 0344
34	48.033 8016	49.131 5099	50.259 9756	51.420 1183	52.612 8853
35	49.994 4776	51.175 5544	52.390 8251	53.641 3461	54.928 2074
36	51.994 3672	53.263 0350	54.569 6186	55.915 3281	57.301 4126
37	54.034 2545	55.394 8745	56.797 4351	58.243 3171	59.733 9479
38	56.114 9396	57.572 0156	59.075 3774	60.626 5959	62.227 2966
39	58.237 2384	59.795 4209	61.404 5733	63.066 4776	64.782 9791
40	60.401 9832	62.066 0736	63.786 1762	65.564 3064	67.402 5535
41	62.610 0228	64.384 9776	66.221 3652	68.121 4587	70.087 6174
42	64.862 2233	66.753 1584	68.711 3459	70.739 3433	72.839 8078
43	67.159 4678	69.171 6630	71.257 3512	73.419 4027	75.660 8030
44	69.502 6571	71.641 5609	73.860 6416	76.163 1135	78.552 3231
45	71.892 7103	74.163 9440	76.522 5060	78.971 9875	81.516 1312
46	74.330 5645	76.739 9278	79.244 2624	81.847 5722	84.554 0344
47	76.817 1758	79.370 6513	82.027 2583	84.791 4520	87.667 8853
48	79.353 5193	82.057 2777	84.872 8716	87.805 2490	90.859 5824
49	81.940 5897	84.800 9948	87.782 5113	90.890 6237	94.131 0720
50	84.579 4014	87.603 0160	90.757 6178	94.049 2760	97.484 3488
60	114.051 5394	119.116 0045	124.450 4349	130.070 2048	135.991 5900
70	149.977 9111	158.003 5281	166.539 6176	175.620 8000	185.284 1142
80	193.771 9578	205.991 3438	219.117 5688	233.222 2217	248.382 7126
90	247.156 6563	265.209 0632	284.798 1256	306.062 6266	329.154 2533
100	312.232 3059	338.284 6605	366.846 5021	398.173 6268	432.548 6540

TABLE III. AMOUNT OF 1 PER ANNUM

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

<i>n</i>	3 per cent	3½ per cent	4 per cent	4¼ per cent	4½ per cent
1	1.000 0000	1.000 0000	1.000 0000	1.000 0000	1.000 0000
2	2.030 0000	2.035 0000	2.040 0000	2.042 5000	2.045 0000
3	3.090 9000	3.106 2250	3.121 6000	3.129 3062	3.137 0250
4	4.183 6270	4.214 9429	4.246 4640	4.262 3018	4.278 1911
5	5.309 1358	5.362 4659	5.416 3226	5.443 4496	5.470 7097
6	6.468 4099	6.550 1522	6.632 9755	6.674 7962	6.716 8917
7	7.662 4622	7.779 4075	7.898 2945	7.958 4750	8.019 1518
8	8.892 3360	9.051 6868	9.214 2263	9.296 7102	9.380 0136
9	10.159 1061	10.368 4958	10.582 7953	10.691 8204	10.802 1142
10	11.463 8793	11.731 3932	12.006 1071	12.146 2228	12.288 2094
11	12.807 7957	13.141 9919	13.486 3514	13.662 4372	13.841 1788
12	14.192 0296	14.601 9616	15.025 8055	15.243 0908	15.464 0318
13	15.617 7904	16.113 0303	16.626 8377	16.890 9222	17.159 9133
14	17.086 3242	17.676 9864	18.291 9112	18.608 7864	18.932 1094
15	18.598 9139	19.295 6809	20.023 5876	20.399 6598	20.784 0543
16	20.156 8813	20.971 0297	21.824 5311	22.266 6453	22.719 3367
17	21.761 5877	22.705 0158	23.697 5124	24.212 9778	24.741 7069
18	23.414 4354	24.499 6913	25.645 4129	26.242 0293	26.855 0837
19	25.116 8684	26.357 1805	27.671 2294	28.357 3156	29.063 5625
20	26.870 3745	28.279 6818	29.778 0786	30.562 5015	31.371 4228
21	28.676 4857	30.269 4707	31.969 2017	32.861 4078	33.783 1368
22	30.536 7803	32.328 9022	34.247 9698	35.258 0176	36.303 3780
23	32.452 8837	34.460 4137	36.617 8886	37.756 4834	38.937 0300
24	34.426 4702	36.666 5282	39.082 6041	40.361 1339	41.689 1963
25	36.459 2643	38.949 8567	41.645 9083	43.076 4821	44.565 2102
26	38.553 0422	41.313 1017	44.311 7446	45.907 2326	47.570 6446
27	40.709 6335	43.759 0602	47.084 2144	48.858 2900	50.711 3236
28	42.930 9225	46.290 6273	49.967 5830	51.934 7673	53.993 3332
29	45.218 8502	48.910 7993	52.966 2863	55.141 9949	57.423 0332
30	47.575 4157	51.622 6773	56.084 9378	58.485 5297	61.007 0697
31	50.002 6782	54.429 4710	59.328 3353	61.971 1647	64.752 3878
32	52.502 7585	57.334 5025	62.701 4687	65.604 9392	68.666 2452
33	55.077 8413	60.341 2100	66.209 5274	69.393 1491	72.756 2263
34	57.730 1765	63.453 1524	69.857 9085	73.342 3580	77.030 2565
35	60.462 0818	66.674 0127	73.652 2249	77.459 4082	81.496 6180
36	63.275 9443	70.007 6032	77.598 3138	81.751 4330	86.163 9658
37	66.174 2226	73.457 8693	81.702 2464	86.225 8690	91.041 3443
38	69.159 4493	77.028 8947	85.970 3363	90.890 4684	96.138 2048
39	72.234 2328	80.724 9060	90.409 1497	95.753 3133	101.464 4240
40	75.401 2597	84.550 2778	95.025 5157	100.822 8291	107.030 3231
41	78.663 2975	88.509 5375	99.826 5363	106.107 7993	112.846 6876
42	82.023 1964	92.607 3713	104.819 5978	111.617 3808	118.924 7885
43	85.483 8923	96.848 6293	110.012 3817	117.361 1195	125.276 4040
44	89.048 4091	101.238 3313	115.412 8770	123.348 9671	131.913 8422
45	92.719 8614	105.781 6729	121.029 3920	129.591 2982	138.849 9651
46	96.501 4572	110.484 0314	126.870 5677	136.098 9283	146.098 2135
47	100.396 5010	115.350 9726	132.945 3904	142.883 1328	153.672 6331
48	104.408 3960	120.388 2566	139.263 2060	149.955 6659	161.587 9016
49	108.540 6478	125.601 8456	145.833 7343	157.328 7817	169.859 3572
50	112.796 8673	130.997 9102	152.667 0837	165.015 2550	178.503 0283
60	163.053 4368	196.516 8829	237.990 6852	262.344 7398	289.497 9540
70	230.594 0637	288.937 8646	364.290 4588	409.917 1129	461.869 6796
80	321.363 0186	419.306 7868	551.244 9768	633.668 4800	729.557 6985
90	443.348 9036	603.205 0276	827.983 3335	972.923 5402	1145.269 0066
100	607.287 7327	862.611 6567	1237.623 7046	1487.306 9707	1790.855 9563

TABLE III. AMOUNT OF 1 PER ANNUM—*Continued*

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

<i>n</i>	4½ per cent	5 per cent	6 per cent	7 per cent	8 per cent
1	1.000 0000	1.000 0000	1.000 0000	1.000 0000	1.000 0000
2	2.047 5000	2.050 0000	2.060 0000	2.070 0000	2.080 0000
3	3.144 7562	3.152 5000	3.183 6000	3.214 9000	3.246 4000
4	4.294 1322	4.310 1250	4.374 6160	4.439 9430	4.506 1120
5	5.498 1034	5.525 6312	5.637 0930	5.750 7390	5.866 6010
6	6.759 2634	6.801 9128	6.975 3185	7.153 2907	7.335 9290
7	8.080 3284	8.142 0084	8.393 8376	8.654 0211	8.922 8034
8	9.464 1440	9.549 1089	9.897 4679	10.259 8026	10.636 6276
9	10.913 6908	11.026 5643	11.491 3160	11.977 9888	12.487 5578
10	12.432 0911	12.577 8925	13.180 7949	13.816 4480	14.486 5625
11	14.022 6154	14.206 7872	14.971 6426	15.7835 993	16.645 4875
12	15.688 6897	15.917 1265	16.869 9412	17.888 4513	18.977 1265
13	17.433 9024	17.712 9828	18.882 1377	20.140 6429	21.495 2966
14	19.262 0128	19.598 6320	21.015 0659	22.550 4879	24.214 9203
15	21.176 9584	21.578 5636	23.275 9699	25.129 0220	27.152 1139
16	23.182 8640	23.657 4918	25.672 5281	27.888 0536	30.324 2830
17	25.284 0500	25.840 3664	28.212 8798	30.840 2173	33.750 2257
18	27.485 0424	28.132 3847	30.905 6526	33.999 0325	37.450 2437
19	29.790 5819	30.539 0039	33.759 9917	37.378 9648	41.446 2632
20	32.205 6345	33.065 9541	36.785 5912	40.995 4923	45.761 9643
21	34.735 4022	35.719 2518	39.992 7267	44.865 1768	50.422 9214
22	37.385 3338	38.505 2144	43.392 2903	49.005 7392	55.456 7552
23	40.161 1371	41.430 4751	46.995 8277	53.436 1409	60.893 2956
24	43.068 7911	44.501 9989	50.815 5774	58.176 6708	66.764 7592
25	46.114 5587	47.727 0988	54.864 5120	63.249 0377	73.105 9400
26	49.305 0002	51.113 4538	59.156 3827	68.676 4704	79.954 4152
27	52.646 9877	54.669 1264	63.705 7657	74.483 8233	87.350 7684
28	56.147 7197	58.402 5828	68.528 1116	80.697 9609	95.338 8298
29	59.814 7363	62.322 7119	73.639 7983	87.346 5293	103.965 9362
30	63.655 9363	66.438 8475	79.058 1862	94.460 7863	113.283 2111
31	67.679 5933	70.760 7899	84.801 6774	102.073 0414	123.345 8680
32	71.894 3740	75.298 8294	90.889 7780	110.218 1543	134.213 5374
33	76.309 3567	80.063 7708	97.343 1647	118.933 4251	145.950 6204
34	80.934 0512	85.066 9594	104.183 7546	128.258 7648	158.626 6701
35	85.778 4186	90.320 3074	111.434 7799	138.236 8784	172.316 8037
36	90.852 8935	95.836 3227	119.120 8667	148.913 4598	187.102 1480
37	96.168 4059	101.628 1389	127.268 1187	160.337 4020	203.070 3198
38	101.736 4052	107.709 5458	135.904 2058	172.561 0202	220.315 9454
39	107.568 8845	114.095 0231	145.058 4581	185.640 2916	238.941 2210
40	113.678 4065	120.799 7742	154.761 9656	199.635 1120	259.056 5187
41	120.078 1308	127.839 7630	165.047 6836	214.609 5698	280.781 0402
42	126.781 8420	135.231 7511	175.950 5446	230.632 2397	304.213 5234
43	133.803 9795	142.993 3387	187.507 5772	247.776 4965	329.583 0053
44	141.159 6685	151.143 0056	199.758 0319	266.120 8512	356.919 6457
45	148.864 7528	159.700 1559	212.743 5138	285.749 3108	386.505 6174
46	156.935 8285	168.685 1637	226.508 1246	306.751 7626	418.426 0668
47	165.390 2804	178.119 4218	241.098 6121	329.224 3860	452.900 1521
48	174.246 3187	188.025 3929	256.564 5288	353.270 0930	490.132 1643
49	183.523 0188	198.426 6626	272.958 4006	378.998 9995	530.342 7374
50	193.240 3622	209.347 9957	290.335 9046	406.528 9295	573.770 1564
60	319.785 5885	353.583 7179	533.128 1809	813.520 3834	1253.213 2958
70	521.058 8495	588.528 5107	967.932 1696	1614.134 1742	2720.080 0738
80	841.188 8678	971.228 8213	1746.599 8914	3189.062 6797	5886.935 4283
90	1350.363 4500	1594.607 3010	3141.075 1872	6287.185 4268	12723.938 6160
100	2160.218 0106	2610.025 1569	5638.368 0586	12381.661 7938	27484.515 7043

TABLE IV. PRESENT VALUE OF 1 PER ANNUM

$$a_n = \frac{1 - v^n}{i}$$

<i>n</i>	½ per cent	1 per cent	1¼ per cent	1½ per cent	1¾ per cent
1	0.995 0249	0.990 0990	0.987 6543	0.985 2217	0.982 8010
2	1.985 0994	1.970 3951	1.963 1154	1.955 8834	1.948 6988
3	2.970 2481	2.940 9852	2.926 5337	2.912 2004	2.897 9840
4	3.950 4957	3.901 9656	3.878 0580	3.854 3846	3.830 9425
5	4.925 8663	4.853 4312	4.817 8350	4.782 6450	4.747 8551
6	5.896 3844	5.795 4765	5.746 0099	5.697 1872	5.648 9976
7	6.862 0740	6.728 1945	6.662 7258	6.598 2140	6.534 6414
8	7.822 9592	7.651 6778	7.568 1243	7.485 9251	7.405 0530
9	8.779 0639	8.566 0176	8.462 3450	8.360 5173	8.260 4943
10	9.730 4119	9.471 3045	9.345 5259	9.222 1846	9.101 2229
11	10.677 0267	10.367 6282	10.217 8034	10.071 1178	9.927 4918
12	11.618 9321	11.255 0775	11.079 3120	10.907 5052	10.739 5497
13	12.556 1513	12.133 7401	11.930 1847	11.731 5322	11.537 6410
14	13.488 7078	13.003 7030	12.770 5528	12.543 3815	12.322 0059
15	14.416 6246	13.865 0525	13.600 5459	13.343 2330	13.092 8805
16	15.339 9250	14.717 8738	14.420 2923	14.131 2640	13.850 4968
17	16.258 6319	15.562 2513	15.229 9183	14.907 6493	14.595 0828
18	17.172 7680	16.398 2686	16.029 5489	15.672 5609	15.326 8627
19	18.082 3562	17.226 0085	16.819 3076	16.426 1684	16.046 0567
20	18.987 4192	18.045 5530	17.599 3161	17.168 6388	16.752 8813
21	19.887 9792	18.856 9831	18.369 6950	17.900 1367	17.447 5492
22	20.784 0590	19.660 3793	19.130 5629	18.620 8244	18.130 2695
23	21.675 6806	20.455 8211	19.882 0374	19.330 8614	18.801 2476
24	22.562 8662	21.243 3873	20.624 2345	20.030 4054	19.460 6856
25	23.445 6380	22.023 1557	21.357 2686	20.719 6112	20.108 7820
26	24.324 0179	22.795 2037	22.081 2530	21.398 6317	20.745 7317
27	25.198 0278	23.559 6076	22.796 2992	22.067 6175	21.371 7264
28	26.067 6894	24.316 4432	23.502 5178	22.726 7167	21.986 9547
29	26.933 0242	25.065 7853	24.200 0176	23.376 0756	22.591 6017
30	27.794 0540	25.807 7082	24.888 9062	24.015 8380	23.185 8493
31	28.650 8000	26.542 2854	25.569 2901	24.646 1458	23.769 8765
32	29.503 2836	27.269 5895	26.241 2742	25.267 1387	24.343 8590
33	30.351 5259	27.989 6926	26.904 9622	25.878 9544	24.907 9695
34	31.195 5482	28.702 6659	27.560 4564	26.481 7285	25.462 3779
35	32.035 3713	29.408 5801	28.207 8582	27.075 5946	26.007 2510
36	32.871 0162	30.107 5050	28.847 2674	27.660 6843	26.542 7528
37	33.702 5037	30.799 5099	29.478 7826	28.237 1274	27.069 0446
38	34.529 8544	31.484 6633	30.102 5013	28.805 0516	27.586 2846
39	35.353 0890	32.163 0330	30.718 5198	29.364 5829	28.094 6286
40	36.172 2279	32.834 6861	31.326 9332	29.915 8452	28.594 2296
41	36.987 2914	33.499 6892	31.927 8352	30.458 9608	29.085 2379
42	37.798 2999	34.158 1081	32.521 3187	30.994 0500	29.567 8014
43	38.605 2735	34.810 0081	33.107 4753	31.521 2316	30.042 0652
44	39.408 2324	35.455 4535	33.686 3954	32.040 6222	30.508 1722
45	40.207 1964	36.094 5084	34.258 1682	32.552 3372	30.966 2626
46	41.002 1855	36.727 2361	34.822 8822	33.056 4898	31.416 4743
47	41.793 2194	37.353 6991	35.380 6244	33.553 1920	31.858 9428
48	42.580 3178	37.973 9595	35.931 4809	34.042 5536	32.293 8013
49	43.363 5003	38.588 0787	36.475 5367	34.524 6834	32.721 1806
50	44.142 7864	39.196 1175	37.012 8757	34.999 6881	33.141 2095
60	51.725 5608	44.955 0384	42.034 5918	39.380 2689	36.963 9855
70	58.939 4176	50.168 5144	46.469 6756	43.154 8718	40.177 9027
80	65.802 3054	54.888 2061	50.386 6571	46.407 3235	42.879 9347
90	72.331 2996	59.160 8815	53.846 0604	49.209 8545	45.151 6104
100	78.542 6448	63.028 8788	56.901 3394	51.624 7037	47.061 4730

TABLE IV. PRESENT VALUE OF 1 PER ANNUM—*Continued*

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

<i>n</i>	2 per cent	2½ per cent	2¾ per cent	2⅔ per cent	2½ per cent
1	0.980 3922	0.979 1922	0.977 9951	0.97 68010	0.975 6098
2	1.941 5609	1.938 0095	1.934 4696	1.93 09411	1.927 4242
3	2.883 8833	2.876 8758	2.869 8969	2.86 29462	2.856 0236
4	3.807 7287	3.796 2065	3.784 7402	3.77 33296	3.761 9742
5	4.713 4595	4.696 4078	4.679 4525	4.66 25930	4.645 8285
6	5.601 4309	5.577 8779	5.554 4768	5.531 2264	5.508 1254
7	6.471 9911	6.441 0065	6.410 2463	6.379 7083	6.349 3906
8	7.325 4814	7.286 1753	7.247 1846	7.208 5063	7.170 1372
9	8.162 2367	8.113 7579	8.065 7062	8.018 0769	7.970 8655
10	8.982 5850	8.924 1204	8.866 2164	8.808 8664	8.752 0639
11	9.786 8480	9.717 6209	9.649 1113	9.581 3102	9.514 2087
12	10.575 3412	10.494 6104	10.414 7788	10.335 8342	10.257 7646
13	11.348 3738	11.255 4325	11.163 5979	11.072 8539	10.983 1850
14	12.106 2488	12.000 4235	11.895 9392	11.792 7755	11.690 9122
15	12.849 2635	12.729 9129	12.612 1655	12.495 9956	12.381 3777
16	13.577 7093	13.444 2231	13.312 6313	13.182 9017	13.055 0027
17	14.291 8719	14.143 6701	13.997 6834	13.853 8722	13.712 1977
18	14.992 0312	14.828 5632	14.667 6611	14.509 2769	14.353 3636
19	15.678 4620	15.499 2051	15.322 8959	15.149 4768	14.978 8913
20	16.351 4333	16.155 8923	15.963 7124	15.774 8247	15.589 1623
21	17.011 2092	16.798 9154	16.590 4278	16.385 6652	16.184 5486
22	17.658 0482	17.428 5585	17.203 3523	16.982 3347	16.765 4132
23	18.292 2041	18.045 1002	17.802 7896	17.565 1621	17.332 1105
24	18.913 9256	18.648 8129	18.389 0362	18.134 6685	17.884 9858
25	19.523 4565	19.239 9636	18.962 3826	18.690 5675	18.424 3764
26	20.121 0358	19.818 8138	19.523 1126	19.233 7656	18.950 6111
27	20.706 8978	20.385 6194	20.071 5038	19.764 3620	19.464 0109
28	21.281 2724	20.940 6310	20.607 8276	20.282 6491	19.964 8887
29	21.844 3847	21.484 0940	21.132 3498	20.788 9124	20.453 5499
30	22.396 4556	22.016 2487	21.645 3298	21.283 4309	20.930 2926
31	22.937 7015	22.537 3305	22.147 0219	21.766 4771	21.395 4074
32	23.468 3348	23.047 5696	22.637 6742	22.238 3171	21.849 1780
33	23.988 5636	23.547 1918	23.117 5298	22.699 2108	22.291 8809
34	24.498 5917	24.036 4179	23.586 8262	23.149 4123	22.723 7863
35	24.998 6193	24.515 4643	24.045 7958	23.589 1695	23.145 1573
36	25.488 8425	24.984 5428	24.494 6658	24.018 7248	23.556 2511
37	25.969 4534	25.443 8607	24.933 6585	24.438 3148	23.957 3181
38	26.440 6406	25.893 6213	25.362 9912	24.848 1707	24.348 6030
39	26.902 5888	26.334 0233	25.782 8765	25.248 5184	24.730 3444
40	27.355 4792	26.765 2615	26.193 5222	25.639 5784	25.102 7750
41	27.799 4894	27.187 5265	26.595 1317	26.021 5662	25.466 1220
42	28.234 7936	27.601 0052	26.987 9039	26.394 6923	25.820 6068
43	28.661 5623	28.005 8802	27.372 0332	26.759 1622	26.166 4457
44	29.079 9631	28.402 3307	27.747 7097	27.115 1768	26.503 8494
45	29.490 1599	28.790 5319	28.115 1195	27.462 9321	26.833 0239
46	29.892 3136	29.170 6555	28.474 4445	27.802 6199	27.154 1696
47	30.286 5820	29.542 8695	28.825 8626	28.134 4272	27.467 4826
48	30.673 1196	29.907 3385	29.169 5478	28.458 5370	27.773 1537
49	31.052 0780	30.264 2238	29.505 6702	28.775 1277	28.071 3695
50	31.423 6059	30.613 6830	29.834 3963	29.084 3738	28.362 3117
60	34.760 8867	33.732 2993	32.748 9528	31.808 4816	30.908 6565
70	37.498 6193	36.259 5088	35.082 0849	33.962 6772	32.897 8570
80	39.744 5136	38.307 4645	36.949 7808	35.666 1923	34.451 8172
90	41.586 9292	39.967 0508	38.444 8902	37.013 3140	35.665 7685
100	43.098 3516	41.311 9173	39.641 7405	38.078 6036	36.614 1053

TABLE IV. PRESENT VALUE OF 1 PER ANNUM—*Continued*

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

<i>n</i>	3 per cent	3½ per cent	4 per cent	4¼ per cent	4½ per cent
1	0.970 8738	0.966 1836	0.961 5385	0.959 2326	0.956 9378
2	1.913 4697	1.899 6943	1.886 0947	1.879 3598	1.872 6678
3	2.828 6114	2.801 6370	2.775 0910	2.761 9758	2.748 9644
4	3.717 0984	3.673 0792	3.629 8952	3.608 6099	3.587 5257
5	4.579 7072	4.515 0524	4.451 8223	4.420 7290	4.389 9767
6	5.417 1914	5.328 5530	5.242 1369	5.199 7400	5.157 8725
7	6.230 2830	6.114 5440	6.002 0547	5.946 9928	5.892 7009
8	7.019 6922	6.873 9555	6.732 7449	6.663 7821	6.595 8861
9	7.786 1089	7.607 6865	7.435 3316	7.351 3497	7.268 7905
10	8.530 2028	8.316 6053	8.110 8958	8.010 8870	7.912 7182
11	9.252 624	9.001 5510	8.760 4767	8.643 5367	8.528 9169
12	9.954 0040	9.663 3343	9.385 0738	9.250 3949	9.118 5808
13	10.634 9553	10.302 7385	9.985 6478	9.832 5131	9.682 8524
14	11.296 0731	10.920 5203	10.563 1229	10.390 8999	10.222 8253
15	11.937 9351	11.517 4109	11.118 3874	10.926 5226	10.739 5457
16	12.561 1020	12.094 1168	11.652 2956	11.440 3095	11.234 0150
17	13.166 1185	12.651 3206	12.165 6688	11.933 1506	11.707 1914
18	13.753 5131	13.189 6817	12.659 2970	12.405 8998	12.159 9918
19	14.323 7991	13.709 8374	13.133 9394	12.859 3764	12.593 2936
20	14.877 4749	14.212 4033	13.590 3263	13.294 3658	13.007 9364
21	15.415 0241	14.697 9742	14.029 1600	13.711 6219	13.404 7239
22	15.936 9166	15.167 1248	14.451 1153	14.111 8675	13.784 4248
23	16.443 6084	15.620 4105	14.856 8417	14.495 7962	14.147 7749
24	16.935 5421	16.058 3676	15.246 9631	14.864 0731	14.495 4784
25	17.413 1477	16.481 5146	15.622 0799	15.217 3363	14.828 2090
26	17.876 8424	16.890 3523	15.982 7692	15.556 1979	15.146 6114
27	18.327 0315	17.285 3645	16.329 5858	15.881 2450	15.451 3028
28	18.764 1082	17.667 0188	16.663 0632	16.193 0407	15.742 8735
29	19.188 4546	18.035 7670	16.983 7146	16.492 1254	16.021 8885
30	19.600 4414	18.392 0454	17.292 0333	16.779 0172	16.288 8885
31	20.000 4285	18.736 2758	17.588 4936	17.054 2131	16.544 3910
32	20.388 7655	19.068 8655	17.873 5515	17.318 1900	16.788 8909
33	20.765 7918	19.390 2082	18.147 6457	17.571 4053	17.022 8621
34	21.131 8367	19.700 6842	18.411 1978	17.814 2977	17.246 7580
35	21.487 2201	20.000 6611	18.664 6132	18.047 2879	17.461 0124
36	21.832 2525	20.290 4938	18.908 2820	18.270 7798	17.666 0406
37	22.167 2354	20.570 5254	19.142 5788	18.485 1605	17.862 2398
38	22.492 4616	20.841 0874	19.367 8642	18.690 8014	18.049 9902
39	22.808 2151	21.102 4999	19.584 4848	18.888 0589	18.229 6557
40	23.114 7720	21.355 0723	19.792 7739	19.077 2747	18.401 5844
41	23.412 4000	21.599 1037	19.993 0518	19.258 7767	18.566 1095
42	23.701 3592	21.834 8828	20.185 6267	19.432 8793	18.723 5498
43	23.981 9021	22.062 6887	20.370 7949	19.599 8843	18.874 2103
44	24.254 2739	22.282 7910	20.548 8413	19.760 0808	19.018 3830
45	24.518 7125	22.495 4503	20.720 0397	19.913 7466	19.156 3474
46	24.775 4491	22.700 9181	20.884 6536	20.061 1478	19.288 3707
47	25.024 7078	22.899 4378	21.042 9361	20.202 5399	19.414 7088
48	25.266 7066	23.091 2442	21.195 1309	20.338 1677	19.535 6065
49	25.501 6569	23.276 5645	21.341 4720	20.468 2664	19.651 2981
50	25.729 7640	23.455 6179	21.482 1846	20.593 0613	19.762 0078
60	27.675 5637	24.944 7341	22.623 4900	21.592 7791	20.638 0220
70	29.123 4214	26.000 3966	23.394 5150	22.252 1303	21.202 1119
80	30.200 7634	26.748 7757	23.915 3918	22.686 9970	21.565 3449
90	31.002 4071	27.279 3156	24.267 2776	22.973 8078	21.799 2408
100	31.598 9053	27.655 4254	24.504 9990	23.162 9702	21.949 8527

TABLE IV. PRESENT VALUE OF 1 PER ANNUM—*Continued*

$$a_n = \frac{1 - v^n}{i}$$

<i>n</i>	4½ per cent	5 per cent	6 per cent	7 per cent	8 per cent
1	0.954 6539	0.952 3810	0.943 3962	0.934 5794	0.925 9259
2	1.866 0181	1.859 4104	1.833 3927	1.808 0182	1.783 2648
3	2.736 0554	2.723 2480	2.673 0120	2.624 3160	2.577 0970
4	3.566 6400	3.545 9505	3.465 1056	3.387 2113	3.312 1268
5	4.359 5609	4.329 4767	4.212 3638	4.100 1974	3.992 7100
6	5.116 5259	5.075 6921	4.917 3243	4.766 5397	4.622 8797
7	5.839 1656	5.786 3734	5.582 3814	5.389 2894	5.206 3701
8	6.529 0363	6.463 2128	6.209 7938	5.971 2985	5.746 6389
9	7.187 6242	7.107 8217	6.801 6923	6.515 2322	6.246 8879
10	7.816 3477	7.721 7349	7.360 0870	7.023 5816	6.710 0814
11	8.416 5610	8.306 4142	7.886 8746	7.498 6744	7.138 9643
12	8.989 5571	8.863 2516	8.383 8439	7.942 6863	7.536 0780
13	9.536 5700	9.393 5730	8.852 6830	8.357 6508	7.903 7759
14	10.058 7780	9.898 6409	9.294 9839	8.745 4680	8.244 2370
15	10.557 3060	10.379 6580	9.712 2490	9.107 9140	8.559 4787
16	11.033 2277	10.837 7696	10.105 8953	9.446 6486	8.851 3692
17	11.487 5682	11.274 0662	10.477 2597	9.763 2230	9.121 6381
18	11.921 3062	11.689 5869	10.827 6035	10.059 0869	9.371 8871
19	12.335 3758	12.085 3209	11.158 1165	10.335 5952	9.603 5992
20	12.730 6690	12.462 2103	11.469 9212	10.594 0143	9.818 1474
21	13.108 0372	12.821 1527	11.764 0766	10.835 5273	10.016 8032
22	13.468 2933	13.163 0026	12.041 5817	11.061 2405	10.200 7437
23	13.812 2132	13.488 5739	12.303 3790	11.272 1874	10.371 0590
24	14.140 5376	13.798 6418	12.550 3575	11.469 3340	10.528 7583
25	14.453 9739	14.093 9446	12.783 3562	11.653 5832	10.674 7762
26	14.753 1970	14.375 1853	13.003 1662	11.825 7787	10.809 9780
27	15.038 8516	14.643 0336	13.210 5341	11.986 7090	10.935 1648
28	15.311 5528	14.898 1273	13.406 1643	12.137 1113	11.051 0785
29	15.571 8881	15.141 0736	13.590 7210	12.277 6741	11.158 4060
30	15.820 4183	15.372 4510	13.764 8312	12.409 0412	11.257 7833
31	16.057 6785	15.592 8105	13.929 0860	12.531 8142	11.349 7994
32	16.284 1800	15.802 6767	14.084 0434	12.646 5553	11.434 9994
33	16.500 4105	16.002 5492	14.230 2296	12.753 7900	11.513 8884
34	16.706 8358	16.192 9040	14.368 1411	12.854 0094	11.586 9337
35	16.903 9005	16.374 1943	14.498 2446	12.947 6723	11.654 5682
36	17.092 0291	16.546 8517	14.620 9871	13.035 2078	11.717 1928
37	17.271 6269	16.711 2873	14.736 7803	13.117 0166	11.775 1785
38	17.443 0805	16.867 8927	14.846 0192	13.193 4735	11.828 8690
39	17.606 7595	17.017 0407	14.949 0747	13.264 9285	11.878 5824
40	17.763 0162	17.159 0864	15.046 2969	13.331 7088	11.924 6133
41	17.912 1873	17.294 3680	15.138 0159	13.394 1204	11.967 2346
42	18.054 5941	17.423 2076	15.224 5433	13.452 4490	12.006 6987
43	18.190 5433	17.545 9120	15.306 1729	13.506 9617	12.043 2395
44	18.320 3277	17.662 7733	15.383 1820	13.557 9081	12.077 0736
45	18.444 2269	17.774 0698	15.455 8321	13.605 5216	12.108 4015
46	18.562 5078	17.880 0665	15.524 3699	13.650 0202	12.137 4088
47	18.675 4251	17.981 0157	15.589 0282	13.691 6076	12.164 2674
48	18.783 2221	18.077 1578	15.650 0266	13.730 4744	12.189 1365
49	18.886 1308	18.168 7217	15.707 5723	13.766 7986	12.212 1634
50	18.984 3731	18.255 9255	15.761 8606	13.800 7463	12.233 4846
60	19.752 2689	18.929 2895	16.161 4277	14.039 1812	12.376 5518
70	20.235 0630	19.342 6766	16.384 5439	14.160 3893	12.442 8196
80	20.538 6070	19.596 4605	16.509 1308	14.222 0054	12.473 5144
90	20.729 4523	19.752 2617	16.578 6994	14.253 3279	12.487 7320
100	20.849 4412	19.847 9102	16.617 5462	14.269 2507	12.494 3176

TABLE V. ANNUITY WHICH 1 WILL BUY

$$\frac{1}{a_n} = \frac{1}{s_n} + i$$

<i>n</i>	½ per cent	1 per cent	1¼ per cent	1½ per cent	1¾ per cent
1	1.005 0000	1.010 0000	1.012 5000	1.015 0000	1.017 5000
2	0.503 7531	0.507 5124	0.509 3944	0.511 2779	0.513 1630
3	0.336 6722	0.340 0221	0.341 7012	0.343 3830	0.345 0675
4	0.253 1328	0.256 2811	0.257 8610	0.259 4448	0.261 0324
5	0.203 0100	0.206 0398	0.207 5621	0.209 0893	0.210 6214
6	0.169 5955	0.172 5484	0.174 0338	0.175 5252	0.177 0226
7	0.145 7285	0.148 6283	0.150 0887	0.151 5562	0.153 0306
8	0.127 8289	0.130 6903	0.132 1331	0.133 5840	0.135 0429
9	0.113 9074	0.116 7404	0.118 1706	0.119 6098	0.121 0581
10	0.102 7706	0.105 5821	0.107 0031	0.108 4342	0.109 8754
11	0.093 6590	0.096 4541	0.097 8684	0.099 2938	0.100 7304
12	0.086 0664	0.088 8488	0.090 2583	0.091 6800	0.093 1138
13	0.079 6422	0.082 4148	0.083 8210	0.085 2404	0.086 6728
14	0.074 1361	0.076 9012	0.078 3052	0.079 7233	0.081 1556
15	0.069 3644	0.072 1238	0.073 5265	0.074 9444	0.076 3774
16	0.065 1894	0.067 9446	0.069 3467	0.070 7651	0.072 1996
17	0.061 5058	0.064 2581	0.065 6602	0.067 0796	0.068 5162
18	0.058 2317	0.060 9820	0.062 3848	0.063 8058	0.065 2449
19	0.055 3025	0.058 0518	0.059 4555	0.060 8785	0.062 3206
20	0.052 6664	0.055 4153	0.056 8204	0.058 2457	0.059 6912
21	0.050 2816	0.053 0308	0.054 4375	0.055 8655	0.057 3146
22	0.048 1138	0.050 8637	0.052 2724	0.053 7033	0.055 1564
23	0.046 1346	0.048 8858	0.050 2967	0.051 7308	0.053 1880
24	0.044 3206	0.047 0735	0.048 4866	0.049 9241	0.051 3856
25	0.042 6519	0.045 4068	0.046 8225	0.048 2634	0.049 7295
26	0.041 1116	0.043 8689	0.045 2873	0.046 7320	0.048 2021
27	0.039 6856	0.042 4455	0.043 8668	0.045 3153	0.046 7908
28	0.038 3617	0.041 1244	0.042 5486	0.044 0011	0.045 4815
29	0.037 1291	0.039 8950	0.041 3223	0.042 7788	0.044 2642
30	0.035 9789	0.038 7481	0.040 1785	0.041 6392	0.043 1298
31	0.034 9030	0.037 6757	0.039 1094	0.040 5743	0.042 0700
32	0.033 8945	0.036 6709	0.038 1079	0.039 5771	0.041 0781
33	0.032 9473	0.035 7274	0.037 1679	0.038 6414	0.040 1478
34	0.032 0559	0.034 8400	0.036 2839	0.037 7619	0.039 2736
35	0.031 2155	0.034 0037	0.035 4511	0.036 9336	0.038 4508
36	0.030 4219	0.033 2143	0.034 6653	0.036 1524	0.037 6751
37	0.029 6714	0.032 4680	0.033 9227	0.035 4144	0.036 9426
38	0.028 9604	0.031 7615	0.033 2198	0.034 7161	0.036 2499
39	0.028 2861	0.031 0916	0.032 5536	0.034 0546	0.035 5940
40	0.027 6455	0.030 4556	0.031 9214	0.033 4271	0.034 9721
41	0.027 0363	0.029 8510	0.031 3206	0.032 8311	0.034 3817
42	0.026 4562	0.029 2756	0.030 7491	0.032 2643	0.033 8206
43	0.025 9032	0.028 7274	0.030 2047	0.031 7246	0.033 2867
44	0.025 3754	0.028 2044	0.029 6856	0.031 2104	0.032 7781
45	0.024 8712	0.027 7050	0.029 1901	0.030 7198	0.032 2932
46	0.024 3889	0.027 2278	0.028 7168	0.030 2512	0.031 8304
47	0.023 9273	0.026 7711	0.028 2641	0.029 8034	0.031 3884
48	0.023 4850	0.026 3338	0.027 8307	0.029 3750	0.030 9657
49	0.023 0609	0.025 9147	0.027 4156	0.028 9648	0.030 5612
50	0.022 6538	0.025 5127	0.027 0176	0.028 5717	0.030 1739
60	0.019 3328	0.022 2444	0.023 7899	0.025 3934	0.027 0534
70	0.016 9666	0.019 9328	0.021 5194	0.023 1724	0.024 8893
80	0.015 1970	0.018 2188	0.019 8465	0.021 5483	0.023 3209
90	0.013 8253	0.016 9031	0.018 5715	0.020 3211	0.022 1476
100	0.012 7319	0.015 8657	0.017 5743	0.019 3706	0.021 2488

TABLE V. ANNUITY WHICH 1 WILL BUY—*Continued*

$$\frac{1}{a|n|} = \frac{1}{s|n|} + i$$

<i>n</i>	2 per cent	2 1/8 per cent	2 1/4 per cent	2 3/8 per cent	2 1/2 per cent
1	1.020 0000	1.021 2500	1.022 5000	1.023 7500	1.025 0000
2	0.515 0495	0.515 9934	0.516 9376	0.517 8822	0.518 8272
3	0.346 7547	0.347 5993	0.348 4446	0.349 2905	0.350 1372
4	0.262 6238	0.263 4209	0.264 2189	0.265 0179	0.265 8179
5	0.212 1584	0.212 9287	0.213 7002	0.214 4729	0.215 2469
6	0.178 5258	0.179 2796	0.180 0350	0.180 7917	0.181 5500
7	0.154 5120	0.155 2552	0.156 0002	0.156 7470	0.157 4954
8	0.136 5098	0.137 2462	0.137 9846	0.138 7250	0.139 4674
9	0.122 5154	0.123 2474	0.123 9817	0.124 7182	0.125 4569
10	0.111 3265	0.112 0559	0.112 7877	0.113 5220	0.114 2588
11	0.102 1779	0.102 9058	0.103 6365	0.104 3699	0.105 1060
12	0.094 5596	0.095 2870	0.096 0174	0.096 7508	0.097 4871
13	0.088 1184	0.088 8460	0.089 5769	0.090 3110	0.091 0483
14	0.082 6020	0.083 3304	0.084 0623	0.084 7977	0.085 5365
15	0.077 8255	0.078 5551	0.079 2885	0.080 0256	0.080 7665
16	0.073 6501	0.074 3814	0.075 1166	0.075 8558	0.076 5990
17	0.069 9698	0.070 7030	0.071 4404	0.072 1820	0.072 9278
18	0.066 7021	0.067 4374	0.068 1772	0.068 9214	0.069 6701
19	0.063 7818	0.064 5194	0.065 2618	0.066 0089	0.066 7606
20	0.061 1567	0.061 8969	0.062 6421	0.063 3922	0.064 1471
21	0.058 7848	0.059 5277	0.060 2757	0.061 0290	0.061 7873
22	0.056 6314	0.057 3771	0.058 1282	0.058 8847	0.059 6466
23	0.054 6681	0.055 4167	0.056 1710	0.056 9309	0.057 6964
24	0.052 8711	0.053 6227	0.054 3802	0.055 1436	0.055 9128
25	0.051 2204	0.051 9752	0.052 7360	0.053 5029	0.054 2759
26	0.049 6992	0.050 4571	0.051 2213	0.051 9919	0.052 7688
27	0.048 2931	0.049 0542	0.049 8219	0.050 5961	0.051 3769
28	0.046 9897	0.047 7540	0.048 5253	0.049 3032	0.050 0879
29	0.045 7784	0.046 5461	0.047 3208	0.048 1026	0.048 8913
30	0.044 6499	0.045 4210	0.046 1993	0.046 9849	0.047 7776
31	0.043 5964	0.044 3708	0.045 1528	0.045 9422	0.046 7390
32	0.042 6106	0.043 3885	0.044 1742	0.044 9674	0.045 7683
33	0.041 6865	0.042 4679	0.043 2572	0.044 0544	0.044 8594
34	0.040 8187	0.041 6035	0.042 3966	0.043 1976	0.044 0068
35	0.040 0022	0.040 7906	0.041 5873	0.042 3923	0.043 2056
36	0.039 2328	0.040 0248	0.040 8252	0.041 6342	0.042 4516
37	0.038 5068	0.039 3022	0.040 1064	0.040 9194	0.041 7409
38	0.037 8206	0.038 6196	0.039 4275	0.040 2444	0.041 0701
39	0.037 1711	0.037 9737	0.038 7854	0.039 6063	0.040 4362
40	0.036 5558	0.037 3619	0.038 1774	0.039 0022	0.039 8362
41	0.035 9719	0.036 7816	0.037 6009	0.038 4297	0.039 2679
42	0.035 4173	0.036 2306	0.037 0536	0.037 8864	0.038 7288
43	0.034 8899	0.035 7068	0.036 5336	0.037 3704	0.038 2169
44	0.034 3879	0.035 2084	0.036 0390	0.036 8797	0.037 7304
45	0.033 9096	0.034 7336	0.035 5680	0.036 4127	0.037 2675
46	0.033 4534	0.034 2810	0.035 1192	0.035 9678	0.036 8268
47	0.033 0179	0.033 8491	0.034 6911	0.035 5436	0.036 4067
48	0.032 6018	0.033 4366	0.034 2823	0.035 1388	0.036 0060
49	0.032 2040	0.033 0423	0.033 8918	0.034 7522	0.035 6235
50	0.031 8232	0.032 6651	0.033 5184	0.034 3827	0.035 2581
60	0.028 7680	0.029 6452	0.030 5353	0.031 4382	0.032 3534
70	0.026 6676	0.027 5790	0.028 5046	0.029 4441	0.030 3971
80	0.025 1607	0.026 1046	0.027 0638	0.028 0378	0.029 0260
90	0.024 0460	0.025 0206	0.026 0113	0.027 0173	0.028 0381
100	0.023 2027	0.024 2061	0.025 2259	0.026 2615	0.027 3119

TABLE V. ANNUITY WHICH 1 WILL BUY—*Continued*

$$\frac{1}{a_{\overline{n}|}} = \frac{1}{s_{\overline{n}|}} + i$$

<i>n</i>	3 per cent	3½ per cent	4 per cent	4¼ per cent	4½ per cent
1	1.030 0000	1.035 0000	1.040 0000	1.042 5000	1.045 0000
2	0.522 6108	0.526 4005	0.530 1961	0.532 0961	0.533 9976
3	0.353 5304	0.356 9342	0.360 3485	0.362 0596	0.363 7734
4	0.269 0270	0.272 2511	0.275 4900	0.277 1150	0.278 7436
5	0.218 3546	0.221 4814	0.224 6271	0.226 2070	0.227 7916
6	0.184 5975	0.187 6682	0.190 7619	0.192 3173	0.193 8784
7	0.160 5064	0.163 5445	0.166 6096	0.168 1522	0.169 7015
8	0.142 4564	0.145 4766	0.148 5278	0.150 0649	0.151 6096
9	0.128 4339	0.131 4460	0.134 4930	0.136 0294	0.137 5745
10	0.117 2305	0.120 2414	0.123 2909	0.124 8301	0.126 3788
11	0.108 0774	0.111 0920	0.114 1490	0.115 6934	0.117 2482
12	0.100 4621	0.103 4840	0.106 5522	0.108 1035	0.109 6662
13	0.094 0295	0.097 0616	0.100 1437	0.101 7034	0.103 2754
14	0.088 5263	0.091 5707	0.094 6690	0.096 2381	0.097 8203
15	0.083 7666	0.086 8251	0.089 9411	0.091 5204	0.093 1138
16	0.079 6108	0.082 6848	0.085 8200	0.087 4102	0.089 0154
17	0.075 9525	0.079 0431	0.082 1985	0.083 8002	0.085 4176
18	0.072 7087	0.075 8168	0.078 9933	0.080 6068	0.082 2369
19	0.069 8139	0.072 9403	0.076 1386	0.077 7643	0.079 4073
20	0.067 2157	0.070 3611	0.073 5818	0.075 2198	0.076 8761
21	0.064 8718	0.068 0366	0.071 2801	0.072 9308	0.074 6006
22	0.062 7474	0.065 9321	0.069 1988	0.070 8623	0.072 5456
23	0.060 8139	0.064 0188	0.067 3091	0.068 9855	0.070 6825
24	0.059 0474	0.062 2728	0.065 5868	0.067 2763	0.068 9870
25	0.057 4279	0.060 6740	0.064 0120	0.065 7145	0.067 4390
26	0.055 9383	0.059 2054	0.062 5674	0.064 2831	0.066 0214
27	0.054 5642	0.057 8524	0.061 2385	0.062 9674	0.064 7195
28	0.053 2932	0.056 6026	0.060 0130	0.061 7549	0.063 5208
29	0.052 1147	0.055 4454	0.058 8799	0.060 6350	0.062 4146
30	0.051 0193	0.054 3713	0.057 8301	0.059 5982	0.061 3915
31	0.049 9989	0.053 3724	0.056 8554	0.058 6365	0.060 4434
32	0.049 0466	0.052 4415	0.055 9486	0.057 7428	0.059 5632
33	0.048 1561	0.051 5724	0.055 1036	0.056 9106	0.058 7445
34	0.047 3220	0.050 7597	0.054 3148	0.056 1347	0.057 9819
35	0.046 5393	0.049 9984	0.053 5773	0.055 4100	0.057 2704
36	0.045 8038	0.049 2842	0.052 8869	0.054 7322	0.056 6058
37	0.045 1116	0.048 6132	0.052 2396	0.054 0974	0.055 9840
38	0.044 4593	0.047 9821	0.051 6319	0.053 5023	0.055 4017
39	0.043 8438	0.047 3878	0.051 0608	0.052 9435	0.054 8557
40	0.043 2624	0.046 8273	0.050 5235	0.052 4184	0.054 3432
41	0.042 7124	0.046 2982	0.050 0174	0.051 9244	0.053 8616
42	0.042 1917	0.045 7983	0.049 5402	0.051 4592	0.053 4087
43	0.041 6981	0.045 3254	0.049 0899	0.051 0207	0.052 9824
44	0.041 2298	0.044 8777	0.048 6645	0.050 6071	0.052 5807
45	0.040 7852	0.044 4534	0.048 2625	0.050 2166	0.052 2020
46	0.040 3625	0.044 0511	0.047 8820	0.049 8476	0.051 8447
47	0.039 9605	0.043 6692	0.047 5219	0.049 4987	0.051 5073
48	0.039 5778	0.043 3065	0.047 1806	0.049 1686	0.051 1886
49	0.039 2131	0.042 9617	0.046 8571	0.048 8561	0.050 8872
50	0.038 8655	0.042 6337	0.046 5502	0.048 5600	0.050 6022
60	0.036 1330	0.040 0886	0.044 2018	0.046 3118	0.048 4543
70	0.034 3366	0.038 4610	0.042 7451	0.044 9395	0.047 1651
80	0.033 1118	0.037 3849	0.041 8141	0.044 0781	0.046 3707
90	0.032 2556	0.036 6578	0.041 2078	0.043 5278	0.045 8732
100	0.031 6467	0.036 1593	0.040 8080	0.043 1724	0.045 5584

TABLE V. ANNUITY WHICH 1 WILL BUY—*Continued*

$$\frac{1}{a_n} = \frac{1}{s_n} + i$$

<i>n</i>	4¾ per cent	5 per cent	6 per cent	7 per cent	8 per cent
1	1.047 5000	1.050 0000	1.060 0000	1.070 0000	1.080 0000
2	0.535 9005	0.537 8049	0.545 4369	0.553 0918	0.560 7692
3	0.365 4897	0.367 2086	0.374 1098	0.381 0517	0.388 0335
4	0.280 3759	0.282 0118	0.288 5915	0.295 2281	0.301 9208
5	0.229 3809	0.230 9748	0.237 3964	0.243 8907	0.250 4564
6	0.195 4451	0.197 0175	0.203 3626	0.209 7958	0.216 3154
7	0.171 2574	0.172 8198	0.179 1350	0.185 5532	0.192 0724
8	0.153 1620	0.154 7218	0.161 0359	0.167 4678	0.174 0148
9	0.139 1280	0.140 6901	0.147 0222	0.153 4865	0.160 0797
10	0.127 9370	0.129 5046	0.135 8680	0.142 3775	0.149 0295
11	0.118 8134	0.120 3889	0.126 7929	0.133 3569	0.140 0763
12	0.111 2402	0.112 8254	0.119 2770	0.125 9020	0.132 6950
13	0.104 8595	0.106 4558	0.112 9601	0.119 6508	0.126 5218
14	0.099 4156	0.101 0240	0.107 5849	0.114 3449	0.121 2968
15	0.094 7211	0.096 3423	0.102 9628	0.109 7946	0.116 8295
16	0.090 6353	0.092 2699	0.098 9521	0.105 8576	0.112 9769
17	0.087 0506	0.088 6991	0.095 4448	0.102 4252	0.109 6294
18	0.083 8834	0.085 5462	0.092 3565	0.099 4126	0.106 7021
19	0.081 0677	0.082 7450	0.089 6209	0.096 7530	0.104 1276
20	0.078 5505	0.080 2426	0.087 1846	0.094 3929	0.101 8522
21	0.076 2891	0.077 9961	0.085 0046	0.092 2890	0.099 8322
22	0.074 2485	0.075 9705	0.083 0456	0.090 4058	0.098 0321
23	0.072 3997	0.074 1368	0.081 2785	0.088 7139	0.096 4222
24	0.070 7187	0.072 4709	0.079 6790	0.087 1890	0.094 9780
25	0.069 1851	0.070 9525	0.078 2267	0.085 8105	0.093 6788
26	0.067 7819	0.069 5643	0.076 9044	0.084 5610	0.092 5071
27	0.066 4944	0.068 2919	0.075 6972	0.083 4257	0.091 4481
28	0.065 3102	0.067 1225	0.074 5926	0.082 3919	0.090 4889
29	0.064 2183	0.066 0455	0.073 5796	0.081 4486	0.089 6185
30	0.063 2095	0.065 0514	0.072 6489	0.080 5864	0.088 8274
31	0.062 2755	0.064 1321	0.071 7922	0.079 7969	0.088 1073
32	0.061 4093	0.063 2804	0.071 0023	0.079 0729	0.087 4508
33	0.060 6046	0.062 4900	0.070 2729	0.078 4081	0.086 8516
34	0.059 8557	0.061 7554	0.069 5984	0.077 7967	0.086 3041
35	0.059 1579	0.061 0717	0.068 9739	0.077 2340	0.085 8033
36	0.058 5068	0.060 4345	0.068 3948	0.076 7153	0.085 3447
37	0.057 8984	0.059 8398	0.067 8574	0.076 2368	0.084 9244
38	0.057 3293	0.059 2842	0.067 3581	0.075 7950	0.084 5389
39	0.056 7964	0.058 7646	0.066 8938	0.075 3868	0.084 1851
40	0.056 2968	0.058 2782	0.066 4615	0.075 0091	0.083 8602
41	0.055 8279	0.057 8223	0.066 0589	0.074 6596	0.083 5615
42	0.055 3876	0.057 3947	0.065 6834	0.074 3359	0.083 2868
43	0.054 9736	0.056 9933	0.065 3331	0.074 0359	0.083 0341
44	0.054 5842	0.056 6162	0.065 0061	0.073 7577	0.082 8015
45	0.054 2175	0.056 2617	0.064 7005	0.073 4996	0.082 5873
46	0.053 8720	0.055 9282	0.064 4148	0.073 2600	0.082 3899
47	0.053 5463	0.055 6142	0.064 1477	0.073 0374	0.082 2080
48	0.053 2390	0.055 3184	0.063 8977	0.072 8307	0.082 0403
49	0.052 9489	0.055 0396	0.063 6636	0.072 6385	0.081 8856
50	0.052 6749	0.054 7767	0.063 4443	0.072 4598	0.081 7429
60	0.050 6271	0.052 8282	0.061 8757	0.071 2292	0.080 7980
70	0.049 4192	0.051 6992	0.061 0331	0.070 6195	0.080 3676
80	0.048 6888	0.051 0296	0.060 5725	0.070 3136	0.080 1699
90	0.048 2405	0.050 6271	0.060 3184	0.070 1590	0.080 0786
100	0.047 9629	0.050 3831	0.060 1774	0.070 0808	0.080 0364

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and twelfths, 12

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Present value of 1 per annum, 296-
299Ratio of decimal and integral
aliquot parts, 15

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fineness, 8

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Grains, vegetables, etc., 25

United States coins, 8

ANSWERS TO

VAN TUYL'S BUSINESS MATHEMATICS

Page 11. — 1. \$57. 2. \$160. 3. \$289. 4. \$82.75. 5. \$10,070.25.
6. \$527.25. 7. \$90.50. 8. \$186. 9. \$320.50. 10. \$96. 11. \$271.
12. \$81.64.

Page 12. — 13. \$1385. 14. \$2033.07. 15. \$1734.75. 16. \$1016.73.
17. \$1360.62. 18. \$1117.65.

Page 13. — 1. \$135. 2. \$127. 3. \$765.50. 4. \$146. 5. \$57.25.
6. \$494. 7. \$116. 8. \$200.50. 9. \$170.42. 10. \$116. 11. \$493.17.
12. \$1047.

Page 14. — 13. \$2244.92. 14. \$2025.50. 15. \$1420.92. 16. \$1197.85.
17. \$1508.88. 18. \$1133.26.

Page 16. — 1. \$5647.30. 2. \$345,625. 3. \$116.33. 4. 2,506,653.13.
5. 41,880.68. 6. 10,535.78. 7. \$59,660. 8. \$4487.50. 9. \$1237.97.

Page 18. — 1. \$41.88. 2. \$137.37. 3. \$37.97. 4. \$100.67.
5. \$173.57. 6. \$513.47. 7. \$466.41. 8. \$759.26. 9. \$418.39.
1. \$360.40. 2. \$240.92. 3. \$623.02.

Page 19. — 4. \$958.12. 5. \$143.72. 6. \$386.85. 7. \$3701.44.
8. \$2550.48. 9. \$688.07. 10. \$149.74. 11. \$101.76. 12. \$321.44.

Page 22. — 13. \$25,690. 14. \$15,670.

Page 24. — 1. \$2.97; \$3.08. 2. \$8.75; \$7.88. 3. \$37.13; \$38.81.
4. \$7; \$7.70. 5. \$59.38; \$64.13. 6. \$79.75; \$84.70. 7. \$100; \$90.
8. \$116.25; \$98.81. 9. \$576.60; \$624.65. 10. \$1357.40; \$1419.10.
11. \$65.63; \$84.38. 12. \$36.25; \$31.90. 13. \$438; \$408.80.
14. \$81.56; \$76.13. 15. \$11.42; \$10.81. 16. \$879.94; \$791.94.
17. \$538.43; \$598.25. 18. \$170.16; \$185.63. 19. \$52.99; \$58.88.
20. \$14.36; \$17.55. 21. \$23.94; \$29.93. 22. \$45.16; \$41.69.
23. \$22.45; \$21.29. 24. \$50.02; \$45.48. 25. \$20.66; \$23.44.
26. \$55.74; \$57.89. 27. \$95.49; \$86.81. 28. \$323.84; \$269.87.
29. \$8.98; \$8.32. 30. \$3.50; \$3.78.

Page 26. — 1. \$6451.78. 2. \$4719.96. 3. \$22,211.86. 4. \$9805.70.

Page 28. — 1. \$26.51. 2. \$68.11. 3. \$4.07.

Page 29. — 4. \$13.89. 5. \$6.67. 6. \$11.48. 7. \$13.92. 8. \$17.78.
9. \$47.48.

Page 32. — 1. \$173.81. 2. \$134.87. 3. \$303.22. 4. \$299.60.
5. \$70.50. 6. \$240.42. 7. \$13.39. 8. \$12.26. 9. \$5.26. 10. \$17.42.

Page 33. —

	COMPOUND TIME	EXACT TIME	BANKERS' TIME		COMPOUND TIME	EXACT TIME	BANKERS' TIME
1.				9.			
Int.	\$25.95	\$26.21	\$26.08	Int.	\$1174.00	\$1178.00	\$1174.00
Amt.	\$824.35	\$824.61	\$824.48	Amt.	\$13,174.00	\$13,178.00	\$13,174.00
2.				10.			
Int.	\$53.13	\$53.76	\$53.13	Int.	\$51.17	\$51.83	\$51.17
Amt.	\$1313.13	\$1313.76	\$1313.13	Amt.	\$1051.17	\$1051.83	\$1051.17
3.				11.			
Int.	\$61.88	\$63.25	\$61.88	Int.	\$349.80	\$352.20	\$349.80
Amt.	\$1436.88	\$1438.25	\$1436.88	Amt.	\$3949.80	\$3952.20	\$3949.80
4.				12.			
Int.	\$61.31	\$62.19	\$61.31	Int.	\$126.70	\$129.50	\$127.40
Amt.	\$1821.31	\$1822.19	\$1821.31	Amt.	\$4326.70	\$4329.50	\$4327.40
5.				13.			
Int.	\$80.65	\$81.57	\$80.65	Int.	\$40.26	\$40.90	\$40.26
Amt.	\$1920.65	\$1921.57	\$1920.65	Amt.	\$467.00	\$467.64	\$467.00
6.				14.			
Int.	\$284.90	\$291.50	\$286.00	Int.	\$28.49	\$28.98	\$28.49
Amt.	\$6884.90	\$6891.50	\$6886.00	Amt.	\$615.99	\$616.48	\$615.99
7.				15.			
Int.	\$140.00	\$142.50	\$140.00	Int.	\$38.54	\$39.03	\$38.54
Amt.	\$7640.00	\$7642.50	\$7640.00	Amt.	\$774.94	\$775.43	\$774.94
8.							
Int.	\$372.00	\$378.67	\$372.00				
Amt.	\$8372.00	\$8378.67	\$8372.00				
	COMPOUND TIME 5%	7%	EXACT TIME 5%	7%	BANKERS' TIME 5%	7%	
16.							
Int.	\$21.62	\$30.27	\$21.85	\$30.58	\$21.73	\$30.43	
Amt.	\$820.02	\$828.67	\$820.25	\$828.98	\$820.13	\$828.83	
17.							
Int.	\$44.28	\$61.99	\$44.80	\$62.72	\$44.28	\$61.99	
Amt.	\$1304.28	\$1321.99	\$1304.80	\$1322.72	\$1304.28	\$1321.99	
18.							
Int.	\$51.56	\$72.19	\$52.71	\$73.79	\$51.56	\$72.19	
Amt.	\$1426.56	\$1447.19	\$1427.71	\$1448.79	\$1426.56	\$1447.19	
19.							
Int.	\$51.09	\$71.52	\$51.82	\$72.55	\$51.09	\$71.52	
Amt.	\$1811.09	\$1831.52	\$1811.82	\$1832.55	\$1811.09	\$1831.52	
20.							
Int.	\$67.21	\$94.10	\$67.98	\$95.17	\$67.21	\$94.10	
Amt.	\$1907.21	\$1934.10	\$1907.98	\$1935.17	\$1907.21	\$1934.10	

	COMPOUND TIME		EXACT TIME		BANKERS' TIME	
	5½ %	6½ %	5½ %	6½ %	5½ %	6½ %
21.						
Int.	\$261.16	\$308.64	\$267.21	\$315.79	\$262.17	\$309.83
Amt.	\$6861.16	\$6908.64	\$6867.21	\$6915.79	\$6862.17	\$6909.83
22.						
Int.	\$128.33	\$151.67	\$130.63	\$154.38	\$128.33	\$151.67
Amt.	\$7628.33	\$7651.67	\$7630.63	\$7654.38	\$7628.33	\$7651.67
23.						
Int.	\$341.00	\$403.00	\$347.11	\$410.22	\$341.00	\$403.00
Amt.	\$8341.00	\$8403.00	\$8347.11	\$8410.22	\$8341.00	\$8403.00
24.						
Int.	\$1076.17	\$1271.83	\$1079.83	\$1276.17	\$1076.17	\$1271.83
Amt.	\$13,076.17	\$13,271.83	\$13,079.83	\$13,276.17	\$13,076.17	\$13,271.83
25.						
Int.	\$46.90	\$55.43	\$47.51	\$56.15	\$46.90	\$55.43
Amt.	\$1046.90	\$1055.43	\$1047.51	\$1056.15	\$1046.90	\$1055.43
26.	7½ %	4 %	7½ %	4 %	7½ %	4 %
Int.	\$437.25	\$233.20	\$440.25	\$234.80	\$437.25	\$233.20
Amt.	\$4037.25	\$3833.20	\$4040.25	\$3834.80	\$4037.25	\$3833.20
27.						
Int.	\$158.38	\$84.47	\$161.88	\$86.33	\$159.25	\$84.93
Amt.	\$4358.38	\$4284.47	\$4361.88	\$4286.33	\$4359.25	\$4284.93
28.						
Int.	\$50.32	\$26.84	\$51.12	\$27.26	\$50.32	\$26.84
Amt.	\$477.06	\$453.58	\$477.86	\$454.00	\$477.06	\$453.58
29.						
Int.	\$35.62	\$19.00	\$36.23	\$19.32	\$35.62	\$19.00
Amt.	\$623.12	\$606.50	\$623.73	\$606.82	\$623.12	\$606.50
30.						
Int.	\$48.17	\$25.69	\$48.79	\$26.02	\$48.17	\$25.69
Amt.	\$784.57	\$762.09	\$785.19	\$762.42	\$784.57	\$762.09

Page 34. — 1. \$163.60. 2. \$152.52.

Page 35. — 3. \$253.21. 4. \$322.65. 5. \$1080.28. 6. \$178.55.

Page 36. — 1. \$15.03. 2. \$41.25. 3. \$13.33. 4. \$35.26. 5. \$100.91.

6. \$96.46. 7. \$66.30. 8. \$65.88. 9. \$48.71. 10. \$170.54.

Page 37. — 1. \$29. 2. \$23.50. 3. \$8.30. 4. \$3.80. 5. \$9.25.

6. \$27. 7. \$86. 8. \$79.50. 9. \$105.33. 10. \$5.50. 11. \$1.65.

12. \$1.48. 13. \$3.55. 14. \$7.36. 15. \$6.18. 16. \$3.67. 17. \$16.65.

18. \$13.20. 19. \$1.42. 20. \$1.52. 21. \$3.50. 22. \$16. 23. \$15.40.

24. \$6.30. 25. \$34.10. 26. \$30. 27. \$22.10. 28. \$18.20. 29. \$3.42.

30. \$4.31. 31. \$.42. 32. \$.40. 33. \$2.80. 34. \$.25. 35. \$400.

36. \$275.10.

Page 40. — 1. \$177.67. 2. \$29.94.

Page 49. — 1. 4,304,608. 2. 4,178,438. 3. 9,013,763. 4. 4,493,181.

5. 7,875,593. 6. 5,810,508. 7. 6,463,083. 8. 6,302,516. 9. 9,543,116.

10. 5,424,034. 11. 4,287,976. 12. 9,043,617. 13. 8,982,303.

14. 13,235,497. 15. 10,468,623.

Page 50. — 1. 173,953. 2. 19,208,217. 3. 19,001,114. 4. 11,265,491.

5. 17,177,652. 6. 23,575. 7. 25,317. 8. 1,765,614. 9. 1,875,819.

10. 22,954,117. 11. 323.26.

Page 51. — 12. 11,958.6958. 13. 9,527,114.3235. 14. 88,919.933.
15. 418.7063499. 16. 881.95. 17. 2%; 59.2%; 33.7%; .3%; 4.8%.

Page 53. — 1. 19. 2. 8. 3. 43. 4. 17. 5. 97. 6. 31. 7. 197.
8. 24. 9. 361. 10. 3.

Page 54. — 1. 60. 2. 90. 3. 120. 4. 840. 5. 900. 6. 975.
7. 5040. 8. 3600. 9. 5760. 10. 12,600.

Page 58. — 1. $3\frac{1}{8}$. 2. $7\frac{1}{2}$. 3. $11\frac{3}{8}$. 4. $115\frac{1}{4}$. 5. $341\frac{1}{2}$.
6. 288.8545. 7. $222\frac{2}{5}$. 8. $269\frac{1}{4}$. 9. $135\frac{1}{5}$. 10. $193\frac{3}{8}$.

Page 62. — 1. $464\frac{4}{5}$. 2. $2156\frac{1}{4}$. 3. $1340\frac{3}{4}$. 4. $1569\frac{9}{10}$. 5. $1210\frac{5}{7}$.
6. $14,357\frac{1}{2}$. 7. $5419\frac{7}{11}$. 8. $2577\frac{1}{7}$. 9. $10,113\frac{1}{2}$. 10. $3513\frac{1}{2}$.
11. $4108\frac{3}{8}$. 12. 5880. 13. $276.96\frac{1}{10}$. 14. 27,173. 15. $148.86\frac{1}{3}$.
16. $2492\frac{7}{8}$. 17. $496\frac{7}{8}$. 18. $963\frac{3}{10}$. 19. $1106\frac{4}{5}$. 20. $1327\frac{3}{8}$.
21. $550\frac{3}{8}$. 22. $42\frac{3}{16}$. 23. $959\frac{3}{8}$. 24. $1571\frac{1}{2}$. 25. $2597\frac{3}{8}$. 26. $2601\frac{5}{8}$.
27. 3782. 28. $1851\frac{1}{18}$. 29. 8125. 30. $1435\frac{1}{2}$.

1. \$19.82. 2. \$21. 3. \$22.36. 4. \$30.52. 5. \$19.44. 6. \$20.97.
7. \$29.12. 8. \$45.66. 9. \$21.74.

Page 63. — 1. \$1071.46 2. \$12.56. 3. \$6905. 4. \$3.70. 5. \$84.04.
6. \$104.

Page 66. — 1. $\frac{4}{31}$. 2. .0125. 3. .082. 4. .00001. 5. .00008.
6. $\frac{1}{84}$. 7. $\frac{3}{28}$. 8. $\frac{1}{10}$. 9. $\frac{5}{8}$. 10. $\frac{1}{24}$. 11. $\frac{3}{52}$. 12. $\frac{4}{66}$. 13. .0025.
14. .000025. 15. .1056. 16. $\frac{3}{4}$. 17. $\frac{3}{8}$. 18. $1\frac{1}{4}$. 19. $\frac{1}{11}$. 20. $\frac{7}{10}$.
21. $2\frac{1}{2}$. 22. $6\frac{2}{3}$. 23. $23\frac{7}{8}$. 24. $71\frac{88}{99}$. 25. $13\frac{66}{101}$. 26. $12\frac{1}{253}$.
27. $42\frac{239}{330}$. 28. $59\frac{5}{9}$. 29. $79\frac{5}{8}$. 30. $86\frac{2}{81}$.

1. $25\frac{3}{8}$. 2. 49. 3. 51. 4. $133\frac{1}{2}$. 5. $65\frac{1}{3}$. 6. 40. 7. 77. 8. 34.
9. $53\frac{1}{2}$. 10. $73\frac{1}{2}$. 11. $111\frac{1}{2}$. 12. 75. 13. 150. 14. 112.5.
15. $59\frac{47}{130}$. 16. $111\frac{1}{78}$. 17. $23\frac{4}{13}$. 18. $37\frac{1}{158}$. 19. $40\frac{37}{172}$.
20. 2. 21. $39\frac{7}{132}$. 22. $62\frac{20}{123}$. 23. $59\frac{53}{102}$. 24. $5.57\frac{9}{61}$. 25. 5.
26. 241. 27. 119,370. 28. 3000. 29. 15,720. 30. 27,450.
31. 500. 32. $.008\frac{1}{3}$. 33. 120. 34. 13.5. 35. 3712.5. 36. $10\frac{1}{3}$.
37. $6\frac{5}{7}$. 38. $7\frac{5}{8}$. 39. $13\frac{7}{8}$. 40. $91\frac{1}{2}$. 41. $61\frac{1}{6}$. 42. $4\frac{22}{33}$. 43. $31\frac{7}{15}$.
44. $6\frac{2}{3}\frac{1}{2}$. 45. $2\frac{81}{121}$. 46. $33.152\frac{1}{2}$. 47. 2.519. 48. $6.391\frac{1}{4}$.
49. 22.525. 50. 1.9. 51. .000004. 52. 250,000. 53. $4\frac{23}{450}$. 54. $16\frac{2}{150}$.

Page 69. — 1. (1) \$25.20; (2) \$27.75; (3) \$31.20; (4) \$28.52; (5) \$20.25;
(6) \$28.50; (7) \$22; (8) \$32; Total, \$215.42. 2. Addison, James, \$36.40;
Buell, Henry, \$34.78; Crane, John, \$34.78; Crawford, William, \$35.25;
Jones, Albert, \$31.22; Landis, Frank, \$27.34; Total, \$199.77.

Page 70. — 3. Total time, 65 hr. 20 min. Total wages, \$35.93.
4. Bigelow, A. E., \$49.92; Arnold, A. W., \$43.31; Babcock, C. D., \$43.31;
Comstock, L. O., \$43.97; Evans, M. R., \$42.66; Geraghty, P. O., \$44.63;
Morrow, G. W., \$42.33; Peters, A. B., \$36.84; Rogers, H. F., \$37.41; Simler,
H., \$36; Watson, F. G., \$39.94; Whiteley, I., \$38.25; Total, \$498.57.

Page 71. — 5. \$31.88.

Page 72. — 1. Brown, John, \$27.39; Carey, Leon, \$24.73; Dunne, Theo.,
\$21.52; Evers, Mark, \$32.38.

Page 76. — 1. \$2174.80, 1st year; \$2664.13, 2d year. 2. \$2.25. 3. \$2.92.

Page 77. — 4. \$2.40 5. \$81. 6. \$24.80. 7. \$3.75. 8. \$562.50;
\$1312.50. 9. $2\frac{2}{3}$ hr. 10. 32 additional men. 11. Overcoat, \$52.50;
hat, \$7.50; suit, \$60. 12. 56 additional men.

Page 81. — 1. 1,322,831; 1,076,560; 928,796; 1,252,468.

Page 82. — 2. Butter production 256% greater. 3. From Atlantic to Pacific 39.33%; from Pacific to Atlantic 60.67%. 4. 36.61%. 5. 1918, 48,832,684; 1919, 48,581,684. 6. 7,035,981. 7. 16%. 8. \$14.96.

Page 83. — 9. \$22,947.89. 10. 150 doz. 11. \$16,825. 12. \$29,145.

Page 87. — 1. \$391.20. 2. \$603.29. 3. \$2316.52. 4. \$22,107.10.

Page 88. — 5. Roll-top desks, \$33.75; flat-top desks, \$18.75; china closets, \$28.13; sideboards, \$46.88; dining tables, \$18.75; dining chairs, \$15 a set. 6. \$66.97; smoothing planes, \$1; screw drivers, \$.45; carriage bolts, $\frac{3}{8}'' \times 2''$, \$3.73 per C; $\frac{3}{8}'' \times 2\frac{1}{2}''$, \$3.87 per C; $\frac{1}{2}'' \times 4\frac{1}{2}''$, \$4.17 per C; machine bolts, $\frac{1}{4}'' \times 6''$, \$5.04 per C. 7. \$50. 8. \$42. 9. \$9.

Page 89. — 10. \$.33; \$1.11; \$.78; \$1.33; \$1.17. 11. 20%. 12. \$132. 13. 5%. 14. $5\frac{5}{8}\%$ gain.

Page 91. — 1. (a) \$32.72; (b) \$40.90.

Page 92. — 2. 65%. 3. \$3, S. P. 4. \$60. 5. 43.9%. 6. \$.22. 7. \$4.68. 8. 11%. 9. \$400,000.

Page 93. — 10. (a) Sales, 172.73%; discount, 5.18%; net sales, 167.55%; cost of sales, 100%; gross selling profit, 67.55%; selling expense, 21.59%; net profit on sales, 45.96%; general expense, 14.34%; net profit, 31.62%; other income, 5.70%; total income, 37.32%; allowances, etc. 7.78%; profit and loss surplus, 29.54%; (b) sales, 100%; discount, 3%; net sales, 97%; cost of sales, 57.89%; gross selling profit, 39.11%; selling expense, 12.50%; net profit on sales, 26.61%; general expense, 8.30%; net profit, 18.31%; other income, 3.30%; total income, 21.61%; allowances, etc. 4.50%; profit and loss surplus, 17.11%. 11. \$2.40. 12. $33\frac{1}{3}\%$. 13. \$4. 14. \$52. 15. \$28.80.

Page 94. — 16. 17.7%.

1. 1-29 da. 4.7%; 30-59 da. 19.4%; 60-79 da. 57.7%; 80 da. or more 18.2%; total 100%. 2. \$.30 3. \$3500, cost of first lot; \$4200, cost of second lot. 4. \$25, cost of pineapples; \$60, cost of oranges. 5. 34.4%. 6. \$40.

Page 95. — 7. 16.4%, gain. 8. 10.5%, loss. 9. $6\frac{2}{3}\%$. 10. \$.50. 11. \$135, gain. 12. $11\frac{1}{2}\%$, gain. 13. 12.2%, C.D.B.; 14.2%, net profit on sales.

Page 96. — 14. 3.5% rent; .6% fuel, heat, and light; 9.8% salaries; 2% delivery expense; .3% insurance; .6% general expense; 1.1% depreciation; .5% bad debts; 18.4% C.D.B. 15. 50%.

16.

INDUSTRY	NO. RE- PORTED	NO. EM- PLOYED	AGGREG. DAYS EMPLOYED	AGGREGATE WAGES	PER CENT EMP.	AV. DAYS EMP.	AV. DAILY WAGE	AV. EARNING PER MAN
Building	116,755	100,304	5,376,294	\$20,161,104	85.9	53.6	\$3.75	\$201.00
Transpor'n	60,367	53,549	4,396,373	12,155,623	88.7	82.1	2.76	226.60
Clothing and textiles	46,481	45,353	2,820,957	7,392,539	97.6	62.2	2.62	162.96
Metals and machinery	30,428	29,850	2,092,485	6,537,150	98.1	70.1	3.12	218.71
Printing and binding	23,995	23,216	1,694,768	5,943,946	96.8	73.	3.51	256.23

Page 99. — 1. \$1766.40. 2. \$1621.41. 3. \$1982.54. 4. \$3641.41.
5. \$8452.57. 6. \$5878.37. 7. \$4658.96. 8. \$5946. 9. \$4471.87.
10. \$405.07.

Page 100. — 11. \$4945. 12. \$2468.95 13. \$1736.35. 14. \$4321.39.
15. \$2562.30. 16. \$924.56. 17. \$412.96. 18. \$1399.98. 19. \$2174.75.
20. \$950.12. 21. \$3146.30. 22. \$5099.88. 23. \$6394.22. 24. \$99.26.

Page 101. — 25. \$986.84. 26. \$.25. 27. \$2705.85. 28. \$15,381.91.
29. \$2542.05. 30. \$222.67.

Page 102. — 31. \$1354.07. 32. \$12,195.15.

1. \$570, gain. 2. \$1915.91. 3. \$368.40. 4. 11.7%. 5. \$75.

Page 103. — 6. $8\frac{1}{2}\%$. 7. \$496.03. 8. 15% on gross amount; $15\frac{1}{5}\%$ of net amount. 9. $21\frac{1}{5}\%$ on net amount; \$156.53. 10. Cash terms; 4% — 30; 5% — 30; 30 da. 11. \$1319.21.

Page 104. — 12. \$6882.35; \$565.57. 13. \$903.02. 14. \$1040.81.
15. 34.23%; 19.77%; 21.18%. 16. Demand note \$10.80 better.

Page 105. — 17. \$2757.64. 18. \$923.36. 19. \$4511.48. 20. \$2447.88 better to borrow at the bank, and pay cash.

Page 106. — 21. \$332.34 worse off by discounting 18 mo. note.

Page 107. — 1. May 22. 2. July 10. 3. Aug. 30.

Page 108. — 4. May 13. 5. Aug. 27. 6. Nov. 1, 1923.

Page 110. — 1. April 21, 1923.

Page 111. — 2. June 29. 3. March 13. 4. Nov. 3, 1923; \$5100.

Page 113. — 1. Aug. 17. 2. Nov. 28.

Page 114. — 3. March 29.

Page 115. — 1. \$1312.87. 2. \$4558.49.

Page 116. — 3. \$4470.56, balance July 1, 1923; \$2594.06, balance Jan. 1, 1924.

Page 119. — 1. \$11,697.85. 2. \$6059.94, balance due the European bank April 30; \$4873.72, balance owed by the European bank May 31; \$6516.88, balance owed by the European bank June 30.

Page 130. — 1. $4.65\frac{5}{8}$. 2. $4.63\frac{5}{8}$. 3. $5.23\frac{3}{4} - \frac{1}{32}$. 4. $5.21\frac{1}{4} - \frac{3}{64}$.

Page 134. — 1. \$4173.22. 2. \$5132.66. 3. \$13,881.41. 4. \$181.62.
5. \$452.89. 6. \$2542.45. 7. \$4274.52. 8. \$16,950.02. 9. £558 19s. 5d.
10. £3555 13s. 7d. 11. F. 8643.04. 12. F. 20,987.83. 13. M. 111,433.03.
14. M. 375,782.88. 15. Y. 30,534.35. 16. 11,637.25 florins. 17. 4.2 marks to \$1.
18. 25.22 francs to £1. 19. \$4416.43. 20. \$1578.78.
21. \$2316.04. 22. \$15,394.21. 23. \$127,503.04.

Page 139. — 1. \$56.25 cheaper in Paris. 2. \$2344.24. 3. \$625.36.

Page 140. — 4. \$6.75, profit; £99 18s. 1d. or \$446.32, balance.
5. \$97,752. 6. 9.25. 7. (a) £881 17s. 5d. (b) 12s. 1d. demand bill better.

Page 141. — 9. (a) \$2274.16; (b) F. 16,895.29. 10. (a) \$3,660,825, cost of shipping gold bars. (b) \$3375, coin costs more. 11. \$1,361,315.63; \$4.8479, value of each pound.

Page 142. — 12. \$1,364,315. 13. (a) \$1077.66, loss; (b) 5.72%
 14. (a) £ 24,815 2s.; (b) \$1076.22, loss.

Page 146. — 1. 99; 1275. 2. 1162 $\frac{1}{2}$. 3. $-3\frac{1}{2}$; 1029. 4. -16 ;
 -462 . 5. (a) \$12.44; (b) \$82.29. 6. \$1162.50.

Page 147. — 7. \$63.75, total interest.

Page 149. — 1. 3125; 3905. 2. $\frac{2}{11}$; $40\frac{1}{11}$. 3. \$196.83; \$295.24.
 4. 800,000 bu. 5. 1.89713.

Page 159. — 1. 1.93952; 2.77379; 4.80265; 7.65963 -10 ; .01557.
 2. 390.7; .8676; 72,787.5; .00055475. 3. 7.03058 -10 ; 2.43180; 1.83091;
 9.411 -10 . 4. 10,301.2. 5. .00190857. 6. .33266. 7. 4.1208.
 8. .00766217. 9. .65017. 10. 148,268. 11. 1.32279. 12. .99504.
 13. 79.0854. 14. 16.0456. 15. .01505. 16. .046553. 17. 1367.47.
 18. 2,447,833.

Page 167. — 1. \$79.42, semiannual interest greater. 2. \$5991.40;
 \$14,358.73; \$82,469.23. 3. \$64,780.70. 4. 33 years of age. 5. 5%.

Page 170. — 1. .5853571.

Page 171. — 2. .2953028. 3. \$830.51. 4. \$1227.31. 5. \$628.39.
 6. .187983. 7. \$2854.56.

Page 177. — 1. \$14,698.97. 2. \$403,567.60. 3. \$10,000,000.
 4. \$73,829.33.

Page 178. — 5. \$4827.08, semiannual annuity greater. 6. \$349.68, an-
 nual annuity greater. 7. 57 years of age.

Page 184. — 1. \$27,670.41. 2. \$23,243.22. 3. \$45,475.78.
 4. \$125,000. 5. \$186,716.79. 6. \$367,388.75. 7. \$37,853.29.

Page 185. — 8. \$6721.24. 9. \$457,393.75. 10. \$7182.50.

Page 188. — 11. \$7892.62.

Page 189. — 12. \$70,000. 13. \$2,741,344.44. 14. \$7098.11, cheaper
 to build the bridge. 15. \$3733.79, sprinkler cheaper. 16. \$1.45 per square,
 second kind cheaper. 17. \$8558.38.

Page 193. — 1. \$1934.02. 2. \$199.18. 3. \$2059.26. 4. No difference
 in cost. 5. \$93.72, annuity better.

Page 194. — 6. 22 yr. 7 mo. 7. \$1295.05.

Page 198. — 1. \$81,957.50. 2. \$60,390. 3. \$155.59. 4. 50 years.
 5. \$540.12. 6. \$7510.13, value of fund at end of 10 years. 7. \$594.84.
 8. \$647,068.

Page 199. — 9. No difference on the assumption that the fractional
 part of \$1000 left over after buying in bonds was invested at 5%.
 10. \$408,865.93 more.

Page 201. — 1. \$2120.68, annual payment. 2. \$3332.27, annual payment.

Page 205. — 1. (a) \$1413.79; (b) \$4533.86, redemption money, case 1;
 \$5072.01, redemption money, case 2; \$4813.01, redemption money, case 3.
 2. (a) \$17,419.86; (b) \$9083.26; (c) \$6741.37.

Page 214. — 1. 115.45. 2. 87.14. 3. 91.05. 4. 95.79. 5. 94.27.
 6. 109.76. 7. (a) 105; (b) \$53,341.67. 8. (a) \$1007.87; (b) \$10.83;
 (c) 99.704. 9. 6%. 10. \$1035.263496.

Page 215. — 11. 99.07073, price of 2-year bonds; 98.24508, price of 4-year bonds; 97.17598, price of 7-year bonds; 96.28063, price of 10-year bonds; \$730,079.88, amount received for entire issue. 12. 100.934725, price of 2-year bonds; 102.598666, price of 5-year bonds; 104.849013, price of 8-year bonds; \$514,998.12, amount received for entire issue.

Page 219. — 1. 98.62. 2. 101.60.

Page 220. — 3. 98.046. 4. 87.82. 5. 97.920848. 6. (a) 102.16; (b) \$146,000. 7. 7.45%. 8. 99.14. 9. 87.920665. 10. \$224,089.85. 11. 95.327, correct price.

Page 229. — 1. (a) \$750. (b) \$624.88. 2. 23.282%. 3. 12.41%. 5. \$549,619.66.

Page 232. — 1. 15 years. 2. \$413.97. 3. (a) \$328,705.95. (b) 3.451%. (c) 19.63 years.

Page 239. — 1. \$5072.27, net profit. 2. \$2880.70, net profit.

Page 240. — 3. \$6750, additional cash invested by Jenkins. \$14,250, present worth of each partner.

Page 241. — 4. A and B are each credited with one half of the net profit of \$666.67. A then takes, and is charged, \$1500, and B \$500, of the amount paid in by C. 5. \$42,500, amount Gray should pay Brown.

Page 242. — 1. 63.55%. 2. \$150, Dept. A; \$240, Dept. B; \$112.50, Dept. C; \$350, Dept. D; \$675, Dept. E. 3. \$58,500. 4. \$113,564.11.

Page 243. — 6. \$3269.90, claim too great. 7. \$5310.22, additional cash invested by Branson; \$81,919.44, present worth of new firm.

Page 245. — 8. \$1293.19. 9. (a) 20%; 12.9%; 9.7%; (b) 13.4%. 10. \$14,760.81, younger son's share; \$16,844.54, older son's share; \$18,394.65, daughter's share.

Page 246. — 11. \$58,447.93. 12. \$151.96. 13. 14.2 years at 5%; 11.9 years at 6%. 14. \$558,484.60. 15. (a) \$3474.67; (b) \$69,493.37.

Page 247. — 16. \$357.78. 17. \$1932.13. 19. \$157,191. 20. 9.098647%. 21. \$9456.31.

Page 248. — 22. \$1519.32. 24. (a) $12\frac{1}{2}\%$; (b) 25%. 25. \$1,271,899.57. 26. \$158,464.89.

Page 249. — 27. \$141,866.89. 28. 100.48, price of bonds due in 1 year; 101.407, price of bonds due in 2 years; 102.754, price of bonds due in 3 years; 103.585, price of bonds due in 4 years; 105.506, price of bonds due in 5 years; \$15,411.96, amount received for entire issue. 30. (a) 115.45433; (b) \$115,854.33; (c) \$49,174.50.

Page 250. — 31. \$82.09. 32. \$6922.47. 33. \$19,215.57. 34. \$50,756.92. 35. 981,090 automobiles; 9,810,900 foot passengers.

Page 251. — 36. \$101,903.86. 37. \$4375. 38. April 26; £5 8.6d.

Page 252. — 39. \$102,912.20, invested in bonds. 40. \$1475, amount received by each Helen H. Winter and Sarah H. Samuels; \$140, decrease in the value of the estate left to William B. Hamilton.

Page 253. — 42. \$20.12, due each share of first series; \$40.24, due each share of the second series; \$80.48, due each share of the third series. 43. \$106,750, A's net worth; \$106,250, B's net worth; \$12,000, C's net worth.

Page 254. — 44. \$15,833.34, received by A; \$4166.66, received by B.
45. .682635, Y's ratio of solvency; .778443, X's ratio of solvency.

Page 255. — 46. \$3050 or \$3550.

Page 256. — 47. £ 300, balance; \$49, profit; the profit is not final.
48. \$7988.52. 50. \$13,476.89.

Page 257. — 52. \$8662.38. 53. \$3254.77. 54. \$10,144.19.
55. 103.26 (\$1032.61, exact value). 56. \$11,631.91. 57. (a) \$16.25;
(b) $33\frac{1}{3}\%$; (c) \$14.01.

Page 258. — 58. \$5962.84.

Page 259. — 62. \$5143.95.

Page 260. — 64. (a) \$30.58; (c) \$70.11.

Page 261. — 65. \$7142.86.

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Van Tuyl, George H.
Modern business mathematics



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